Section 10.3 (cont'd.)

17.) $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n}$

$= \lim_{n\to\infty} \frac{3}{n+1} = 0 < 1$ so series converges

18.) $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{(n+1)\left(\frac{2}{3}\right)^{n+1}}{n \cdot \left(\frac{2}{3}\right)^n}$

$= \lim_{n\to\infty} \frac{2}{3}\left(1+\frac{1}{n}\right) = \frac{2}{3} < 1$ so series converges

19.) $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{(n+1)!}{3^{n+1}} \cdot \frac{3^n}{n!}$

$= \lim_{n\to\infty} \frac{n+1}{3} = \infty > 1$ so series diverges

20.) $\lim_{n\to\infty} \frac{(n+1)\left(\frac{3}{2}\right)^{n+1}}{n\left(\frac{3}{2}\right)^n} = \lim_{n\to\infty} \frac{3}{2}\left(1+\frac{1}{n}\right) = \frac{3}{2} > 1$

so series diverges

21.) $\lim_{n\to\infty} \frac{n+1}{4^{n+1}} \cdot \frac{4^n}{n} = \lim_{n\to\infty} \frac{1}{4}\left(1+\frac{1}{n}\right) = \frac{1}{4} < 1$

so series converges

22.) $\lim_{n\to\infty} \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2} = \lim_{n\to\infty} \frac{1}{2}\left(\frac{n+1}{n}\right)^2 = \frac{1}{2} < 1$

so series converges

23.) $\lim_{n\to\infty} \frac{2^{n+1}}{(n+1)^5} \cdot \frac{n^5}{2^n} = \lim_{n\to\infty} 2\left(\frac{n}{n+1}\right)^5 = 2 \cdot 1^5 = 2 > 1$

so series diverges

24.) $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{e^{-(n+1)}}{e^{-n}} = \lim_{n\to\infty} e^{-1} = \frac{1}{e} < 1$

so series converges

25.) $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n}$

$= \lim_{n\to\infty} \frac{2}{n+1} = 0 < 1$ so series converges

26.) $\lim_{n\to\infty} \frac{4^{n+1}}{(n+1)!} \cdot \frac{n!}{4^n} = \lim_{n\to\infty} \frac{4}{n+1} = 0 < 1$

so series converges.

28.) $\lim_{n\to\infty} \frac{3^{n+1}}{(n+1)+1} \cdot \frac{n+1}{3^n} = \lim_{n\to\infty} 3 \cdot \frac{n+1}{n+2}$

$= \lim_{n\to\infty} 3 \cdot \frac{1+1/n}{1+2/n} = 3(1) = 3 > 1$ so

series diverges.

29.) $\lim_{n\to\infty} \frac{(n+1)5^{n+1}}{(n+1)!} \cdot \frac{n!}{n5^n}$

$= \lim_{n\to\infty} 5 \cdot \frac{n+1}{n} \cdot \frac{1}{n+1} = \lim_{n\to\infty} \frac{5}{n} = 0 < 1$

so series converges.

30.) $\lim_{n\to\infty} \frac{2\cdot(n+1)!}{(n+1)^5} \cdot \frac{n^5}{2\cdot n!} = \lim_{n\to\infty} (n+1)\left(\frac{n}{n+1}\right)^5$

$= \lim_{n\to\infty} (n+1)\cdot\left(\frac{1}{1+1/n}\right)^5 = \infty\cdot(1)^5 = \infty > 0$

so series diverges.

41.) $\lim_{n\to\infty} \frac{2n}{n+1} = 2 \neq 0$ so series

diverges by nth term test.

42.) $\sum_{n=1}^{\infty} \frac{10}{3n^{2/3}} = \frac{10}{3}\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$ diverges

by p-series test with $p = \frac{2}{3} \leq 1$.

44.) $\sum_{n=0}^{\infty} \left(\frac{5}{6}\right)^n = 1 + \left(\frac{5}{6}\right) + \left(\frac{5}{6}\right)^2 + \dots$

$= \frac{1}{1-(5/6)} = \frac{1}{1/6} = 6$ so series

converges by geom. series test since $-1 < 5/6 < 1$.

46.) $\lim_{n\to\infty} (\ln n) = \infty \neq 0$ so series

diverges by nth-term test

47.) $\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{n^3}\right) = \left(\sum_{n=1}^{\infty} \frac{1}{n^2}\right) - \left(\sum_{n=1}^{\infty} \frac{1}{n^3}\right)$

converges since each series

converges by p-series test with $p = 2 > 1$ and $p = 3 > 1$.

50.) $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{(n+1)(0.4)^{n+1}}{n(0.4)^n}$

$= \lim_{n\to\infty} (0.4)\left(1+\frac{1}{n}\right) = 0.4 < 1$ so series

converges by ratio test.

51.) $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{(n+1)!}{3^{(n+1)-1}} \cdot \frac{3^{n-1}}{n!}$

$= \lim_{n\to\infty} \frac{1}{3} \cdot \frac{n+1}{1} = \infty > 1$ so series

diverges by ratio test.

52.) $\sum_{n=1}^{\infty} \frac{1}{n^{0.95}}$ diverges by

p-series test with $p = 0.95 \leq 1$.

53.) $\sum_{n=1}^{\infty} \frac{2^n}{2^{n+1}+1}$ diverges by the

nth term test since

$$\lim_{n\to\infty} \frac{2^n}{2^{n+1}+1} \cdot \frac{\frac{1}{2^n}}{\frac{1}{2^n}} = \lim_{n\to\infty} \frac{1}{2+\frac{1}{2^n}}$$

$$= \frac{1}{2+0} = \frac{1}{2} \neq 0 .$$

54.) $\displaystyle\sum_{n=1}^{\infty} 2e^{-n} = 2\sum_{n=1}^{\infty}\left(\frac{1}{e}\right)^n$

$= 2\left[\frac{1}{e} + \left(\frac{1}{e}\right)^2 + \left(\frac{1}{e}\right)^3 + \left(\frac{1}{e}\right)^4 + \cdots\right]$

$= 2\left(\frac{1}{e}\right)\left[1 + \left(\frac{1}{e}\right) + \left(\frac{1}{e}\right)^2 + \left(\frac{1}{e}\right)^3 + \cdots\right]$

$= \frac{2}{e} \cdot \frac{1}{1-\frac{1}{e}} = \frac{2}{e-1}$, so converges

by geometric series test

since $r = \frac{1}{e}$ and $-1 < r < 1$.

55.) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{5^{n-1}} = \sum_{n=1}^{\infty} \frac{2^n}{5^{-1}5^n} = 5\sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^n$

$= 5\left[\frac{2}{5} + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^3 + \cdots\right]$

$= 5\left(\frac{2}{5}\right)\left[1 + \frac{2}{5} + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^3 + \cdots\right] = 2 \cdot \frac{1}{1-\frac{2}{5}}$

$= 2 \cdot \frac{5}{3} = \frac{10}{3}$, so converges by the geometric series test since $r = \frac{2}{5}$ and $-1 < r < 1$.

56.) $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$ diverges by the nth term test since

$$\lim_{n\to\infty} \frac{n^2}{n^2+1} \cdot \frac{1/n^2}{1/n^2} = \lim_{n\to\infty} \frac{1}{1+\frac{1}{n^2}} = \frac{1}{1+0} = 1 \neq 0$$