Section 7.2

2.) $3x+6y+2z=6$;

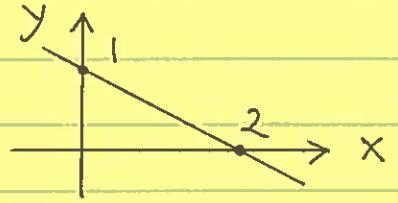
$x=0, y=0 \rightarrow z=3$

$x=0, z=0 \rightarrow y=1$

$y=0, z=0 \rightarrow x=2$

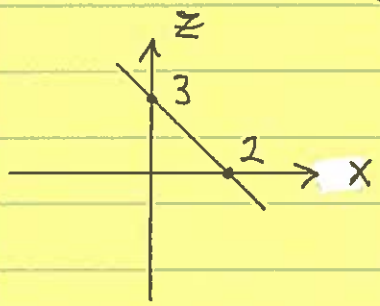
xy-trace ($z=0$):

$3x+6y=6 \rightarrow x+2y=2$ (line)



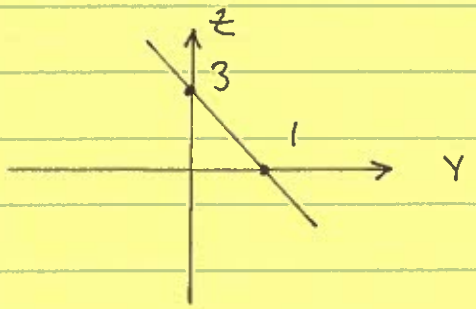
xz-trace ($y=0$)

$3x+2z=6$ (line)



yz-trace ($x=0$):

$6y+2z=6 \rightarrow 3y+z=3$ (line)



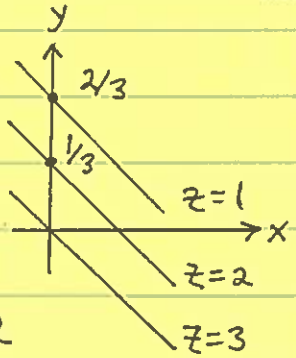
Level Curves:

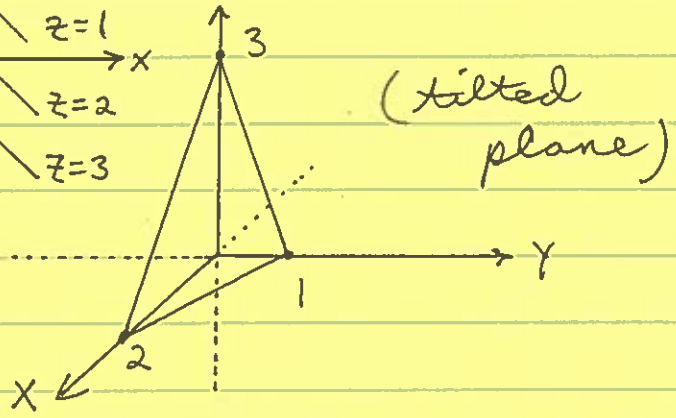
$z=1$: $3x+6y=4$

$z=2$: $3x+6y=2$

$z=3$: $3x+6y=0$

(all are lines.)





(tilted plane)

4.) $x+y+z=3$;

xy-trace ($z=0$):

$x+y=3$ (line)

$x=0, y=0 \rightarrow z=3$

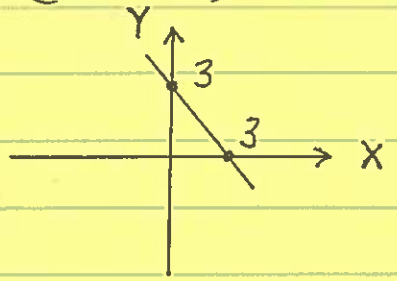
$x=0, z=0 \rightarrow y=3$

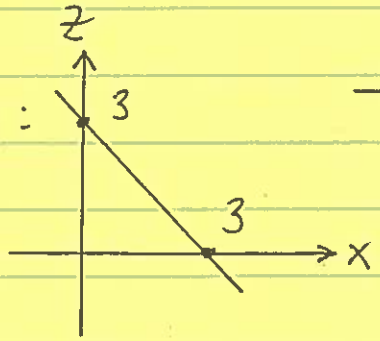
$y=0, z=0 \rightarrow x=3$

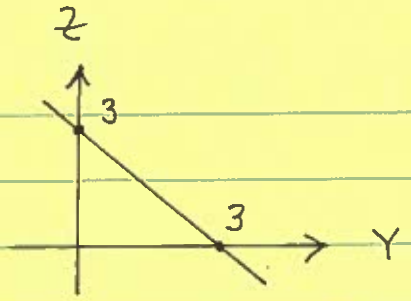
xz-trace ($y=0$):

$x+z=3$

(line)







YZ-trace ($x=0$):

$y+z=3$

(line)

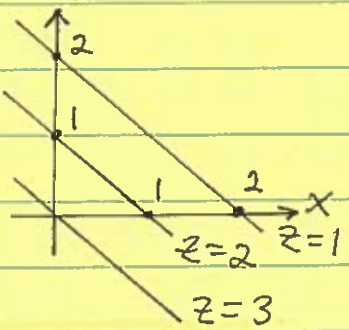
Level Curves:

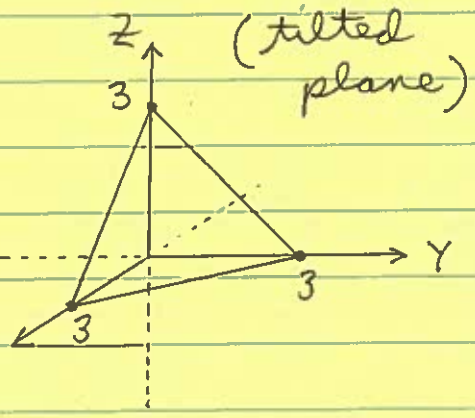
$z=1$: $x+y=2$

$z=2$: $x+y=1$

$z=3$: $x+y=0$

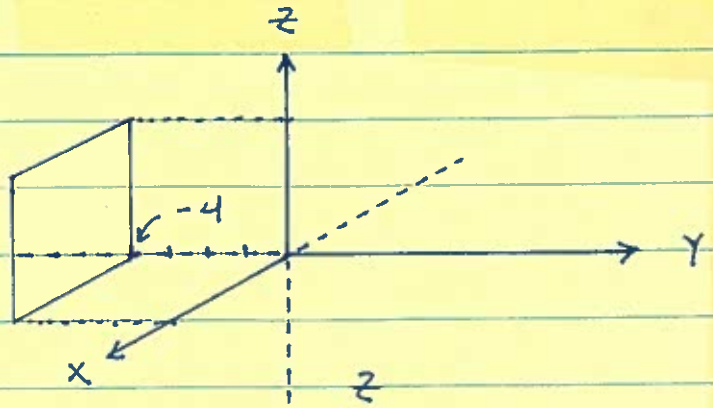
(all are lines.)





8.) $y=-4$

"vertical" plane



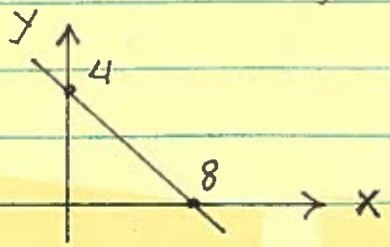
10.) $x+2y=8$

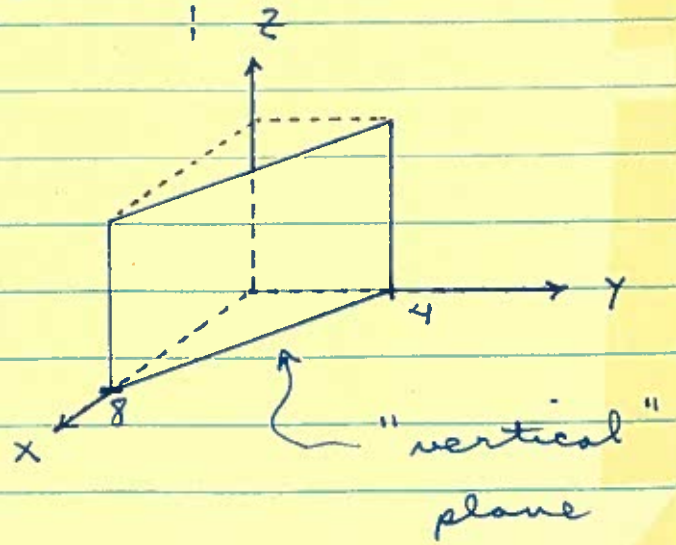
$x=0 \rightarrow y=4$

$y=0 \rightarrow x=8$

XY-trace ($z=0$):

$x+2y=8$ (line)





31.) $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{9} = 1$;

