Section 7.4

1.) $z = 2x - 3y + 5 \longrightarrow$

$z_x = 2, \quad z_y = -3$

2.) $z = x^2 - 3y^2 + 7 \longrightarrow$

$z_x = 2x, \quad z_y = -6y$

3.) $z = 5\sqrt{x} - 6y^2 \longrightarrow$

$z_x = 5 \cdot \frac{1}{2} x^{-1/2}, \quad z_y = -12y$

4.) $z = x^{-1/2} + 4y^{3/2} \longrightarrow$

$z_x = -\frac{1}{2} x^{-3/2} + 4 \cdot \frac{3}{2} y^{1/2}$

5.) $z = \frac{x}{y} \longrightarrow$

$z_x = \frac{1}{y}, \quad z_y = \frac{-x}{y^2}$

6.) $z = x\sqrt{y} \longrightarrow$

$z_x = \sqrt{y}, \quad z_y = x \cdot \frac{1}{2} y^{-1/2}$

7.) $z = \sqrt{x^2 + y^2} \longrightarrow$

$z_x = \frac{1}{2}(x^2+y^2)^{-1/2} \cdot (2x), \quad z_y = \frac{1}{2}(x^2+y^2)^{-1/2} \cdot (2y)$

8.) $z = \frac{xy}{x^2+y^2} \longrightarrow$

$$z_x = \frac{(x^2+y^2) \cdot y - xy(2x)}{(x^2+y^2)^2}, \quad z_y = \frac{(x^2+y^2) \cdot x - xy(2y)}{(x^2+y^2)^2}$$

9.) $z = x^2 e^{2y} \longrightarrow$

$z_x = 2x e^{2y}, \quad z_y = x^2 \cdot 2e^{2y}$

10.) $z = xe^{x+y} \longrightarrow$

$z_x = x \cdot e^{x+y} + 1 \cdot e^{x+y}, \quad z_y = x \cdot e^{x+y}$

11.) $z = e^{-x^2-y^2} \longrightarrow$

$z_x = e^{-x^2-y^2} \cdot (-2x), \quad z_y = e^{-x^2-y^2} \cdot (-2y)$

12.) $z = e^{x/y} \longrightarrow$

$z_x = e^{x/y} \cdot \frac{1}{y}, \quad z_y = e^{x/y} \cdot \frac{-x}{y^2}$

13.) $z = \ln \frac{x-y}{(x+y)^2} \longrightarrow$

$$z_x = \frac{1}{\frac{(x-y)}{(x+y)^2}} \cdot \frac{(x+y)^2\cdot(1) - (x-y)\cdot 2(x+y)}{(x+y)^4},$$

$$z_y = \frac{1}{\frac{(x-y)}{(x+y)^2}} \cdot \frac{(x+y)^2\cdot(-1) - (x-y)\cdot 2(x+y)}{(x+y)^4}$$

14.) $z = \ln(x^2+y^2)^{1/2} = \frac{1}{2}\ln(x^2+y^2) \longrightarrow$

$z_x = \frac{1}{2}\cdot\frac{1}{x^2+y^2}\cdot(2x)$, $z_y = \frac{1}{2}\cdot\frac{1}{x^2+y^2}\cdot(2y)$

$f(x,y) = 3x^2y\cdot e^{x-y}$, $g(x,y) = 3xy^2\cdot e^{y-x} \longrightarrow$

15.) $f_x(x,y) = (3x^2y)\cdot e^{x-y} + (6xy)\cdot e^{x-y}$

16.) $f_y(x,y) = (3x^2y)\cdot e^{x-y}(-1) + (3x^2)\cdot e^{x-y}$

20.) $g_x(x,y) = (3xy^2)\cdot e^{y-x}\cdot(-1) + (3y^2)\cdot e^{y-x} \longrightarrow$

$g_x(-2,-2) = (-24)\cdot \cancel{e^0}^{1}\cdot(-1) + (12)\cdot\cancel{e^0}^{1} = 36$

21.) $z = 3x^2 + xy - y^2 \longrightarrow$

$z_x = 6x + y \longrightarrow z_x(2,1) = 12+1 = 13$,

$z_y = x - 2y \longrightarrow z_y(2,1) = 2-2 = 0$.

