Section 10.4

1.) $f(x) = \sum_{n=0}^{\infty} \left(\frac{x}{4}\right)^n \rightarrow$

ratio test : $\lim_{n\to\infty} \frac{\left|\frac{x}{4}\right|^{n+1}}{\left|\frac{x}{4}\right|^n} = \lim_{n\to\infty} \left|\frac{x}{4}\right| = \left|\frac{x}{4}\right| < 1 \rightarrow$

$-1 < \frac{x}{4} < 1 \rightarrow \boxed{-4 < x < 4}$ and $R = 4$.

3.) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x+1)^n}{n!} \longrightarrow$

ratio test : $\lim_{n\to\infty} \frac{|x+1|^{n+1}}{(n+1)!} \cdot \frac{n!}{|x+1|^n} = \lim_{n\to\infty} \frac{|x+1|}{n+1} = 0 < 1$

for all x-values, $-\infty < x < \infty$, and $R = \infty$.

9.) $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!} \longrightarrow$

ratio test : $\lim_{n\to\infty} \frac{\frac{|x|^{n+1}}{(n+1)!}}{\frac{|x|^n}{n!}} = \lim_{n\to\infty} \frac{|x|^{n+1}}{(n+1)!} \cdot \frac{n!}{|x|^n}$

$= \lim_{n\to\infty} \frac{1}{n+1} |x| = 0 \cdot |x| = 0 < 1$ for $\boxed{\text{all } x\text{-values}}$

so $R = \infty$.

10.) $f(x) = \sum_{n=0}^{\infty} \frac{(3x)^n}{n!} \longrightarrow$

ratio test : $\lim_{n\to\infty} \frac{\frac{|3x|^{n+1}}{(n+1)!}}{\frac{|3x|^n}{n!}} = \lim_{n\to\infty} \frac{|3x|^{n+1}}{(n+1)!} \cdot \frac{n!}{|3x|^n}$

$= \lim_{n\to\infty} \frac{1}{n+1} \cdot |3x| = 0 \cdot |3x| = 0 < 1$ for $\boxed{\text{all } x\text{-values}}$

so $R = \infty$.

11.) $f(x) = \sum_{n=0}^{\infty} n! \left(\frac{x}{2}\right)^n \rightarrow$

ratio test : $\lim_{n\to\infty} \frac{(n+1)! \left|\frac{x}{2}\right|^{n+1}}{n! \left|\frac{x}{2}\right|^n} = \lim_{n\to\infty} (n+1)\left|\frac{x}{2}\right| < 1$

only if $\boxed{x=0}$ and $R=0$.

12.) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n+1)(n+2)} \rightarrow$

ratio test : $\lim_{n\to\infty} \frac{\frac{|x|^{n+1}}{(n+2)(n+3)}}{\frac{|x|^n}{(n+1)(n+2)}} = \lim_{n\to\infty} \frac{|x|^{n+1}}{\cancel{(n+2)}(n+3)} \cdot \frac{(n+1)\cancel{(n+2)}}{|x|^n}$

$= \lim_{n\to\infty} \left(\frac{n+1}{n+3}\right)|x| = 1 \cdot |x| < 1 \rightarrow \boxed{-1 < x < 1}$

and $R=1$.

14.) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n n! (x-4)^n}{3^n} \rightarrow$

ratio test : $\lim_{n\to\infty} \frac{\frac{(n+1)! |x-4|^{n+1}}{3^{n+1}}}{\frac{n! |x-4|^n}{3^n}} = \lim_{n\to\infty} \frac{(n+1)! |x-4|^{n+1}}{3^{n+1}} \cdot \frac{3^n}{n! |x-4|^n}$

$= \lim_{n\to\infty} \frac{n+1}{3} |x-4| < 1$ only if $\boxed{x=4}$ so

$R=0$.

15.) $$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n\,5^n} \longrightarrow$$

ratio test : $$\lim_{n\to\infty} \frac{\left|\dfrac{(-1)^{n+2}(x-5)^{n+1}}{(n+1)\,5^{n+1}}\right|}{\left|\dfrac{(-1)^{n+1}(x-5)^n}{n\,5^n}\right|}$$

$$= \lim_{n\to\infty} \frac{|x-5|^{n+1}}{|x-5|^n} \cdot \frac{5^n}{5^{n+1}} \cdot \frac{n}{n+1} = \lim_{n\to\infty} \frac{|x-5|}{5} \frac{1}{1+\frac{1}{n}} = \frac{|x-5|}{5} < 1$$

$$\to |x-5| < 5 \to -5 < x-5 < 5 \to 0 < x < 10$$

and $R = 5$.