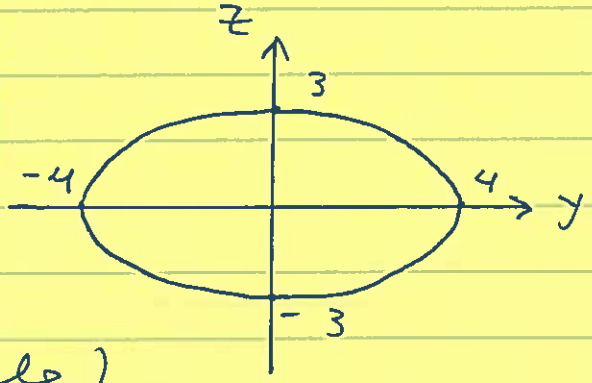
$x=0, y=0 \rightarrow z = \pm 3$

$x=0, z=0 \rightarrow y = \pm 4$

$y=0, z=0 \rightarrow x = \pm 3$

yz-trace

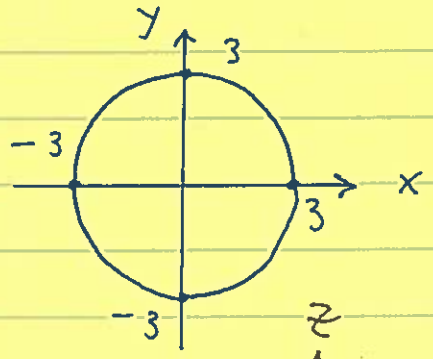
$(x=0)$: $\frac{y^2}{16} + \frac{z^2}{9} = 1$ (ellipse)



xz-trace

$(y=0)$: $\frac{x^2}{9} + \frac{z^2}{9} = 1$

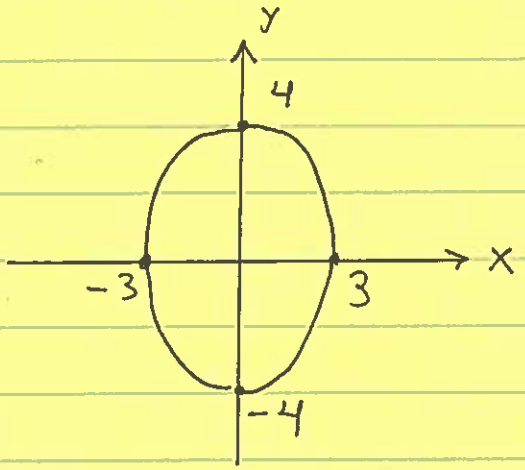
$\rightarrow x^2 + y^2 = 3^2$ (circle)

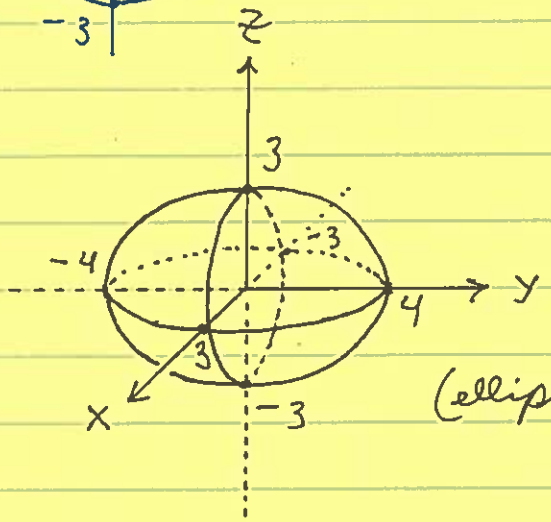


xy-trace

$(z=0)$: $\frac{x^2}{9} + \frac{y^2}{16} = 1$

(ellipse)





(ellipsoid)

33.) $4x^2 + 4y^2 - z^2 = 4$;

$x=0, y=0 \rightarrow -z^2 = 4$ (NO!? ★)

$x=0, z=0 \rightarrow 4y^2 = 4 \rightarrow y = \pm 1$

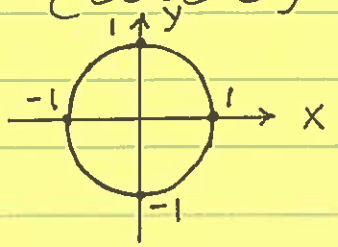
$y=0, z=0 \rightarrow 4x^2 = 4 \rightarrow x = \pm 1$

xy-trace $(z=0)$:

$4x^2 + 4y^2 = 4 \rightarrow$

$x^2 + y^2 = 1$

(circle)

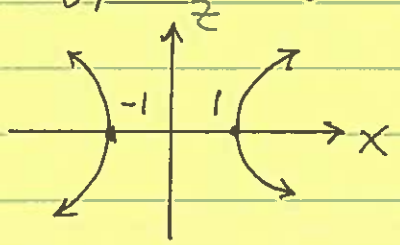


xz-trace ($y=0$):

$4x^2 - z^2 = 4 \rightarrow$

$x^2 - \frac{z^2}{4} = 1$

(hyperbola)

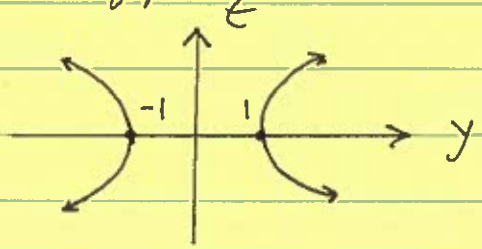


yz-trace ($x=0$):

$4y^2 - z^2 = 4 \rightarrow$

$y^2 - \frac{z^2}{4} = 1$

(hyperbola)



Level Curves:

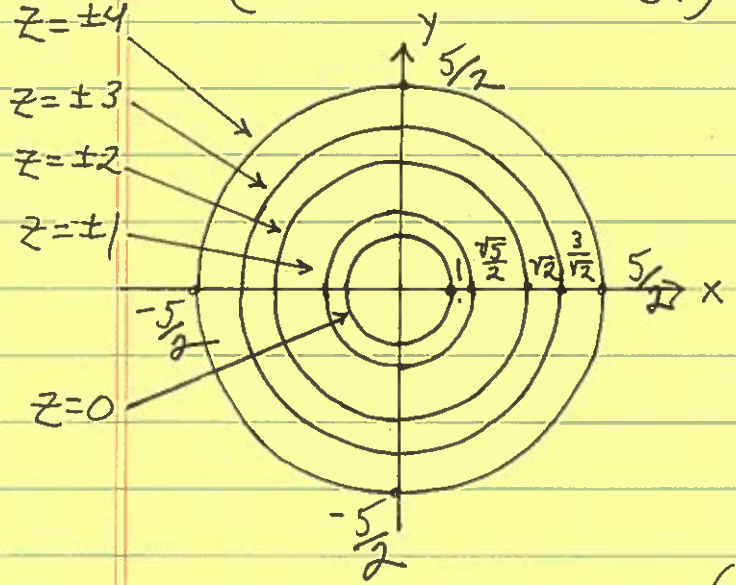
$z = \pm 1$: $4x^2 + 4y^2 - 1 = 4 \rightarrow x^2 + y^2 = \frac{5}{4} = \left(\frac{\sqrt{5}}{2}\right)^2$

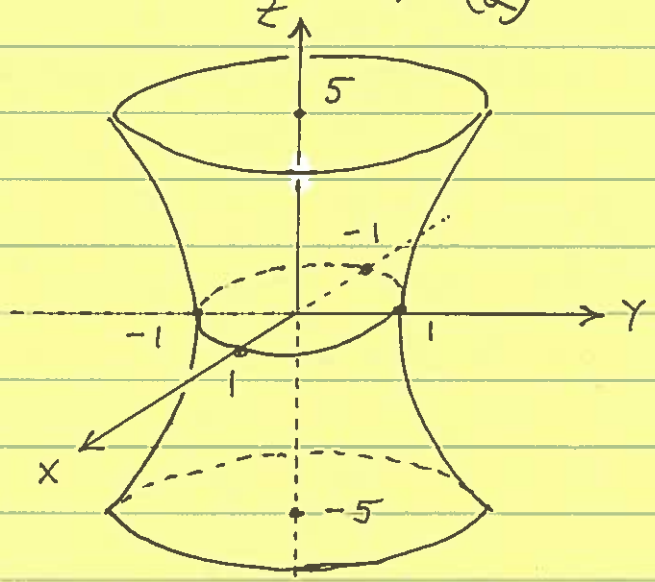
$z = \pm 2$: $4x^2 + 4y^2 - 4 = 4 \rightarrow x^2 + y^2 = 2 = (\sqrt{2})^2$

$z = \pm 3$: $4x^2 + 4y^2 - 9 = 9 \rightarrow x^2 + y^2 = \frac{9}{2} = \left(\frac{3}{\sqrt{2}}\right)^2$

$z = \pm 4$: $4x^2 + 4y^2 - 16 = 9 \rightarrow x^2 + y^2 = \frac{25}{4} = \left(\frac{5}{2}\right)^2$

(all are circles.)



