

Section C.3

$$1.) \quad x^3 - 2x^2 y' + 3y = 0 \rightarrow -2x^2 y' + 3y = -x^3 \rightarrow \\ y' + \frac{3}{-2x^2} y = \frac{-x^3}{-2x^2} \rightarrow y' + \frac{-3}{2x^2} \cdot y = \frac{x}{2}$$

$$2.) \quad y' - 5(2x - y) = 0 \rightarrow y' - 10x + 5y = 0 \rightarrow \\ y' + 5 \cdot y = 10x$$

$$3.) \quad xy' + y = xe^x \rightarrow y' + \frac{1}{x} \cdot y = \frac{xe^x}{x} \rightarrow \\ y' + \frac{1}{x} \cdot y = e^x$$

$$4.) \quad xy' + y = x^3 y \rightarrow xy' + y - x^3 y = 0 \rightarrow \\ xy' + (1 - x^3)y = 0 \rightarrow y' + \frac{1 - x^3}{x} \cdot y = 0$$

$$5.) \quad y + 1 = (x - 1)y' \rightarrow 0 = (x - 1)y' - y - 1 \rightarrow \\ (x - 1)y' - y = 1 \rightarrow y' + \frac{-1}{x - 1} \cdot y = \frac{1}{x - 1}$$

$$6.) \quad x = x^2(y' + y) \rightarrow y' + y = \frac{x}{x^2} \rightarrow \\ y' + (1) \cdot y = \frac{1}{x}$$

$$7.) \quad y' + 3y = 6 \rightarrow \mu = e^{\int 3 dx} = e^{3x} \rightarrow \\ e^{3x} y' + 3e^{3x} \cdot y = 6e^{3x} \rightarrow D(e^{3x} y) = 6e^{3x} \rightarrow$$

$$e^{3x} y = \int 6e^{3x} dx = 6 \cdot \frac{1}{3} e^{3x} + c \rightarrow$$

$$y = 2 + ce^{-3x}$$

$$8.) \quad y' + 5y = 15 \rightarrow \mu = e^{\int 5 dx} = e^{5x} \rightarrow$$

$$e^{5x} y' + 5e^{5x} y = 15e^{5x} \rightarrow D(e^{5x} y) = 15e^{5x} \rightarrow$$

$$e^{5x} y = \int 15e^{5x} dx = 15 \cdot \frac{1}{5} e^{5x} + c \rightarrow$$

$$\boxed{y = 3 + ce^{-5x}}$$

$$9.) \quad y' + (1)y = e^{-x} \rightarrow \mu = e^{\int 1 dx} = e^x \rightarrow$$

$$e^x y' + e^x y = e^x e^{-x} = e^0 = 1 \rightarrow D(e^x y) = 1 \rightarrow$$

$$e^x y = \int 1 dx = x + c \rightarrow y = xe^{-x} + ce^{-x}$$

$$10.) \quad y' + 3y = e^{-3x} \rightarrow \mu = e^{\int 3 dx} = e^{3x} \rightarrow$$

$$e^{3x} y' + 3e^{3x} y = e^{3x} e^{-3x} = e^0 = 1 \rightarrow D(e^{3x} y) = 1 \rightarrow$$

$$e^{3x} y = \int 1 dx = x + c \rightarrow \boxed{y = xe^{-3x} + ce^{-3x}}$$

$$11.) \quad y' + \frac{1}{x} \cdot y = 3x + 4 \rightarrow \mu = e^{\int \frac{1}{x} dx} = e^{\ln x} = x \rightarrow$$

$$xy' + y = 3x^2 + 4x \rightarrow D(xy) = 3x^2 + 4x \rightarrow$$

$$xy = \int (3x^2 + 4x) dx = x^3 + 2x^2 + c \rightarrow$$

$$\boxed{y = x^2 + 2x + \frac{c}{x}}$$

$$12.) \quad y' + \frac{2}{x} \cdot y = 3x + 1 \rightarrow \mu = e^{\int \frac{2}{x} dx} = e^{2 \ln x} = e^{\ln x^2} = x^2$$

$$\rightarrow x^2 y' + 2xy = 3x^3 + x^2 \rightarrow D(x^2 y) = 3x^3 + x^2 \rightarrow$$

$$x^2 y = \int (3x^3 + x^2) dx = \frac{3}{4} x^4 + \frac{1}{3} x^3 + c \rightarrow$$

