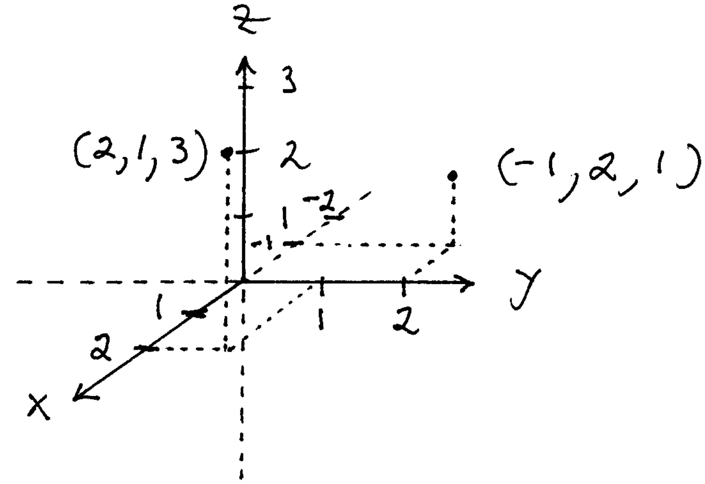
Section 7.1

1.)



6.) Distance $L = \sqrt{(-4-2)^2 + (-1-{}^-1)^2 + (1-5)^2}$
$= \sqrt{36+0+16} = \sqrt{52}$.

8.) Distance $L = \sqrt{(8-8)^2 + (-2-{}^-2)^2 + (2-4)^2}$
$= \sqrt{0+0+4} = 2$

9.) midpoint $= \left(\frac{6-2}{2}, \frac{-9-1}{2}, \frac{1+5}{2}\right) = (2, -5, 3)$

12.) midpoint $= \left(\frac{0+4}{2}, \frac{-2+2}{2}, \frac{5+7}{2}\right) = (2, 0, 6)$

13.) $\left(\frac{x-2}{2}, \frac{y+1}{2}, \frac{z+1}{2}\right) = (2, -1, 3) \rightarrow$

$\frac{x-2}{2} = 2 \rightarrow x-2=4 \rightarrow x=6$;

$\frac{y+1}{2} = -1 \rightarrow y+1=-2 \rightarrow y=-3$;

$\frac{z+1}{2} = 3 \rightarrow z+1=6 \rightarrow z=5$.

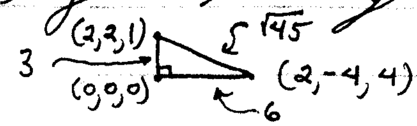
15.) $\left(\frac{x+2}{2}, \frac{y+0}{2}, \frac{z+3}{2}\right) = \left(\frac{3}{2}, 1, 2\right) \rightarrow$

$\frac{x+2}{2} = \frac{3}{2} \rightarrow x+2 = 3 \rightarrow x=1$;

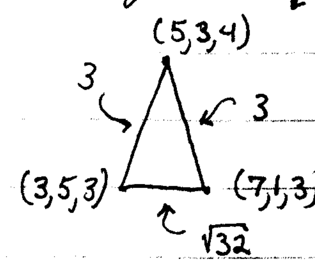
$\frac{y}{2} = 1 \rightarrow y=2$; $\frac{z+3}{2} = 2 \rightarrow z+3=4 \rightarrow$

$z=1$.