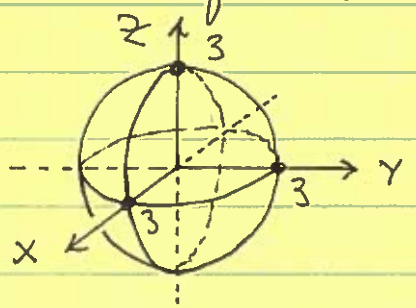


(hyperboloid of one sheet)

38.) $x^2 + y^2 + z^2 = 9 = 3^2$

sphere : center $(0,0,0)$

radius $= 3$



48.) $z = 4x^2 + y^2$;

$x=0, y=0 \rightarrow z=0$

$x=0, z=0 \rightarrow y=0$

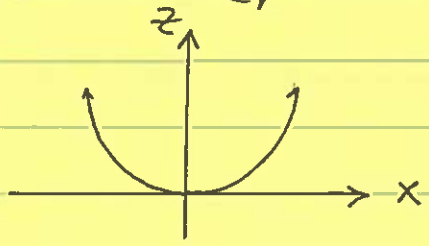
$y=0, z=0 \rightarrow x=0$

xy-trace ($z=0$):

$4x^2 + y^2 = 0 \rightarrow x=0, y=0$

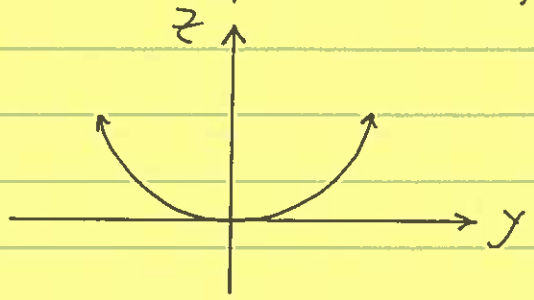
xz-trace ($y=0$)

$z = 4x^2$ (parabola)



yz-trace ($x=0$):

$z = y^2$ (parabola)



Level Curves:

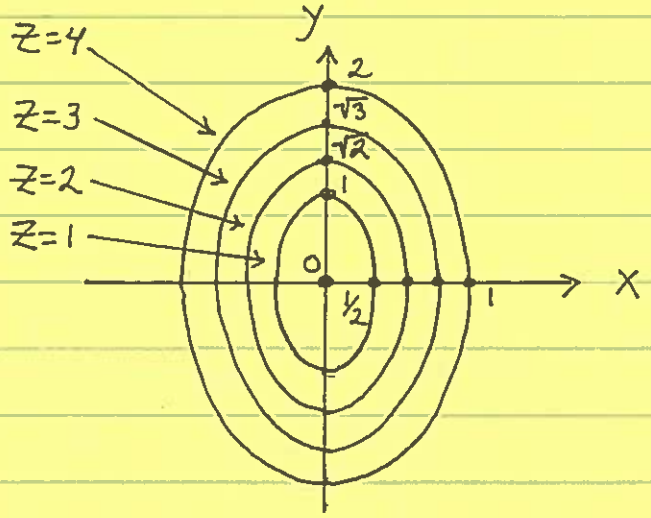
$z=1: \; 1 = 4x^2 + y^2$

$z=2: \; 2 = 4x^2 + y^2$

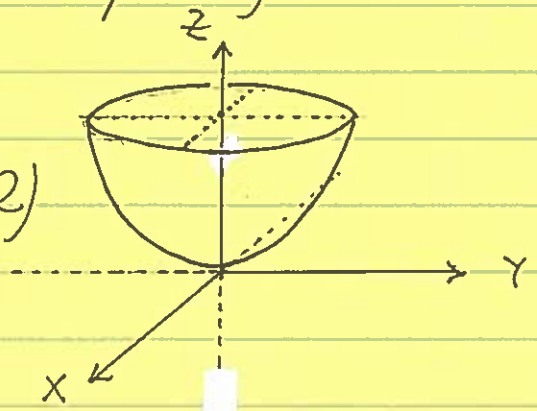
$z=3: \; 3 = 4x^2 + y^2$

$z=4: \; 4 = 4x^2 + y^2$

(all are ellipses)



(ellipsoid)



51.) $4x^2 - y^2 + 4z^2 = -16 \rightarrow 4x^2 + 4z^2 = y^2 - 16$;

$x=0, y=0 \rightarrow 4z^2 = -16$ (NO!? ✗)

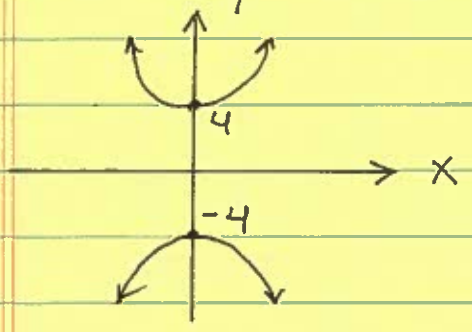
$x=0, z=0 \rightarrow y^2 - 16 = 0 \rightarrow y = \pm 4$

$y=0, z=0 \rightarrow 4x^2 = -16$ (NO!? ✗)

XY-trace ($z=0$):

$4x^2 - y^2 = -16 \rightarrow$

$\frac{y^2}{16} - \frac{x^2}{4} = 1$ (hyperbola)



XZ-trace ($y=0$):

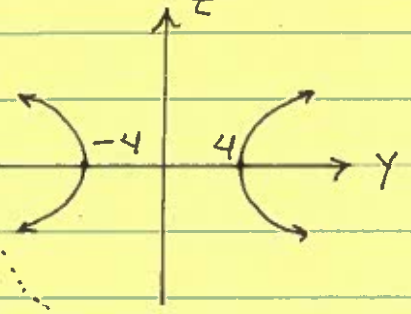
$4x^2 + 4z^2 = -16 \rightarrow$

$x^2 + z^2 = -4$ (NO!? ✱)

YZ-trace ($x=0$):

$-y^2 + 4z^2 = -16 \rightarrow$

$\frac{y^2}{16} - \frac{z^2}{4} = 1$ (hyperbola)



Level Curves (USING Y-values!):

$y = \pm 4: \; 4x^2 + 4z^2 = 0 \rightarrow x=0, y=0$

$y = \pm 5: \; 4x^2 + 4z^2 = 9 \rightarrow x^2 + z^2 = \left(\frac{3}{2}\right)^2$

$y = \pm 6: \; 4x^2 + 4z^2 = 20 \rightarrow x^2 + z^2 = (\sqrt{5})^2$

$y = \pm 7: \; 4x^2 + 4z^2 = 33 \rightarrow x^2 + z^2 = \left(\frac{\sqrt{33}}{2}\right)^2$

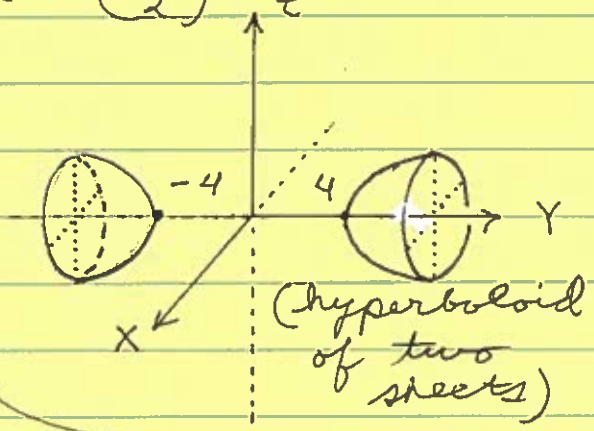
(all are circles)

54.) $4y = x^2 + z^2$;

$x=0, \; y=0 \rightarrow \; z=0$

$x=0, \; z=0 \rightarrow \; y=0$

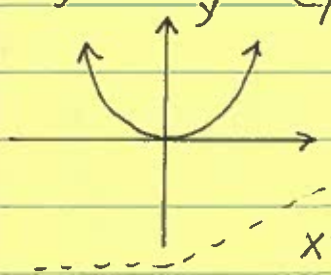
$y=0, \; z=0 \rightarrow \; x=0$;



(hyperboloid of two sheets)

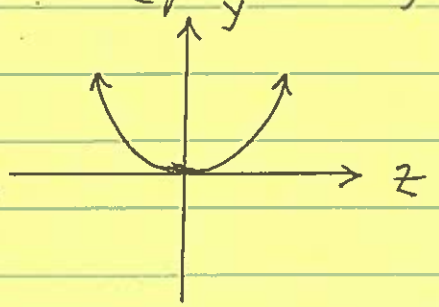
XY-trace ($z=0$):

$4y = x^2$ (parabola)



YZ-trace ($x=0$):

$4y = z^2$ (parabola)



XZ-trace ($y=0$):

$x^2 + z^2 = 0 \rightarrow x=0, \; z=0$

Level Curves (USING Y-values!)