$$\boxed{y = \frac{3}{4} x^2 + \frac{1}{3} x + \frac{c}{x^2}}$$

$$13.) y' + 5x \cdot y = x \rightarrow \mu = e^{\int 5x dx} = e^{\frac{5}{2}x^2} \rightarrow$$

$$e^{\frac{5}{2}x^2} y' + 5x e^{\frac{5}{2}x^2} y = x e^{\frac{5}{2}x^2} \rightarrow$$

$$D(e^{\frac{5}{2}x^2} y) = x e^{\frac{5}{2}x^2} \rightarrow e^{\frac{5}{2}x^2} y = \int x e^{\frac{5}{2}x^2} dx \\ = \frac{1}{5} e^{\frac{5}{2}x^2} + c \rightarrow \boxed{y = \frac{1}{5} + c e^{-\frac{5}{2}x^2}}$$

$$14.) y' + 5 \cdot y = e^{5x} \rightarrow \mu = e^{\int 5 dx} = e^{5x} \rightarrow$$

$$e^{5x} y' + 5 e^{5x} y = e^{5x} e^{5x} = e^{10x} \rightarrow D(e^{5x} y) = e^{10x} \rightarrow$$

$$e^{5x} y = \int e^{10x} dx = \frac{1}{10} e^{10x} + c \rightarrow \boxed{y = \frac{1}{10} e^{5x} + c e^{-5x}}$$

$$15.) y' + \frac{1}{x-1} \cdot y = \frac{x^2-1}{x-1} = x+1 \rightarrow$$

$$\mu = e^{\int \frac{1}{x-1} dx} = e^{\ln(x-1)} = x-1 \rightarrow$$

$$(x-1) y' + y = x^2-1 \rightarrow D((x-1)y) = x^2-1 \rightarrow$$

$$(x-1)y = \int (x^2-1) dx = \frac{1}{3}x^3 - x + c \rightarrow$$

$$\boxed{y = (\frac{1}{3}x^3 - x + c) / (x-1)}$$

$$16.) xy' + y = x^2+1 \rightarrow y' + \frac{1}{x} \cdot y = x + \frac{1}{x} \rightarrow$$

$$\mu = e^{\int \frac{1}{x} dx} = e^{\ln x} = x \rightarrow xy' + y = x^2+1 \rightarrow$$

$$D(xy) = x^2+1 \rightarrow xy = \int (x^2+1) dx = \frac{x^3}{3} + x + c$$

$$\rightarrow \boxed{y = \frac{x^2}{3} + 1 + \frac{c}{x}}$$

$$17.) \quad x^3 y' + 2y = e^{\frac{1}{x^2}} \rightarrow$$

$$y' + \frac{2}{x^3} \cdot y = \frac{1}{x^3} e^{\frac{1}{x^2}} \rightarrow \mu = e^{\int \frac{2}{x^3} dx} = e^{-\frac{1}{x^2}} \rightarrow$$

$$e^{-\frac{1}{x^2}} y' + \frac{2}{x^3} e^{-\frac{1}{x^2}} y = \frac{1}{x^3} \rightarrow D\left(e^{-\frac{1}{x^2}} y\right) = \frac{1}{x^3} \rightarrow$$

$$e^{-\frac{1}{x^2}} y = \int \frac{1}{x^3} dx = -\frac{1}{2x^2} + C \rightarrow \boxed{y = \left(-\frac{1}{2x^2} + C\right) e^{\frac{1}{x^2}}}$$

$$18.) \quad xy' + y = x^2 \ln x \rightarrow y' + \left(\frac{1}{x}\right) \cdot y = x \ln x \rightarrow$$

$$\mu = e^{\int \frac{1}{x} dx} = e^{\ln x} = x \rightarrow xy' + y = x^2 \ln x \rightarrow$$

$$D(xy) = x^2 \ln x \rightarrow xy = \int x^2 \ln x dx$$

$$\left(\text{Let } u = \ln x, \quad dv = x^2 dx\right)$$

$$du = \frac{1}{x} dx, \quad v = \frac{1}{3} x^3$$

$$\rightarrow xy = \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^2 dx = \frac{1}{3} x^3 \ln x - \frac{1}{3} \cdot \frac{1}{3} x^3 + C \rightarrow$$