17.) $d_1 = \sqrt{2^2+2^2+1^2} = 3$, $d_2 = \sqrt{0^2+6^2+3^2} = \sqrt{45}$,
$d_3 = \sqrt{2^2+4^2+4^2} = \sqrt{36} = 6$ so right triangle
since $3^2 + 6^2 = (\sqrt{45})^2$:



18.) $d_1 = \sqrt{2^2+2^2+1^2} = 3$, $d_2 = \sqrt{4^2+4^2+0^2} = \sqrt{32}$,
$d_3 = \sqrt{2^2+2^2+1} = 3$ so isosceles triangle:



21.) $(x-0)^2+(y-2)^2+(z-2)^2 = 2^2$

$\rightarrow x^2+(y-2)^2+(z-2)^2 = 4$

24.) Distance $= \sqrt{(0-{}^-1)^2+(3-{}^-2)^2+(3-1)^2}$
$= \sqrt{1+25+4} = \sqrt{30}$ so radius is
$\frac{\sqrt{30}}{2}$; midpoint $= \left(\frac{0-1}{2}, \frac{3-2}{2}, \frac{3+1}{2}\right) = \left(\frac{-1}{2}, \frac{1}{2}, 2\right)$
is center so sphere is

$\left(x-{}^-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2+(z-2)^2 = \left(\frac{\sqrt{30}}{2}\right)^2 \rightarrow$

$\left(x+\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2+(z-2)^2 = \frac{30}{4} \rightarrow$

$\left(x+\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2+(z-2)^2 = \frac{15}{2}$.

26.) $(x-4)^2+(y--1)^2+(z-1)^2=5^2 \to$
$(x-4)^2+(y+1)^2+(z-1)^2=25$

27.) Distance $=\sqrt{(2-0)^2+(0-6)^2+(0-0)^2}$
$=\sqrt{4+36}=\sqrt{40}$ so radius is
$\frac{\sqrt{40}}{2}$; midpoint $=\left(\frac{2+0}{2}, \frac{0+6}{2}, \frac{0+0}{2}\right)=(1,3,0)$
is center so sphere is
$(x-1)^2+(y-3)^2+(z-0)^2=\left(\frac{\sqrt{40}}{2}\right)^2 \to$
$(x-1)^2+(y-3)^2+z^2=\frac{40}{4}=10$.

29.) radius $=1$ (since $z=1$) so sphere is
$(x--2)^2+(y-1)^2+(z-1)^2=1^2 \to$
$(x+2)^2+(y-1)^2+(z-1)^2=1$.

32.) $x^2+y^2+z^2-8y=0 \to$
$x^2+(y^2-8y+16)+z^2=16 \to$
$(x-0)^2+(y-4)^2+(z-0)^2=4^2 \to$
center is $(0,4,0)$ and radius $=4$.

34.) $x^2+y^2+z^2-4y+6z+4=0 \to$
$x^2+(y^2-4y+4)+(z^2+6z+9)=-4+4+9 \to$
$x^2+(y-2)^2+(z+3)^2=3^2 \to$
$(x-0)^2+(y-2)^2+(z--3)^2=3^2$ so
center is $(0,2,-3)$ and radius $=3$.

36.) $4x^2 + 4y^2 + 4z^2 - 8x + 16y + 11 = 0 \rightarrow$

$4(x^2 - 2x + 1) + 4(y^2 + 4y + 4) + 4z^2 = -11 + 4 + 16 \rightarrow$

$4(x-1)^2 + 4(y+2)^2 + 4z^2 = 9 \rightarrow$

$(x-1)^2 + (y - {}^-2)^2 + (z-0)^2 = \frac{9}{4} = \left(\frac{3}{2}\right)^2 \rightarrow$

center is $(1, {}^-2, 0)$ and radius $= 3/2$.

39.) $x^2 + y^2 + z^2 - 6x - 10y + 6z + 30 = 0$

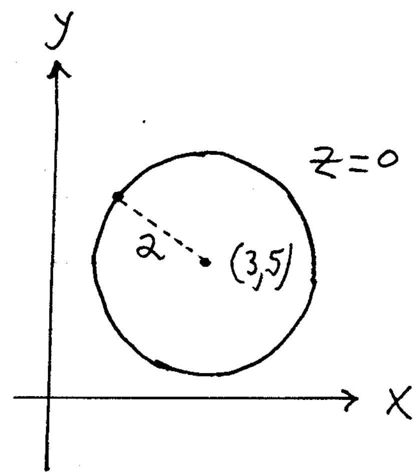
so xy-trace means $z = 0 \rightarrow$

$x^2 + y^2 - 6x - 10y + 30 = 0 \rightarrow$

$(x^2 - 6x + 9) + (y^2 - 10y + 25) = {}^-30 + 9 + 25 \rightarrow$

$\underline{(x-3)^2 + (y-5)^2 = 64 = 2^2} \rightarrow$

circle, center $(3,5)$ and radius $= 2$.