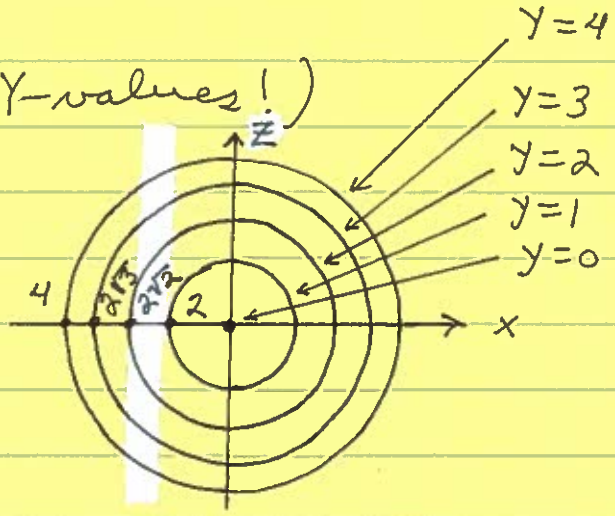
$y=1: \quad x^2+z^2=2^2$

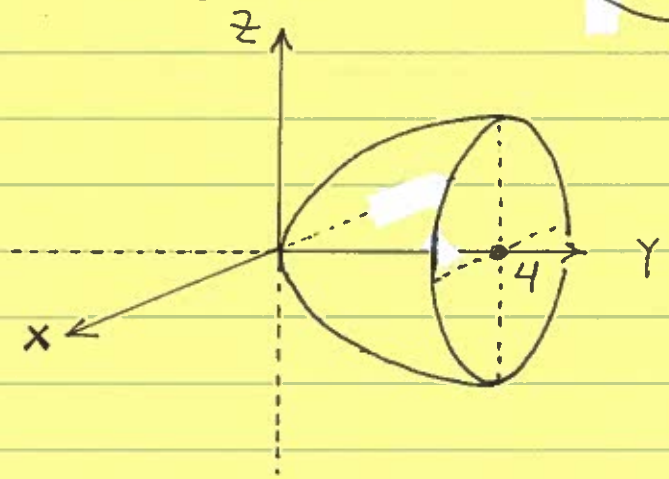
$y=2: \quad x^2+z^2=(2\sqrt{2})^2$

$y=3: \quad x^2+z^2=(2\sqrt{3})^2$

$y=4: \quad x^2+z^2=4^2$

(all are circles)





(paraboloid)

56.) $z^2=2x^2+2y^2$;

$x=0, y=0 \rightarrow z=0$

$x=0, z=0 \rightarrow y=0$

$y=0, z=0 \rightarrow x=0$

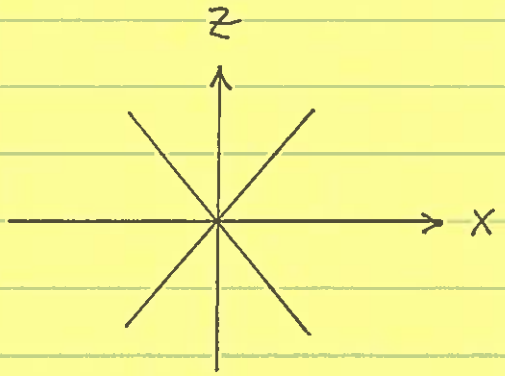
xy-trace ($z=0$):

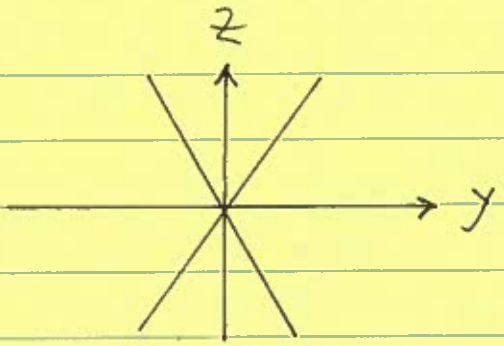
$2x^2+2y^2=0 \rightarrow x=0, y=0$

xz-trace ($y=0$):

$z^2=2x^2 \rightarrow$

$z=\pm\sqrt{2}x$ (lines)





yz-trace ($x=0$):

$z^2 = 2y^2 \rightarrow$

$z = \pm\sqrt{2}\,y$ (lines)

Level curves

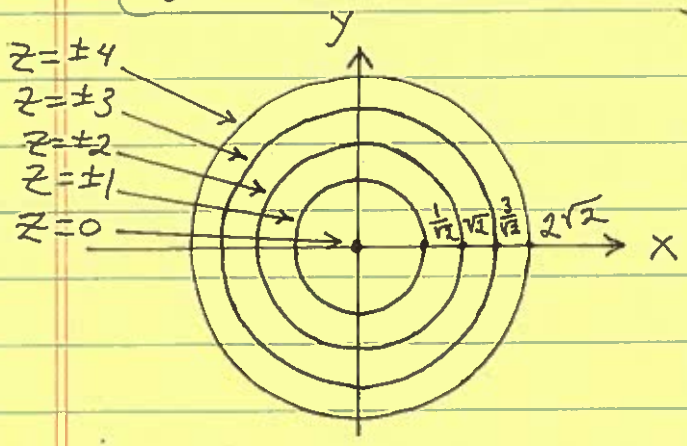
$z = \pm 1: \; 1 = 2x^2 + 2y^2 \rightarrow \; x^2 + y^2 = \left(\frac{1}{\sqrt{2}}\right)^2$

$z = \pm 2: \; 4 = 2x^2 + 2y^2 \rightarrow \; x^2 + y^2 = (\sqrt{2})^2$

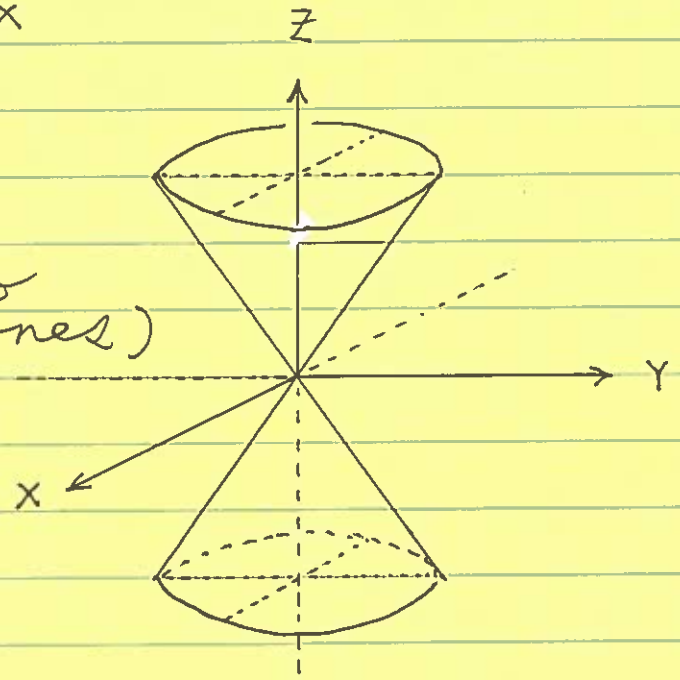
$z = \pm 3: \; 9 = 2x^2 + 2y^2 \rightarrow \; x^2 + y^2 = \left(\frac{3}{\sqrt{2}}\right)^2$

$z = \pm 4: \; 16 = 2x^2 + 2y^2 \rightarrow \; x^2 + y^2 = (2\sqrt{2})^2$

(all are circles.)



(two cones)



Math 16C
Kouba
Worksheet 3

Neatly sketch the level curves in the xy-plane for each of the equations below using the following values for z : $-3, -2, -1, 0, 1, 2, 3$. After determining the level curve, sketch the related surface in 3D-space (or at least describe in words what you think the surface is).