24.) $z = e^x\cdot y^2 \longrightarrow$

$z_x = e^x\cdot y^2 \rightarrow z_x(0,2) = \cancel{e^0}^{1}\cdot(4) = 4$,

$z_y = e^x\cdot 2y \rightarrow z_y(0,2) = \cancel{e^0}^{1}\cdot 4 = 4$

26.) $z = \dfrac{4XY}{(X^2+Y^2)^{1/2}} \rightarrow$

$$z_X = \frac{(X^2+Y^2)^{1/2} \cdot 4Y - 4XY \cdot \frac{1}{2}(X^2+Y^2)^{-1/2} \cdot (2X)}{((X^2+Y^2)^{1/2})^2} \rightarrow$$

$$z_X(1,0) = \frac{(1)(0) - 0}{1} = 0 \quad ,$$

$$z_Y = \frac{(X^2+Y^2)^{1/2} \cdot (4X) - 4XY \cdot \frac{1}{2}(X^2+Y^2)^{-1/2} \cdot (2Y)}{((X^2+Y^2)^{1/2})^2} \rightarrow$$

$$z_Y(1,0) = \frac{(1)(4) - (0)}{1} = 4 .$$

27.) $z = \ln(X^2+Y^2) \longrightarrow$

$$z_X = \frac{2X}{X^2+Y^2} \rightarrow z_X(1,0) = \frac{2}{1} = 2 ,$$

$$z_Y = \frac{2Y}{X^2+Y^2} \rightarrow z_Y(1,0) = \frac{0}{1} = 0$$

29.) $\omega = 3X^2Y - 5XYZ + 10YZ^2 \longrightarrow$

$\omega_X = 6XY - 5YZ ,$

$\omega_Y = 3X^2 - 5XZ + 10Z^2 ,$

$\omega_Z = -5XY + 20YZ$

31.) $\omega = \dfrac{XY}{X+Y+Z} \longrightarrow$

$$\omega_X = \frac{(X+Y+Z)(Y) - (XY)\cdot(1)}{(X+Y+Z)^2} ,$$

$$\omega_Y = \frac{(X+Y+Z)(X) - (XY)\cdot(1)}{(X+Y+Z)^2} ,$$

$$\omega_z = \frac{(x+y+z)(0) - xy(1)}{(x+y+z)^2}$$

38.) $\omega = xye^{z^2} \rightarrow$

$\omega_x = ye^{z^2} \rightarrow \omega_x(2,1,0) = 1 \cdot \cancel{e^0}^{1} = 1,$

$\omega_y = xe^{z^2} \rightarrow \omega_y(2,1,0) = 2 \cdot \cancel{e^0}^{1} = 2,$

$\omega_z = xy \cdot e^{z^2} \cdot 2z \rightarrow \omega_z(2,1,0) = (2)(1)e^0(0) = 0$

39.) $z = x^2 + 4xy + y^2 - 4x + 16y + 3 \rightarrow$

$z_x = 2x + 4y - 4 = 2(x + 2y - 2) = 0 \rightarrow$

$\boxed{x + 2y - 2 = 0}$;

$z_y = 4x + 2y + 16 = 2(2x + y + 8) = 0 \rightarrow$

$\boxed{2x + y + 8 = 0}$; then

$x = 2 - 2y \xrightarrow{\text{sub.}} 2(2-2y) + y + 8 = 0 \rightarrow$

$4 - 4y + y + 8 = 0 \rightarrow -3y = -12 \rightarrow$ $y = 4,\ x = -6$

41.) $z = \frac{1}{x} + \frac{1}{y} + xy \longrightarrow$

$z_x = \frac{-1}{x^2} + y = 0 \rightarrow \boxed{y = \frac{1}{x^2}}$;

$z_y = \frac{-1}{y^2} + x = 0 \rightarrow \boxed{x = \frac{1}{y^2}}$; then (sub.)

$x = \frac{1}{\left(\frac{1}{x^2}\right)^2} = \frac{1}{\frac{1}{x^4}} = x^4 \rightarrow x = x^4 \rightarrow$

$0 = x^4 - x = x(x^3 - 1) \rightarrow \cancel{x = 0}$ NO or $x = 1 \rightarrow$

$y = 1$

42.) $z = \ln(x^2 + y^2 + 1) \longrightarrow$

$$z_X = \frac{2X}{X^2+Y^2+1} = 0 \rightarrow 2X = 0 \rightarrow \boxed{X=0} \;;$$

$$z_Y = \frac{2Y}{X^2+Y^2+1} = 0 \rightarrow 2Y = 0 \rightarrow \boxed{Y=0} \;.$$