41.) $x^2 + y^2 + z^2 - 4x - 4y - 6z - 12 = 0$

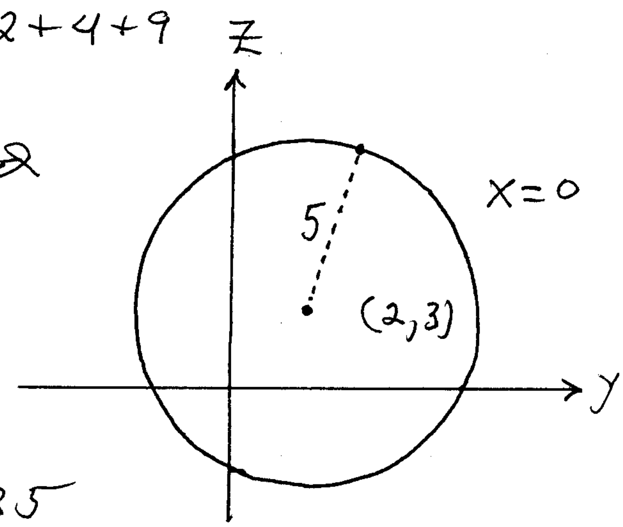
so YZ-trace means $x = 0 \rightarrow$

$y^2 + z^2 - 4y - 6z - 12 = 0 \rightarrow$

$(y^2 - 4y + 4) + (z^2 - 6z + 9) = 12 + 4 + 9$

$\rightarrow \underline{(y-2)^2 + (z-3)^2 = 5^2} \rightarrow$

circle, center $(2,3)$ and radius $= 5$.



43.) $x^2 + y^2 + z^2 = 25$

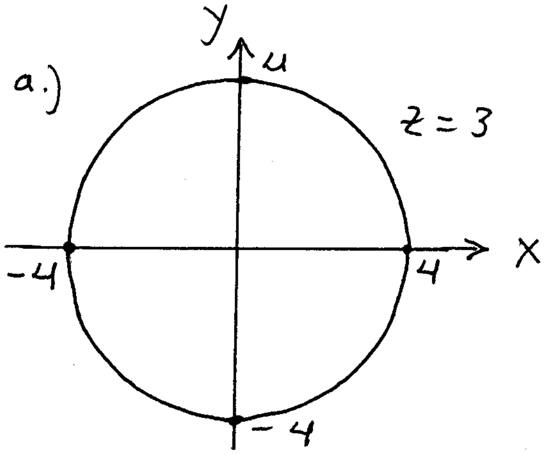
a.) $z = 3 \rightarrow x^2 + y^2 + 9 = 25$

$\rightarrow x^2 + y^2 = 16 = 4^2 \rightarrow$

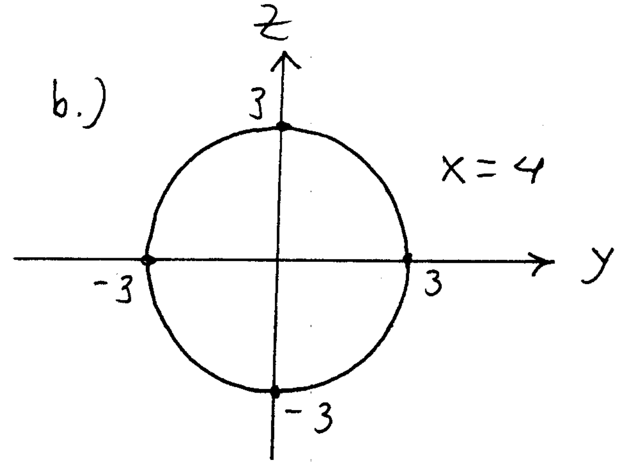
circle, center $(0,0)$ and radius $= 4$.