1.) $z = x - 2y$

2.) $z = x^2 - y$

3.) $z = x^2 + y^2$

4.) $z = e^{-x} + y$

5.) $x^2 + y^2 + z^2 = 9$

6.) $z = \dfrac{x+y}{x-y}$

7.) $z = \ln x - y$

8.) $z^2 = x^2 + y^2$

9.) $z^2 = x^2 + y^2 + 1$

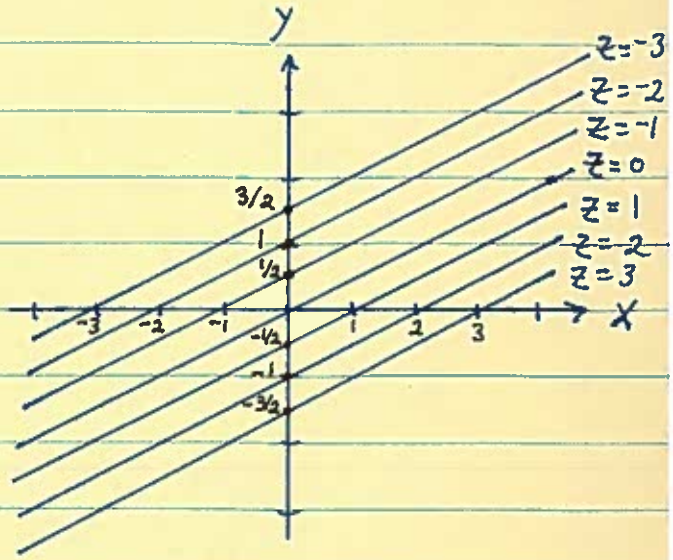
10.) $z^2 = x^2 + y^2 - 1$

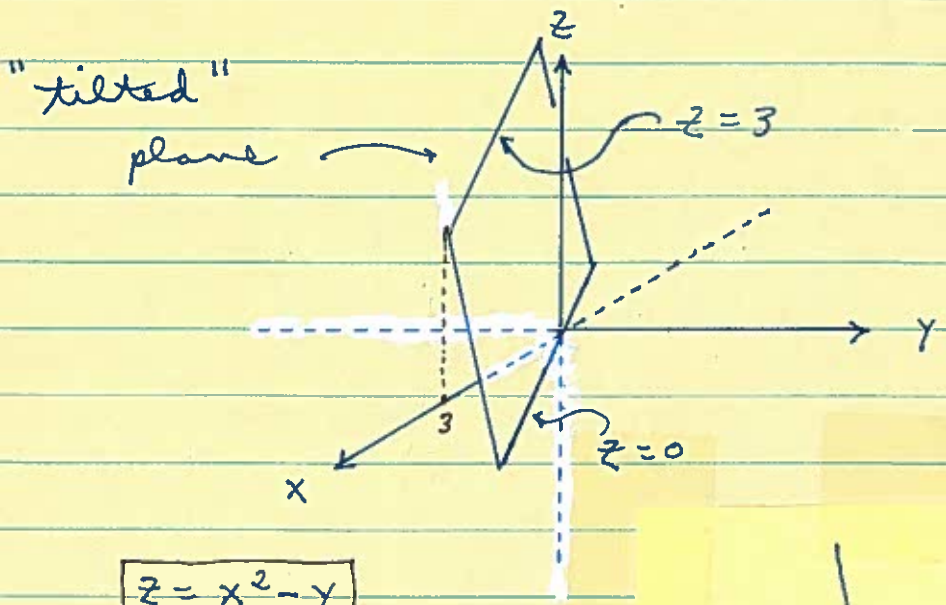
Worksheet 3

$z = x - 2y$

1.)

z-value	level curve
3	$3 = x - 2y$
2	$2 = x - 2y$
1	$1 = x - 2y$
0	$0 = x - 2y$
-1	$-1 = x - 2y$
-2	$-2 = x - 2y$
-3	$-3 = x - 2y$

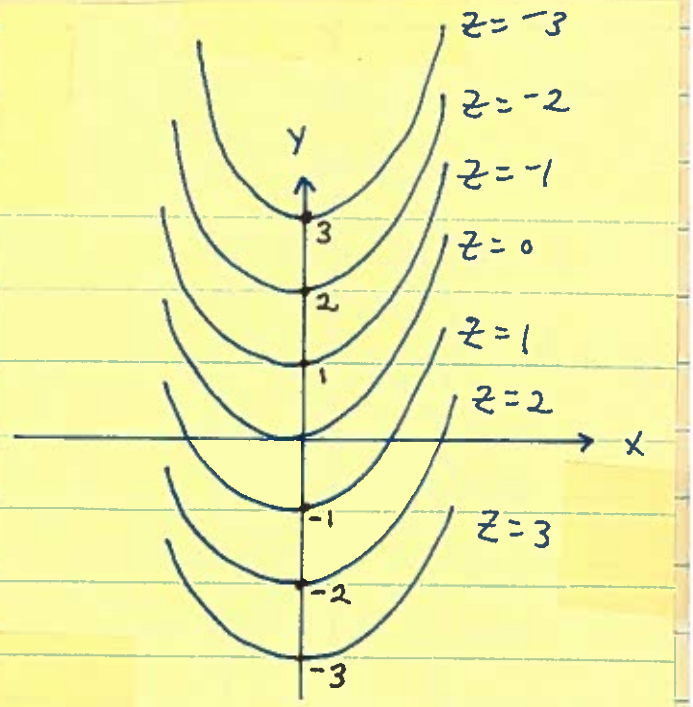


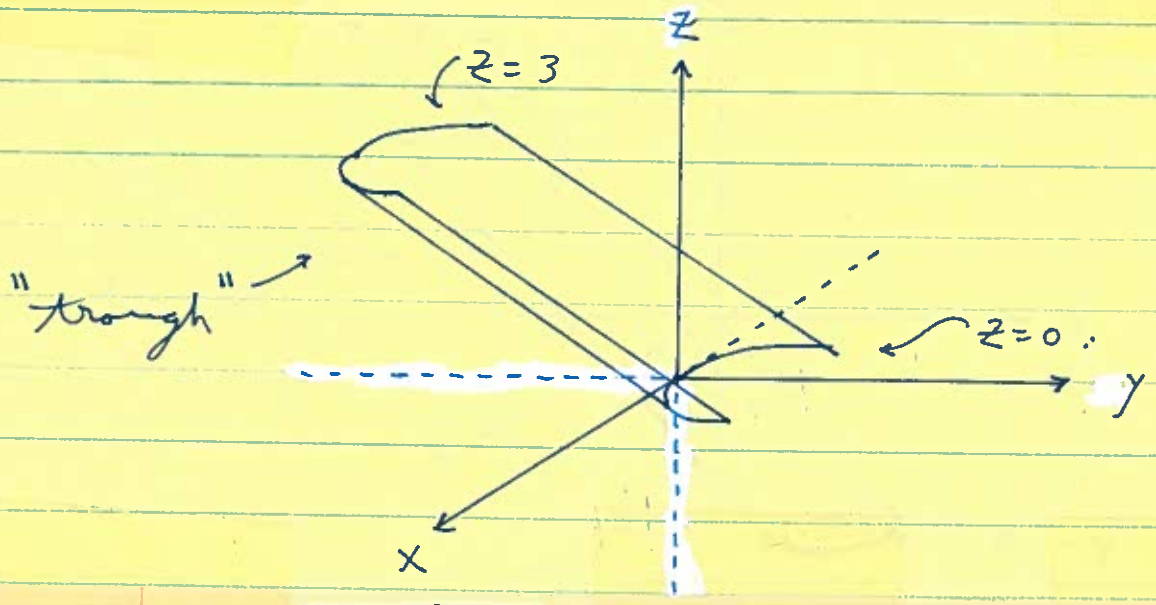


$z = x^2 - y$

2.)

z-value	level curve
3	$3 = x^2 - y \rightarrow y = x^2 - 3$
2	$2 = x^2 - y \rightarrow y = x^2 - 2$
1	$1 = x^2 - y \rightarrow y = x^2 - 1$
0	$0 = x^2 - y \rightarrow y = x^2$
-1	$-1 = x^2 - y \rightarrow y = x^2 + 1$
-2	$-2 = x^2 - y \rightarrow y = x^2 + 2$
-3	$-3 = x^2 - y \rightarrow y = x^2 + 3$

