Math 16C Kouba

Lagrange Multipliers – Two Constraints

Ex: Maximize and Minimize

$S = xy + yz$ s.t.

$x^2 + y^2 = 1$ and $z = x$.

$\downarrow$ $\downarrow$

$x^2 + y^2 - 1 = 0$ $z - x = 0$

Let $F = (xy + yz) - \mu(x^2 + y^2 - 1) - \lambda(z - x)$

$$\xrightarrow{\partial} \left.\begin{array}{l} F_x = y - 2\mu x + \lambda = 0 \\ F_y = x + z - 2\mu y = 0 \end{array}\right\} \longrightarrow$$

$$F_z = y - \lambda = 0 \longrightarrow \lambda = y \quad \text{(SUB)}$$

$$\longrightarrow \left.\begin{array}{l} y - 2\mu x + (y) = 0 \\ x + z - 2\mu y = 0 \end{array}\right\} \longrightarrow \left.\begin{array}{l} 2y = 2\mu x \\ x + z = 2\mu y \end{array}\right\}$$

$$\longrightarrow \mu = \frac{y}{x} \text{ and } \mu = \frac{x+z}{2y} \longrightarrow$$

$$\frac{Y}{X} = \frac{X+Z}{2Y} \longrightarrow \boxed{2Y^2 = X^2 + XZ}$$; and

$$\xrightarrow{\partial} F_\mu = -(X^2+Y^2-1) = 0 \longrightarrow \boxed{Y^2 = 1-X^2}$$

$$F_\lambda = -(Z-X) = 0 \longrightarrow \boxed{Z = X}$$

$$\longrightarrow 2(1-X^2) = X^2 + X(X) = 2X^2$$

$$\longrightarrow 2 - 2X^2 = 2X^2 \longrightarrow 2 = 4X^2 \longrightarrow$$

$$X^2 = 1/2 \longrightarrow X = \pm 1/\sqrt{2}$$;

X	Y	Z	S	
$\frac{1}{\sqrt{2}}$	$1/\sqrt{2}$	$1/\sqrt{2}$	1	MAX
$1/\sqrt{2}$	$-1/\sqrt{2}$	$1/\sqrt{2}$	-1	MIN
$-1/\sqrt{2}$	$1/\sqrt{2}$	$-1/\sqrt{2}$	-1	MIN
$-1/\sqrt{2}$	$-1/\sqrt{2}$	$-1/\sqrt{2}$	1	MAX

and Lagrange Multipliers are $\mu = \pm 1$ and $\lambda = \pm\frac{1}{\sqrt{2}}$.