b.) $x = 4 \to 16 + y^2 + z^2 = 25 \to$
$y^2 + z^2 = 9 = 3^2 \to$ circle, center
$(0,0)$ and radius $= 3$.

a.)



b.)

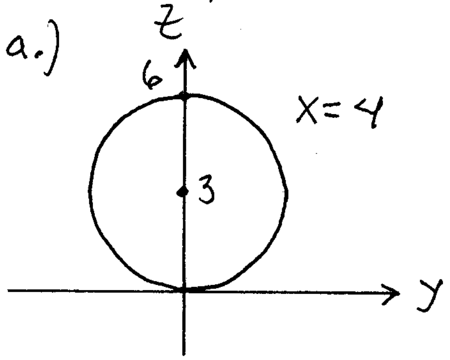


46.) $x^2 + y^2 + z^2 - 8x - 6z + 16 = 0$

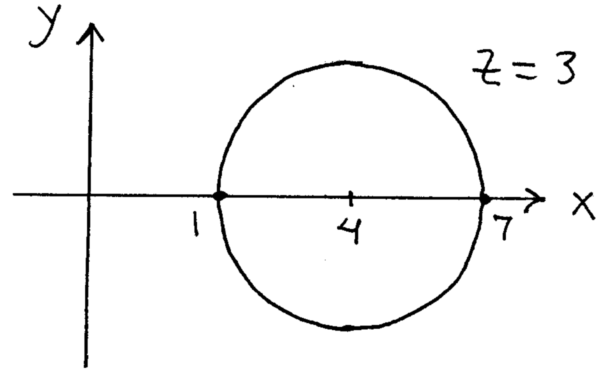
a.) $x = 4 \to 16 + y^2 + z^2 - 32 - 6z + 16 = 0 \to$
$y^2 + (z^2 - 6z + 9) = 0 + 9 \to y^2 + (z-3)^2 = 3^2 \to$
circle, center $(0,3)$ and radius $= 3$.

b.) $z = 3 \to x^2 + y^2 + 9 - 8x - 18 + 16 = 0 \to$
$(x^2 - 8x + 16) + y^2 = -7 + 16 \to (x-4)^2 + y^2 = 3^2 \to$
circle, center $(4,0)$ and radius $= 3$.

a.)



b.)



47.) $(x, y, z) = (3, 3, 3)$

Math 16C
Kouba
Worksheet 2

Use any method to solve the following differential equations.

1.) $y' + y^3 = y$ with $x = 0$ and $y = 2$

2.) $y' + 2y = e^{-2x} \cot^2(7x)$

3.) $y' \cdot \cos^2 x + y = 1$

4.) $xy' + 2y = x \cos x$

5.) $\tan x \cdot y' = y^2(y+1) \cot x$

6.) $\cos(5x^2) \cdot y' = x \cdot \sec^2(3y)$

7.) $(e^{2x} - e^x) \cdot e^{2y} \cdot \sin(e^y) \cdot y' = (1 + e^x)e^x$

8.) $\cos^3 y \cdot \sin y \cdot dy = \tan^3(10x) \cdot dx$

Worksheet 2

1.) $Y' + Y^3 = Y \rightarrow \quad Y' = Y - Y^3 = Y(1-Y)(1+Y) \rightarrow$

$\int \frac{1}{Y(1-Y)(1+Y)}\, dy = \int 1\, dx \quad ;$

$\frac{1}{Y(1-Y)(1+Y)} = \frac{A}{Y} + \frac{B}{1-Y} + \frac{C}{1+Y} \rightarrow$