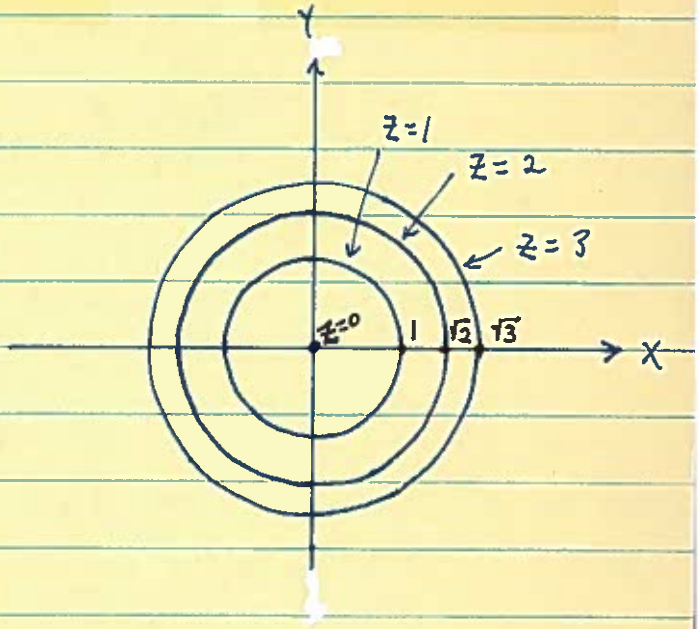


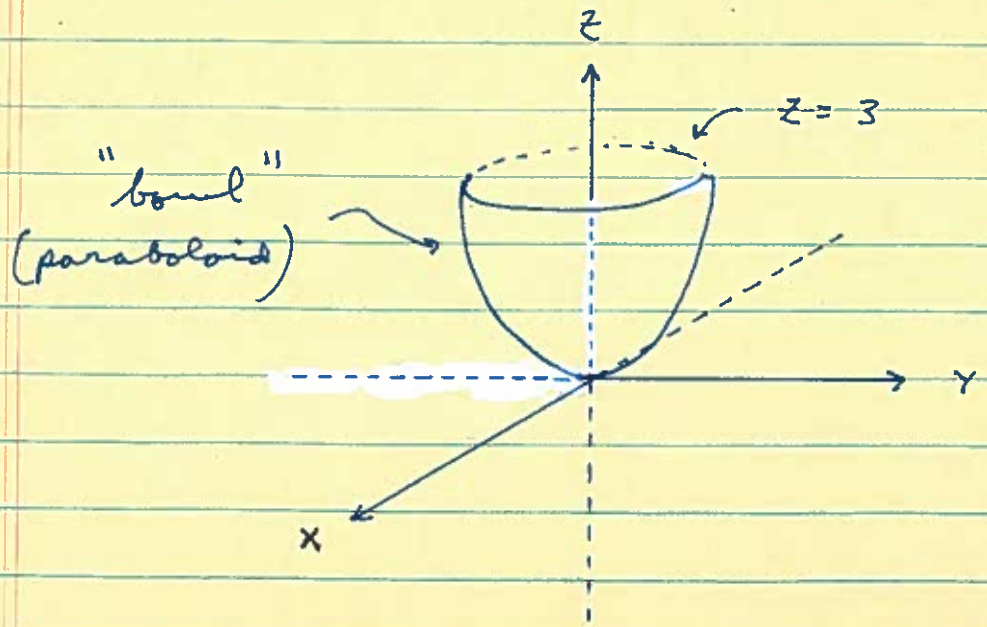


$z = x^2 + y^2$

3.)

z-value	level curve
3	$3 = x^2 + y^2$
2	$2 = x^2 + y^2$
1	$1 = x^2 + y^2$
0	$0 = x^2 + y^2$
-1	impossible
-2	impossible
-3	impossible

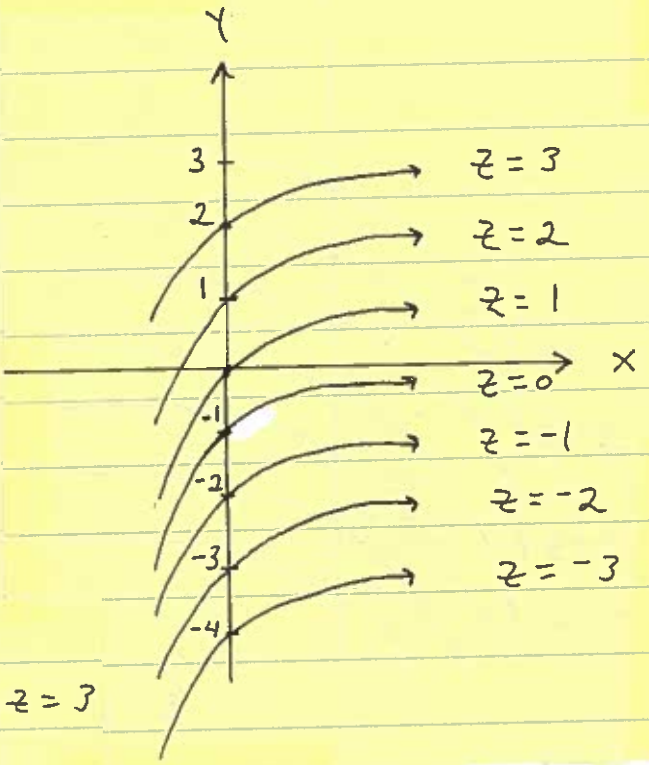


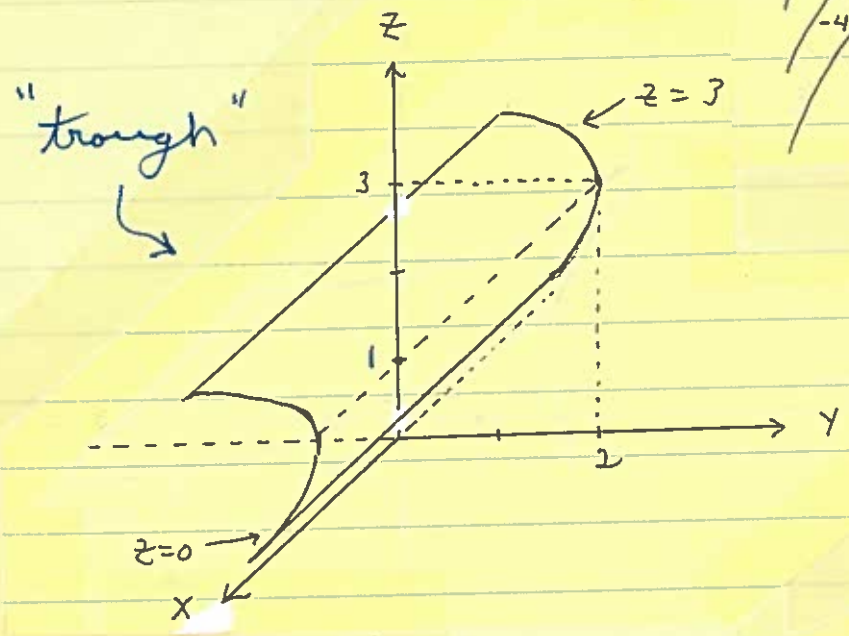


$$z = e^{-x} + y$$

4.)

z-value	level curve
3	$y = 3 - e^{-x}$
2	$y = 2 - e^{-x}$
1	$y = 1 - e^{-x}$
0	$y = -e^{-x}$
-1	$y = -1 - e^{-x}$
-2	$y = -2 - e^{-x}$
-3	$y = -3 - e^{-x}$

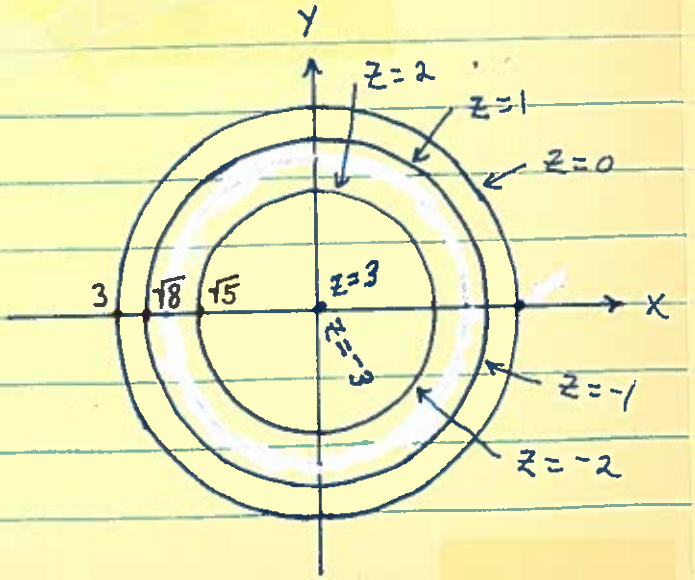


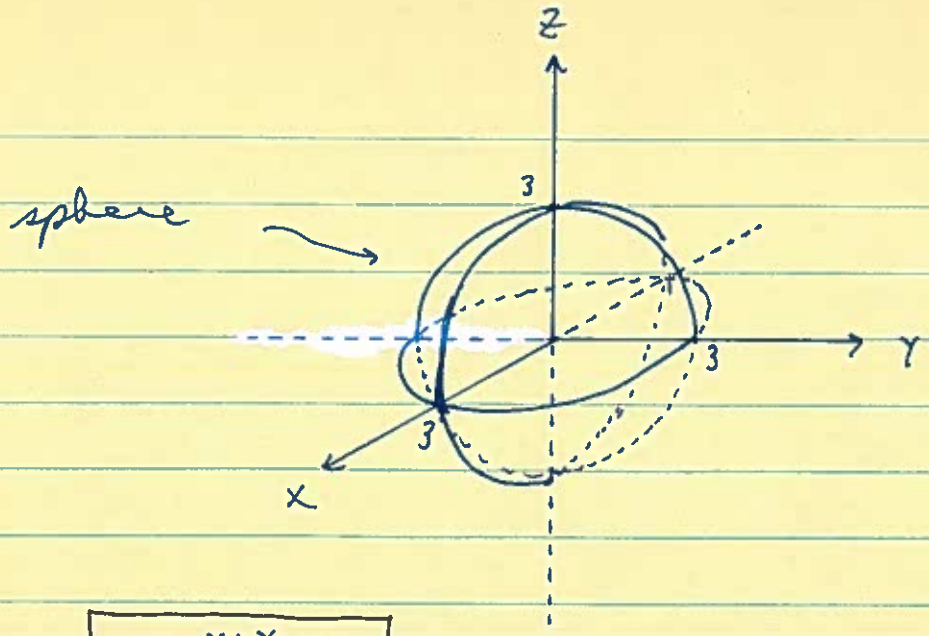


$$x^2 + y^2 + z^2 = 9$$

5.)

