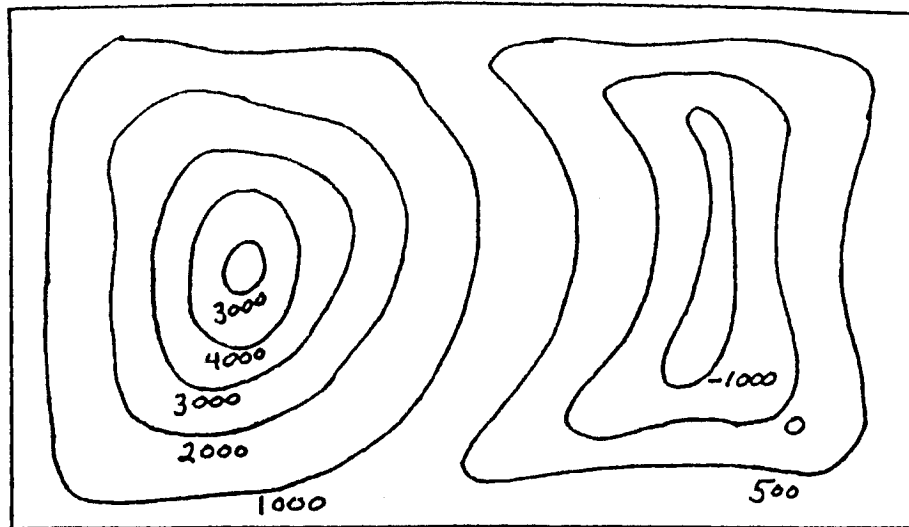Math 16C
Kouba
Sketching Surfaces in 3D-Space Using Level Curves and Traces

RECALL : Topographical Map

Assume that the numbers represent height in feet relative to sea level of a particular region.

DEFINITION : The intersection of a horizontal plane at a particular height $z = c$ with a given surface is called a level curve.

EXAMPLE : Sketch the surface $z = x^2 + y$ using level curves.

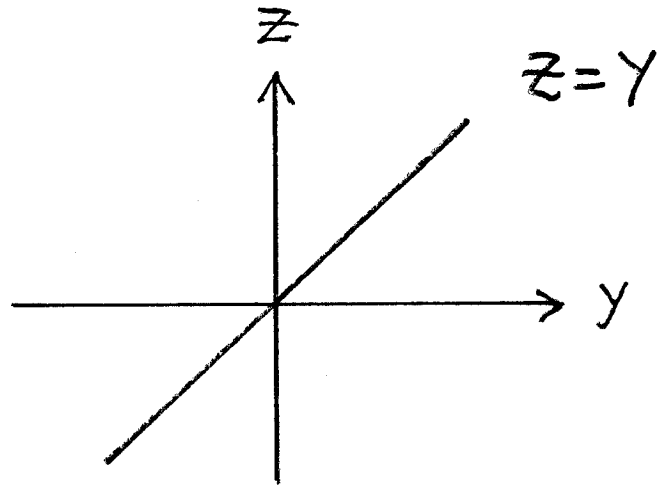| Values for z	Level curves	
:	:	
-3	$-3 = x^2 + y$	$y = -x^2 - 3$
-2	$-2 = x^2 + y$	$y = -x^2 - 2$
-1	$-1 = x^2 + y$	$y = -x^2 - 1$
0	$0 = x^2 + y$	$y = -x^2$
1	$1 = x^2 + y$	$y = -x^2 + 1$
2	$2 = x^2 + y$	$y = -x^2 + 2$
3	$3 = x^2 + y$	$y = -x^2 + 3$
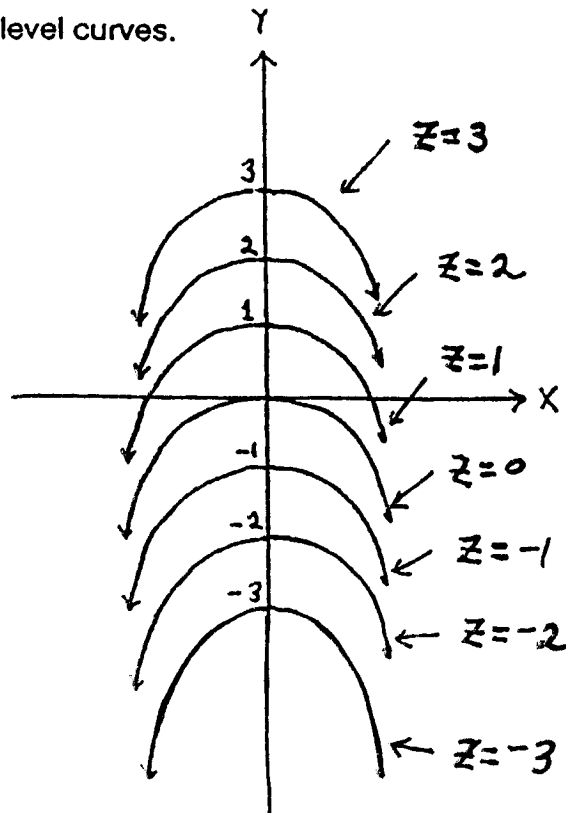
<u>DEFINITION</u> : The intersection of each coordinate plane (xy-plane (z=0), xz-plane (y=0), and yz-plane (x=0)) with a given surface is called a <u>trace</u>.

<u>EXAMPLE</u> : Sketch the traces (on separate coordinate axes) for the surface $z = x^2 + y$.
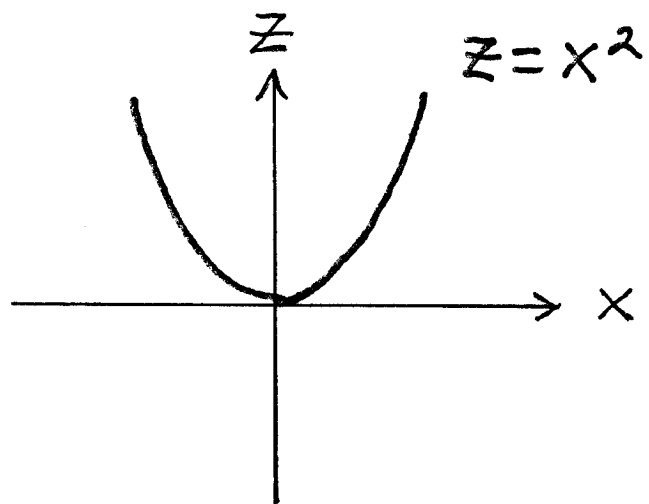
x= 0 : (yz-trace)

$z = y$
(line)

$z = y$

y=0 : (xz-trace)

$z = x^2$
(parabola)

$z = x^2$

z=0 : (xy-trace)

$0 = x^2 + y$

$\rightarrow y = -x^2$

(parabola)

$y = -x^2$