43.) $z = 2X - 3Y + 5 \rightarrow$

$z_X = 2 \;,\; z_Y = -3$

a.) $z_X(2,1) = 2$ b.) $z_Y(2,1) = -3$

49.) $z = 4 - X^2 - Y^2 \rightarrow$

$z_X = -2 \;,\; z_Y = -2Y$

a.) $z_X(1,1) = -2$ b.) $z_Y(1,1) = -2$

52.) $z = X^4 - 3X^2Y^2 + Y^4 \rightarrow$

$$\frac{\partial z}{\partial y} = -6x^2y + 4y^3 \rightarrow \frac{\partial^2 z}{\partial x\,\partial y} = -12xy \;,$$

$$\frac{\partial z}{\partial x} = 4X^3 - 6XY^2 \rightarrow \frac{\partial^2 z}{\partial Y\,\partial X} = -12XY$$

55.) $z = X^3 - 4y^2 \rightarrow$

$$\frac{\partial z}{\partial x} = 3X^2 \rightarrow \frac{\partial^2 z}{\partial x^2} = 6X \text{ and } \frac{\partial^2 z}{\partial Y\,\partial X} = 0 \;,$$

$$\frac{\partial z}{\partial y} = -8y \rightarrow \frac{\partial^2 z}{\partial y^2} = -8 \text{ and } \frac{\partial^2 z}{\partial x\,\partial y} = 0$$

59.) $z = \dfrac{XY}{X-Y} \longrightarrow$

$$\frac{\partial z}{\partial x} = \frac{(X-Y)(Y)-(XY)(1)}{(X-Y)^2} = \frac{XY - Y^2 - XY}{(X-Y)^2} = \frac{-y^2}{(X-Y)^2} \;;$$

$$\frac{\partial z}{\partial Y} = \frac{(X-Y)(X)-(XY)(-1)}{(X-Y)^2} = \frac{X^2 - XY + XY}{(X-Y)^2} = \frac{X^2}{(X-Y)^2} \;;$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{(x-y)^2(0) - (-y^2)\cdot 2(x-y)}{(x-y)^4} = \frac{2y^2}{(x-y)^3} \;;$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{(x-y)^2(0) - (x^2)\cdot 2(x-y)(-1)}{(x-y)^4} = \frac{2x^2}{(x-y)^3} \;;$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{(x-y)^2(-2y) - (-y^2)\cdot 2(x-y)(-1)}{(x-y)^4}$$

$$= \frac{2\cancel{(x-y)}\cdot[-y(x-y) - y^2]}{(x-y)^{\cancel{4}\,3}} = \frac{-2xy}{(x-y)^3} \;;$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{(x-y)^2(2x) - x^2\cdot 2(x-y)}{(x-y)^4}$$

$$= \frac{2\cancel{(x-y)}\cdot[(x-y)x - x^2]}{(x-y)^{\cancel{4}\,3}} = \frac{-2xy}{(x-y)^3} .$$

62.) $z = xe^y + ye^x \longrightarrow$

$\frac{\partial z}{\partial x} = e^y + ye^x$, $\frac{\partial z}{\partial y} = xe^y + e^x \longrightarrow$

$\frac{\partial^2 z}{\partial x^2} = ye^x$, $\frac{\partial^2 z}{\partial y^2} = xe^y$,

$\frac{\partial^2 z}{\partial y \partial x} = e^y + e^x$, $\frac{\partial^2 z}{\partial x \partial y} = e^y + e^x$

72.) $IQ(M,C) = \frac{100M}{C} \longrightarrow$

$IQ_C = \frac{-100M}{C^2}$, $IQ_M = \frac{100}{C} \longrightarrow$

$IQ_C(12,10) = \frac{-1200}{100} = -12$ (IQ pts. per chron. years) ,

$IQ_M(12,10) = \frac{100}{10} = +10$ (IQ pts. per mental years)

For a fixed M an increase in chronological age decreases IQ; for a fixed C an increase in mental age M increases IQ.

73.) $N = f(p,t)$; $\frac{\partial N}{\partial p} < 0$ means that for a fixed tuition t an <u>increase</u> in food price p <u>decreases</u> the number of applicants ; $\frac{\partial N}{\partial t} < 0$ means that for a fixed food price p an <u>increase</u> in tuition t <u>decreases</u> the number of applicants.

74.) $T(x,y) = 500 - 0.6x^2 - 1.5y^2$ at $(2,3) = (x,y)$:

$T_x = -1.2x$ and $T_y = -3y$ so

a.) Rate of change in x-direction at $(2,3)$ is $T_x = (-1.2)(2) = -2.4$ degrees per meter.

b.) Rate of change in y-direction at $(2,3)$ is $T_y = (-3)(3) = -9$ degrees per meter.