$$\boxed{y = \frac{1}{3} x^2 \ln x - \frac{1}{9} x^2 + \frac{C}{x}}$$

$$y' + y = 4 \rightarrow \mu = e^{\int 1 dx} = e^x \rightarrow$$

$$e^x y' + e^x y = 4e^x \rightarrow$$

$$D(e^x \cdot y) = 4e^x \rightarrow e^x \cdot y = \int 4e^x dx \rightarrow$$

$$e^x y = 4e^x + C \rightarrow \boxed{y = 4 + Ce^{-x}}$$

$$19.) \quad y' + y = 4 \rightarrow \text{(two ways)}$$

$$\text{1st order: } \mu = e^{\int 1 dx} = e^x \rightarrow$$

$$e^x y' + e^x y = 4e^x \rightarrow D(e^x y) = 4e^x \rightarrow$$

$$e^x y = \int 4e^x dx = 4e^x + c \rightarrow \boxed{y = 4 + ce^{-x}}$$

OR

Separable: $y' + y = 4 \rightarrow y' = 4 - y \rightarrow$

$$\frac{dy}{dx} = 4 - y \rightarrow \int \frac{1}{4-y} dy = \int 1 dx \rightarrow$$

$$-\ln|4-y| = x + c \rightarrow \ln|4-y| = -x - c \rightarrow$$

$$|4-y| = e^{-x-c} = e^{-c} e^{-x} = c_1 e^{-x} \text{ where}$$

$$c_1 > 0 \rightarrow 4-y = c_2 e^{-x} \text{ where } c_2 \text{ is } \pm$$

$$\rightarrow \boxed{y = 4 - c_2 e^{-x}}$$

22.) $y' + 4xy = x \rightarrow$ (two ways)

1st Order: $\mu = e^{\int 4x dx} = e^{2x^2} \rightarrow$

$$e^{2x^2} y' + 4xe^{2x^2} y = xe^{2x^2} \rightarrow$$

$$D(e^{2x^2} y) = xe^{2x^2} \rightarrow e^{2x^2} y = \int xe^{2x^2} dx$$

$$= \frac{1}{4} e^{2x^2} + c \rightarrow \boxed{y = \frac{1}{4} + ce^{-2x^2}}$$

OR

Separable: $y' + 4xy = x \rightarrow$

$$y' = x - 4xy = (1 - 4y)x \rightarrow$$

$$\int \frac{1}{1-4y} dy = \int x dx \rightarrow -\frac{1}{4} \ln|1-4y| = \frac{x^2}{2} + c$$

$$\rightarrow \ln|1-4y| = -2x^2 - 4c \rightarrow$$

$$|1-4y| = e^{-2x^2 - 4c} = e^{-2x^2} \cdot e^{-4c}$$

$$= c_1 e^{-2x^2} \rightarrow 1-4y = c_2 e^{-2x^2} \rightarrow$$

$$1 - c_2 e^{-2x^2} = 4y \rightarrow \boxed{y = \frac{1}{4} + c_3 e^{-2x^2}}$$

$$28.) \quad Y' + (2)Y = e^{-2x} \rightarrow$$

$$\mu = e^{\int 2 dx} = e^{2x} \rightarrow e^{2x} Y' + 2e^{2x} Y = e^{2x} e^{-2x} \rightarrow$$

$$D(e^{2x} Y) = e^0 = 1 \rightarrow e^{2x} Y = \int 1 dx \rightarrow$$

$$e^{2x} Y = x + c \rightarrow \underline{Y = x e^{-2x} + c e^{-2x}} ;$$

$$31.) \quad Y' + (3x^2)Y = 3x^2 \rightarrow \mu = e^{\int 3x^2 dx} = e^{x^3} \rightarrow$$

$$e^{x^3} Y' + 3x^2 e^{x^3} Y = 3x^2 e^{x^3} \rightarrow D(e^{x^3} Y) = 3x^2 e^{x^3} \rightarrow$$

$$e^{x^3} Y = \int 3x^2 e^{x^3} dx = e^{x^3} + c \rightarrow$$

$$\underline{Y = 1 + c e^{-x^3}} ; \text{ if } x=0, Y=6 \text{ then}$$

$$6 = 1 + c \rightarrow c = 5 \rightarrow \underline{Y = 1 + 5e^{-x^3}} .$$

$$34.) \quad x^2 Y' - 4xY = 10 \rightarrow Y' + \left(\frac{-4}{x}\right)Y = \frac{10}{x^2} \rightarrow$$