z-value	level curve
3	$x^2 + y^2 = 0$
2	$x^2 + y^2 = 5$
1	$x^2 + y^2 = 8$
0	$x^2 + y^2 = 9$
-1	$x^2 + y^2 = 8$
-2	$x^2 + y^2 = 5$
-3	$x^2 + y^2 = 0$



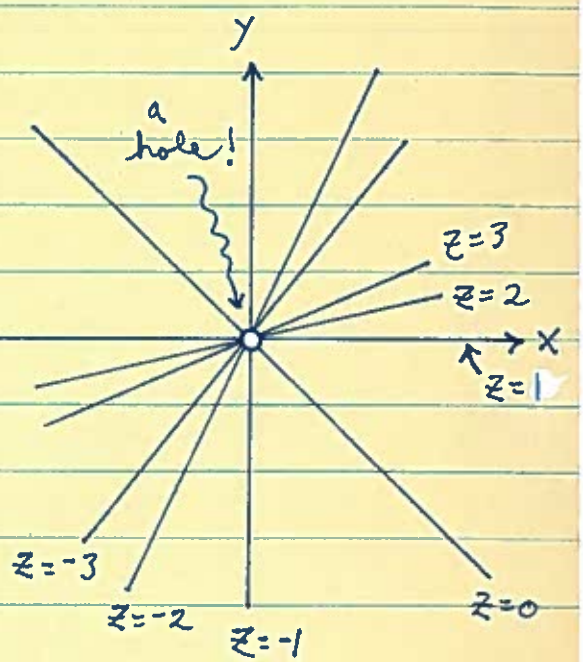


sphere

$z = \frac{x+y}{x-y}$

6.)

z-value	level curve
3	$3 = \frac{x+y}{x-y} \rightarrow y = \frac{1}{2}x$
2	$2 = \frac{x+y}{x-y} \rightarrow y = \frac{1}{3}x$
1	$1 = \frac{x+y}{x-y} \rightarrow y = 0$
0	$0 = \frac{x+y}{x-y} \rightarrow y = -x$
-1	$-1 = \frac{x+y}{x-y} \rightarrow x = 0$
-2	$-2 = \frac{x+y}{x-y} \rightarrow y = 3x$
-3	$-3 = \frac{x+y}{x-y} \rightarrow y = 2x$

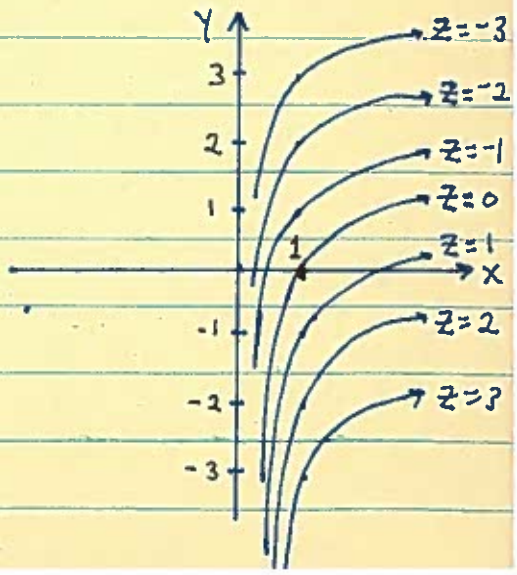


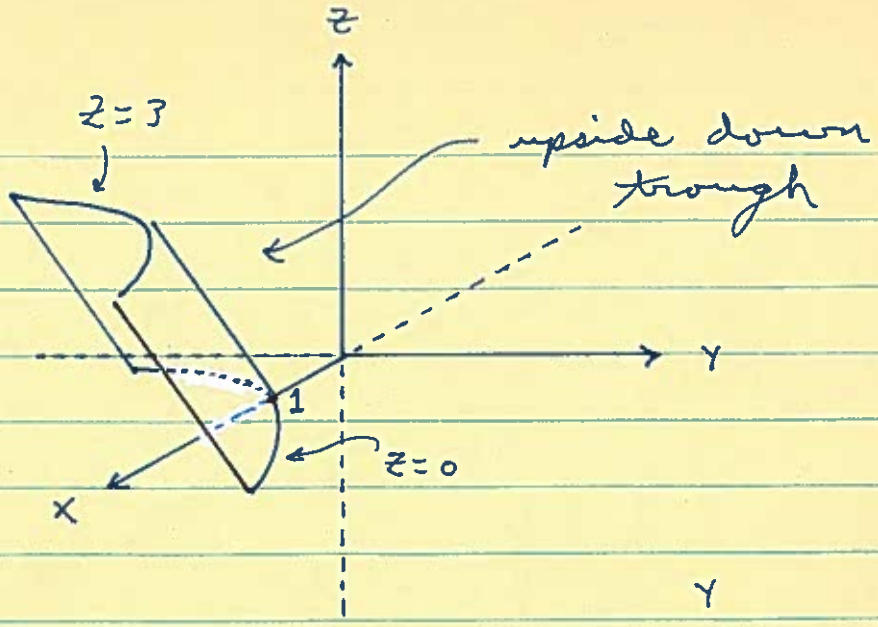
The surface is a "twisted ribbon."

$z = \ln x - y$

7.)

z-value	level curve
3	$3 = \ln x - y \rightarrow y = \ln x - 3$
2	$2 = \ln x - y \rightarrow y = \ln x - 2$
1	$1 = \ln x - y \rightarrow y = \ln x - 1$
0	$0 = \ln x - y \rightarrow y = \ln x$
-1	$-1 = \ln x - y \rightarrow y = \ln x + 1$
-2	$-2 = \ln x - y \rightarrow y = \ln x + 2$
-3	$-3 = \ln x - y \rightarrow y = \ln x + 3$

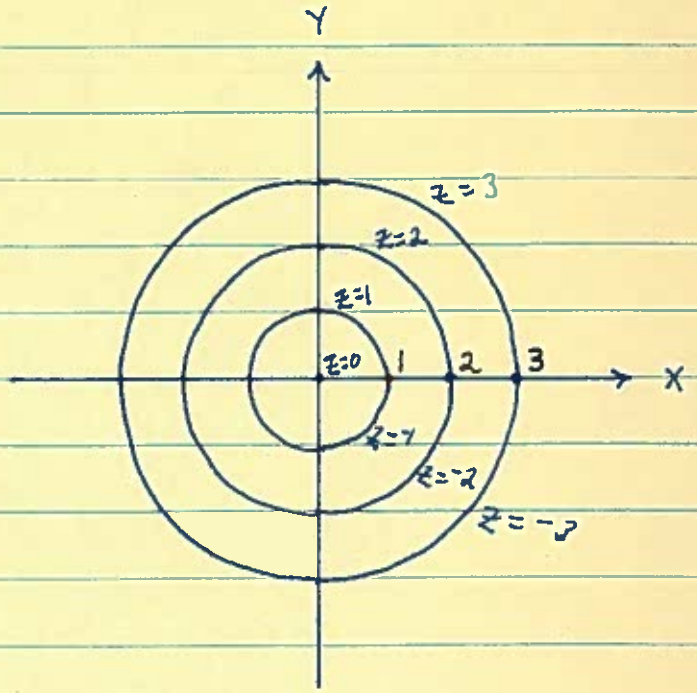


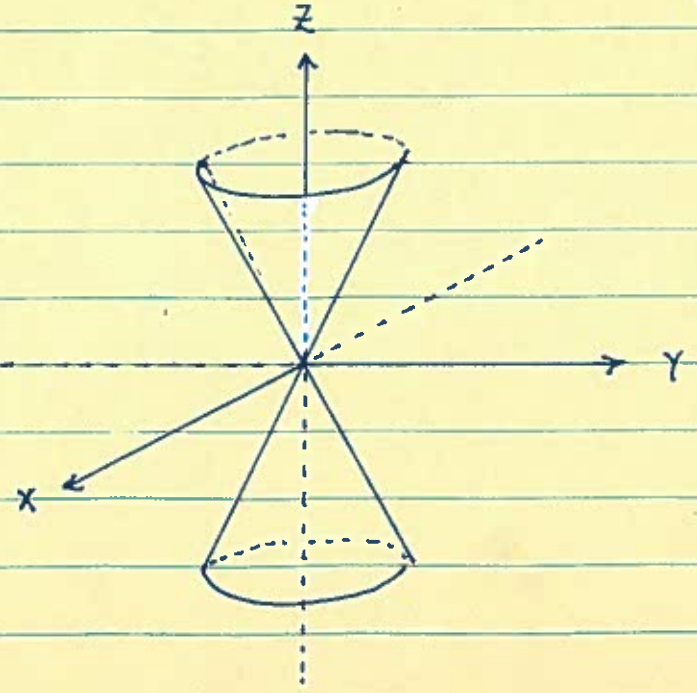


$z^2 = x^2 + y^2$

8.)