16.) $$f(x) = \sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{(n+1)\,3^{n+1}} \longrightarrow$$

ratio test : $$\lim_{n\to\infty} \frac{\left|\dfrac{(x-2)^{n+2}}{(n+2)\,3^{n+2}}\right|}{\left|\dfrac{(x-2)^{n+1}}{(n+1)\,3^{n+1}}\right|}$$

$$= \lim_{n\to\infty} \frac{|x-2|^{n+2}}{|x-2|^{n+1}} \cdot \frac{3^{n+1}}{3^{n+2}} \cdot \frac{n+1}{n+2} = \lim_{n\to\infty} \frac{|x-2|}{3} \cdot \frac{1+\frac{1}{n}}{1+\frac{2}{n}} = \frac{|x-2|}{3} < 1$$

$$\to |x-2| < 3 \to -3 < x-2 < 3 \to -1 < x < 5$$

and $R = 3$

20.) $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{2n-1} \rightarrow$

ratio test: $\lim_{n\to\infty} \frac{|x|^{2(n+1)-1}}{2(n+1)-1} \cdot \frac{2n-1}{|x|^{2n-1}} = \lim_{n\to\infty} \frac{|x|^{2n+1}}{2n+1} \cdot \frac{2n-1}{|x|^{2n-1}}$

$= \lim_{n\to\infty} \left(\frac{2n-1}{2n+1}\right)|x|^2 = \lim_{n\to\infty}\left(\frac{2-\frac{1}{n}}{2+\frac{1}{n}}\right)|x|^2 = |x|^2 < 1 \rightarrow$

$|x| < 1 \rightarrow \boxed{-1 < x < 1}$ so $R = 1$.

22.) $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!} \rightarrow$

ratio test: $\lim_{n\to\infty} \frac{|x|^{2(n+1)}}{(n+1)!} \cdot \frac{n!}{|x|^{2n}} = \lim_{n\to\infty}\left(\frac{1}{n+1}\right)|x|^2$

$= 0 \cdot |x|^2 = 0 < 1$ for $\boxed{\text{all } x\text{-values}}$

so $R = \infty$.

23.) $f(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} \rightarrow$

ratio test: $\lim_{n\to\infty} \frac{|x|^{2(n+1)+1}}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{|x|^{2n+1}}$

$= \lim_{n\to\infty} \frac{|x|^{2n+3}}{(2n+3)!} \cdot \frac{(2n+1)!}{|x|^{2n+1}} = \lim_{n\to\infty} \frac{|x|^2}{(2n+3)(2n+2)} = 0 < 1$

for all x-values so $R = \infty$.

39.) $f(x) = \sum_{n=0}^{\infty} \frac{(x+1)^{n+1}}{n+1}$

$= (x+1) + \frac{(x+1)^2}{2} + \frac{(x+1)^3}{3} + \frac{(x+1)^4}{4} + \cdots$ so

$f'(x) = 1 + (x+1) + (x+1)^2 + (x+1)^3 + \cdots = \sum_{n=0}^{\infty} (x+1)^n$,

$f''(x) = 1 + 2(x+1) + 3(x+1)^2 + 4(x+1)^3 + \cdots \sum_{n=0}^{\infty} (n+1)(x+1)^n$,

and $\int f(x)\,dx = \frac{(x+1)^2}{2\cdot 1} + \frac{(x+1)^3}{3\cdot 2} + \frac{(x+1)^4}{4\cdot 3} + \cdots = \sum_{n=1}^{\infty} \frac{(x+1)^{n+1}}{(n+1)\,n}$;

a.) $\lim_{n\to\infty} \frac{|x+1|^{n+2}}{n+2} \cdot \frac{n+1}{|x+1|^{n+1}} = \lim_{n\to\infty} \left(\frac{n+1}{n+2}\right)|x+1| = |x+1| < 1 \rightarrow$

$-1 < x+1 < 1 \rightarrow \boxed{-2 < x < 0}$ and $R = 1$;

b.) $\lim_{n\to\infty} \frac{|x+1|^{n+1}}{|x+1|^n} = \lim_{n\to\infty} |x+1| = |x+1| < 1 \rightarrow$

$-1 < x+1 < 1 \rightarrow \boxed{-2 < x < 0}$ and $R = 1$;

c.) $\lim_{n\to\infty} \frac{(n+2)|x+1|^{n+1}}{n\,|x+1|^n} = \lim_{n\to\infty} \left(1 + \frac{2}{n}\right)|x+1| = |x+1| < 1 \rightarrow$

$-1 < x+1 < 1 \rightarrow \boxed{-2 < x < 0}$ and $R = 1$;

d.) $\lim_{n\to\infty} \frac{|x+1|^{n+2}}{(n+2)\cancel{(n+1)}} \cdot \frac{\cancel{(n+1)}\,n}{|x+1|^{n+1}} = \lim_{n\to\infty} \left(\frac{n}{n+2}\right)|x+1| = |x+1| < 1 \rightarrow$

$-1 < x+1 < 1 \rightarrow \boxed{-2 < x < 0}$ and $R = 1$.