$$\mu = e^{\int \frac{-4}{x} dx} = e^{-4 \ln x} = e^{\ln x^{-4}} = x^{-4} \rightarrow$$

$$x^{-4} Y' - 4x^{-5} Y = 10x^{-6} \rightarrow D(x^{-4} Y) = 10x^{-6} \rightarrow$$

$$x^{-4} Y = \int 10x^{-6} dx = -2x^{-5} + c \rightarrow \underline{Y = -2x^{-1} + c x^4} ;$$

$$\text{if } x=1, Y=10 \text{ then } 10 = -2 + c \rightarrow c = 12 \rightarrow$$

$$\underline{Y = -2x^{-1} + 12x^4} .$$

$$35.) \quad \frac{dS}{dt} = 20 - 0.2S + 0.2t \rightarrow$$

$$\frac{dS}{dt} + 0.2S = 20 + 0.2t \rightarrow e^{\int 0.2 dt} = e^{0.2t} \rightarrow$$

$$e^{0.2t} \frac{dS}{dt} + 0.2 e^{0.2t} S = (20 + 0.2t) e^{0.2t} \rightarrow$$

$$D(e^{0.2t} S) = 20 e^{0.2t} + 0.2 t e^{0.2t} \rightarrow$$

$$e^{0.2t} S = \int (20 e^{0.2t} + 0.2 t e^{0.2t}) dt$$

$$= 20 \cdot \frac{1}{0.2} e^{0.2t} + 0.2 \int t e^{0.2t} dt$$

$$\left(\text{Let } u = t, \quad dv = e^{0.2t} dt \right. \\ \left. du = 1 dt, \quad v = \frac{1}{0.2} e^{0.2t} \right)$$

$$= 100 e^{0.2t} + 0.2 \left[\frac{1}{0.2} t e^{0.2t} - \frac{1}{0.2} \int e^{0.2t} dt \right]$$

$$= 100 e^{0.2t} + t e^{0.2t} - \frac{1}{0.2} e^{0.2t} + C \rightarrow$$

$$S = 100 + t - 5 + C e^{-0.2t} \rightarrow$$

$$S = 95 + t + C e^{-0.2t} ;$$

$$t=0, S=0 \rightarrow 0 = 95 + C \rightarrow C = -95 \rightarrow$$

$$S = 95 + t - 95 e^{-0.2t}$$

t : 0	1	2	3	4	5	6	7	8	9	10
S : 0	18.2	33.3	45.9	56.3	65.1	72.4	78.6	83.8	88.3	92.1

$$44.) \quad \frac{dy}{dt} = \frac{1-y}{4} = \frac{1}{4} - \frac{1}{4}y \rightarrow$$

$$y' + \frac{1}{4} \cdot y = \frac{1}{4} \rightarrow \mu = e^{\int \frac{1}{4} dt} = e^{\frac{1}{4}t} \rightarrow$$

$$e^{\frac{1}{4}t} y' + \frac{1}{4} e^{\frac{1}{4}t} y = \frac{1}{4} e^{\frac{1}{4}t} \rightarrow D(e^{\frac{1}{4}t} y) = \frac{1}{4} e^{\frac{1}{4}t}$$

$$\rightarrow e^{\frac{1}{4}t} y = \int \frac{1}{4} e^{\frac{1}{4}t} dt = e^{\frac{1}{4}t} + c \rightarrow$$

$$\underline{y = 1 + c e^{-\frac{1}{4}t}} ;$$

$$a.) \quad t=0, y=0 \rightarrow 0 = 1 + c e^0 = 1 + c \rightarrow$$

$$c = -1 \rightarrow$$

$$\underline{y = 1 - e^{-\frac{1}{4}t}}$$

$$b.) \quad \text{If } y = 50\% = 0.5, \text{ then}$$

$$0.5 = 1 - e^{-\frac{1}{4}t} \rightarrow e^{-\frac{1}{4}t} = \frac{1}{2} \rightarrow$$

$$\ln e^{-\frac{1}{4}t} = \ln\left(\frac{1}{2}\right) \rightarrow -\frac{1}{4}t = \ln\left(\frac{1}{2}\right) \rightarrow$$

$$t = -4 \ln\left(\frac{1}{2}\right) \approx \underline{2.77 \text{ yrs.}}$$

$$c.) \quad \text{If } t = 4 \text{ yrs, then}$$

$$y = 1 - e^{-\frac{1}{4}(4)} \approx 0.6321 = \underline{63.21\%}$$