$A(1-Y)(1+Y) + BY(1+Y) + CY(1-Y) = 1$

$Y=0: \quad A=1 \quad , \quad Y=1: \; 2B=1 \rightarrow B=\frac{1}{2}, \quad Y=-1: \; -2C=1 \rightarrow C = \frac{-1}{2}$

then

$\int \left[\frac{1}{Y} + \frac{\frac{1}{2}}{1-Y} + \frac{\frac{-1}{2}}{1+Y}\right] dy = x + c \rightarrow$

$\ln|Y| - \frac{1}{2}\ln|1-Y| - \frac{1}{2}\ln|1+Y| = x + c$ then $x=0, Y=2 \rightarrow$

$\ln 2 - \frac{1}{2}\cancel{\ln 1}^{0} - \frac{1}{2}\ln 3 = c \rightarrow \quad c = \ln 2 - \frac{1}{2}\ln 3$ so

$$\boxed{\ln|Y| - \frac{1}{2}\ln|1-Y| - \frac{1}{2}\ln|1+Y| = x + \left(\ln 2 - \frac{1}{2}\ln 3\right)}$$

2.) $Y' + 2Y = e^{-2x}\cot^2(7x)$, let $\mu = e^{\int 2\,dx} = e^{2x} \rightarrow$

$e^{2x}Y' + 2e^{2x}Y = e^{2x}e^{-2x}\cot^2(7x) = e^0 \cdot \cot^2(7x) = \cot^2(7x) \rightarrow$

$D\left(e^{2x}Y\right) = \cot^2(7x) \rightarrow \quad e^{2x}Y = \int \cot^2(7x)\, dx \rightarrow$

$e^{2x}Y = \int \left[\csc^2(7x) - 1\right] dx \rightarrow$

$$\boxed{e^{2x}Y = \frac{-1}{7}\cot(7x) - x + c}$$

3.) $Y'\cos^2 x + Y = 1 \rightarrow \quad Y'\cos^2 x = 1 - Y \rightarrow$

$\int \frac{1}{1-Y}\, dy = \int \frac{1}{\cos^2 x}\, dx = \int \sec^2 x\, dx \rightarrow$

$$\boxed{-\ln|1-Y| = \tan X + C}.$$

4.) $XY' + 2Y = X\cos X \rightarrow \underbrace{Y' + \left(\frac{2}{X}\right)Y = \cos X}_{} \rightarrow$

$\mu = e^{\int \frac{2}{X}dx} = e^{2\ln X} = e^{\ln X^2} = X^2 \rightarrow$

$X^2Y' + 2XY = X^2\cos X \rightarrow D(X^2Y) = X^2\cos X \rightarrow$

$X^2Y = \int X^2\cos X\,dx$ (Let $u = X^2$, $dv = \cos X\,dx$
$du = 2X\,dx$, $v = \sin X$)

$X^2Y = X^2\sin X - 2\int X\sin X\,dx$ (Let $u = X$, $dv = \sin X\,dx$
$du = dx$, $v = -\cos X$)

$X^2Y = X^2\sin X - 2\left[-X\cos X + \int \cos X\,dx\right]$

$$\boxed{X^2Y = X^2\sin X + 2X\cos X - 2\sin X + C}.$$

5.) $\tan X \cdot Y' = Y^2(Y+1)\cot X \rightarrow$

$\int \frac{1}{Y^2(Y+1)}\,dy = \int \frac{\cot X}{\tan X}\,dx = \int \frac{\cos X}{\sin X}\cdot\frac{\cos X}{\sin X}\,dx = \int \frac{\cos^2 X}{\sin^2 X}\,dx;$

$\frac{1}{Y^2(Y+1)} = \frac{A}{Y} + \frac{B}{Y^2} + \frac{C}{Y+1} \rightarrow AY(Y+1) + B(Y+1) + C(Y^2) = 1 \rightarrow$