z-value	level curve
3	$9 = x^2 + y^2$
2	$4 = x^2 + y^2$
1	$1 = x^2 + y^2$
0	$0 = x^2 + y^2$
-1	$1 = x^2 + y^2$
-2	$4 = x^2 + y^2$
-3	$9 = x^2 + y^2$



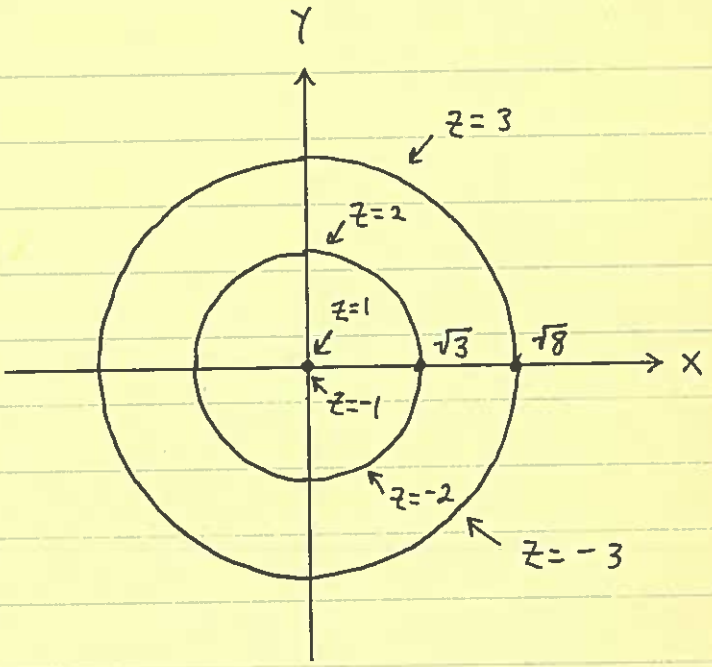


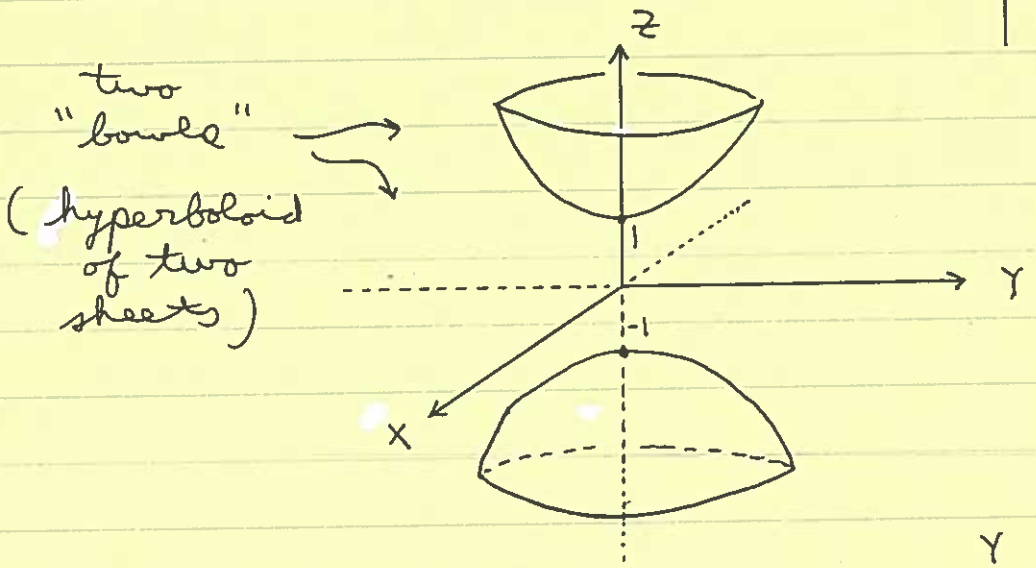
two cones

$$z^2 = x^2 + y^2 + 1$$

9.)

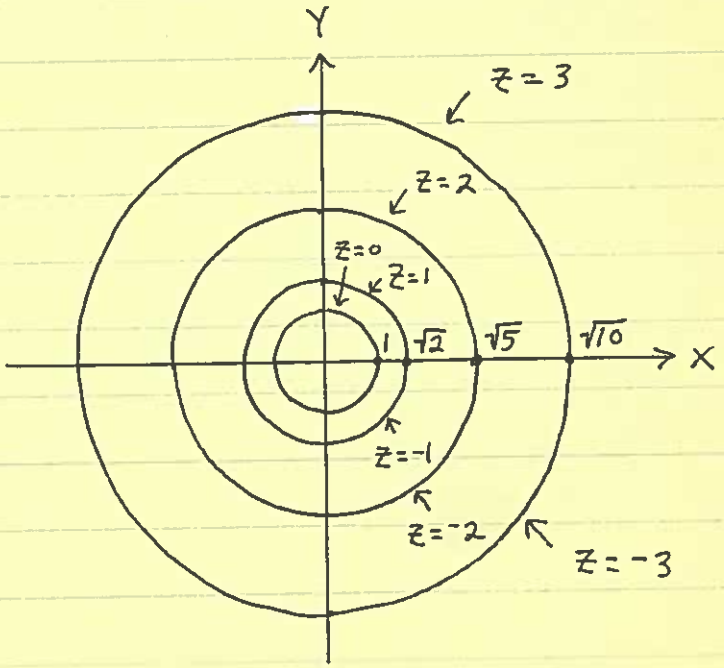
z-value	level curve
3	$8 = x^2 + y^2$
2	$3 = x^2 + y^2$
one point → 1	$0 = x^2 + y^2$
can't! → 0	$-1 = x^2 + y^2$
one point → -1	$0 = x^2 + y^2$
-2	$3 = x^2 + y^2$
-3	$8 = x^2 + y^2$

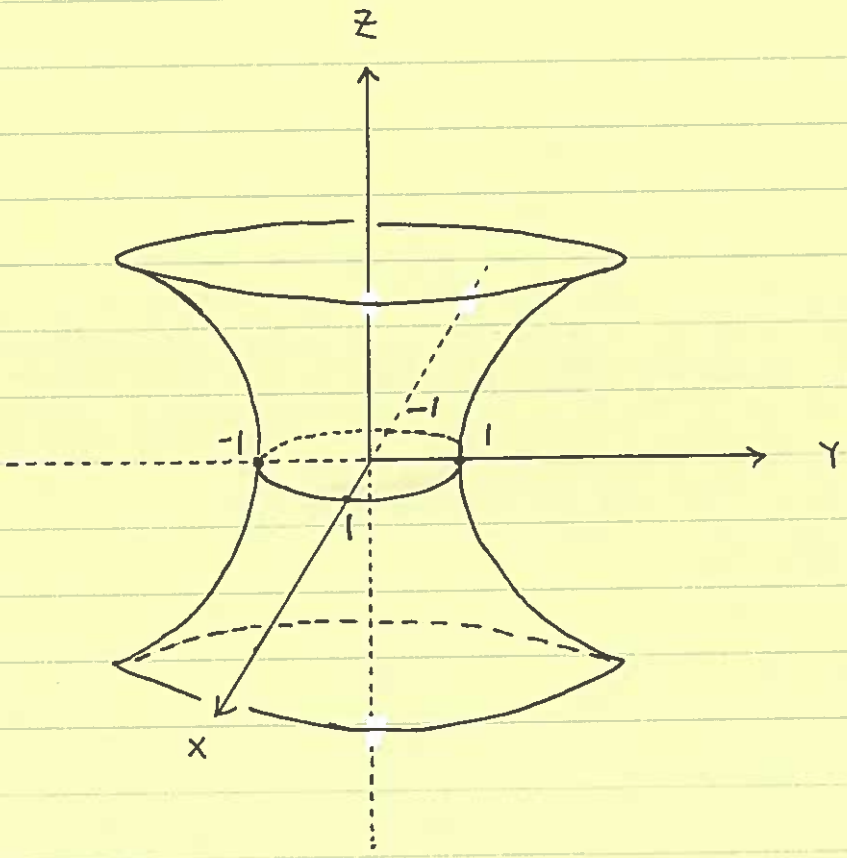




10.) $$z^2 = x^2 + y^2 - 1$$

z-value	level curve
3	$10 = x^2 + y^2$
2	$5 = x^2 + y^2$
1	$2 = x^2 + y^2$
0	$1 = x^2 + y^2$
-1	$2 = x^2 + y^2$
-2	$5 = x^2 + y^2$
-3	$10 = x^2 + y^2$





"tube"

(hyperboloid of one sheet)