$Y=0:\ B=1$, $Y=-1:\ C=1$, $Y=1:\ 2A+2+1=1 \rightarrow A=-1$ so

$\int\left[\frac{-1}{Y} + \frac{1}{Y^2} + \frac{1}{Y+1}\right]dy = \int \frac{1-\sin^2 X}{\sin^2 X}\,dx = \int\left[\csc^2 X - 1\right]dx \rightarrow$

$$\boxed{-\ln|Y| - \frac{1}{Y} + \ln|Y+1| = -\cot X - X + C}.$$

6.) $\cos(5X^2)\cdot Y' = X\sec^2(3Y) \rightarrow$

$\int \frac{1}{\sec^2(3Y)}\,dy = \int \frac{X}{\cos(5X^2)}\,dx \longrightarrow$

$$\int \cos^2(3Y)\,dy = \int x \sec(5x^2)\,dx \rightarrow$$

$\rightarrow$ Let $u = 5x^2 \rightarrow \ldots$

$$\int \frac{1}{2}(1+\cos(6Y))\,dy = \int x \sec(5x^2)\,dx \rightarrow$$

$$\boxed{\frac{1}{2}\left(Y + \frac{1}{6}\sin(6Y)\right) = \frac{1}{10}\cdot \ln\left|\sec(5x^2)+\tan(5x^2)\right| + c}$$

7.) $(e^{2x} - e^{x})\,e^{2Y}\sin(e^{Y})\,Y' = (1+e^{x})\cdot e^{x} \rightarrow$

$$\int e^{Y}e^{Y}\sin(e^{Y})\,dy = \int \frac{(1+e^{x})e^{x}}{e^{x}e^{x} - e^{x}}\,dx \rightarrow$$

$$\int e^{Y}\sin(e^{Y})\cdot e^{Y}\,dy = \int \frac{(1+e^{x})\cancel{e^{x}}}{(e^{x}-1)\cancel{e^{x}}}\,dx = \int \frac{1+e^{x}}{e^{x}-1}\,dx \rightarrow$$

(Let $u = e^{Y} \rightarrow du = e^{Y}\,dy$)

$$\int u \sin u\,du = \int \frac{(e^{x}-1)+1+1}{e^{x}-1}\,dx = \int \left[1 + \frac{2}{e^{x}-1}\right]dx \rightarrow$$

(Let $W = u$, $dV = \sin u\,du$
$dW = du$, $V = -\cos u$)

$$-u\cos u + \int \cos u\,du = x + \int \frac{2e^{-x}}{(e^{x}-1)\,e^{-x}}\,dx \rightarrow$$

$$-u\cos u + \sin u = x + 2\int \frac{e^{-x}}{1-e^{-x}}\,dx \longrightarrow$$

$$\boxed{-e^{Y}\cos(e^{Y}) + \sin(e^{Y}) = x + 2\cdot \ln\left|1-e^{-x}\right| + c}$$

8.) $\int \cos^3 Y \sin Y\,dy = \int \tan^3(10x)\,dx \rightarrow$

$\leftarrow$ Let $u = \cos Y \rightarrow \ldots$

$$-\frac{1}{4}\cos^4 Y = \int \tan(10x)\cdot \tan^2(10x)\,dx \rightarrow$$

$$-\frac{1}{4}\cos^4 Y = \int \tan(10x)\left[\sec^2(10x) - 1\right]dx \longrightarrow$$

$$-\frac{1}{4}\cos^4 Y = \int \left(\sec(10x)\cdot\sec(10x)\tan(10x) - \tan(10x)\right) dx \rightarrow$$

Let $u = \sec(10x)$

$$\boxed{-\frac{1}{4}\cos^4 Y = \frac{1}{10}\cdot\frac{1}{2}\sec^2(10x) - \frac{1}{10}\ln|\sec(10x)| + c}$$