Math 16C
Kouba
P-Series Test Prerequisites

Ex: (from Math 16B)

$$\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \lim_{A\to\infty} \int_1^A x^{-1/2}\,dx$$

$$= \lim_{A\to\infty} 2x^{1/2}\Big|_1^A = \lim_{A\to\infty} (2\sqrt{A} - 2\sqrt{1})$$

$$= \infty - 2 = \infty \qquad \text{(DIVERGES)}$$

Ex: (from Math 16B)

$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{A\to\infty} \int_1^A x^{-2}\,dx$$

$$= \lim_{A\to\infty} -x^{-1}\Big|_1^A = \lim_{A\to\infty} \left(\frac{-1}{A} - \frac{-1}{1}\right)$$

$$= 0 + 1 = 1 \qquad \text{(CONVERGES)}$$

FACTS:

1.) $\int_1^\infty \frac{1}{x^p}\,dx$ converges if $p > 1$.

2.) $\int_1^\infty \frac{1}{x^p}\,dx$ diverges if $p \le 1$.

Math 16C
Kouba
p- Series Test

Def: A series of the form

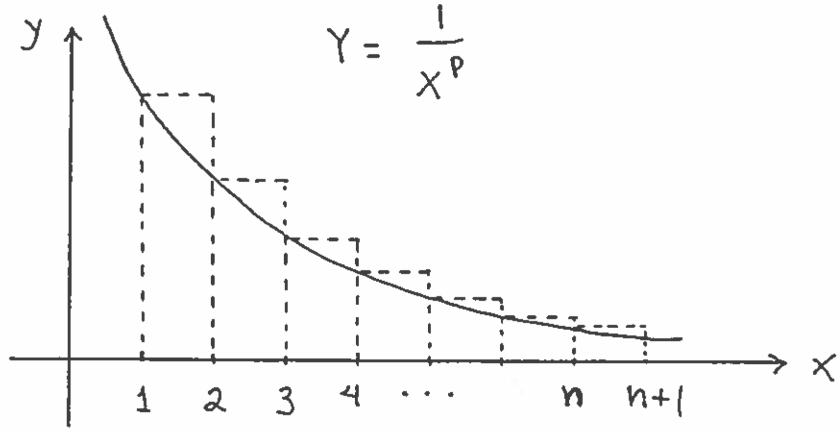
$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots ,$$

where p is a constant, is called a p-series.

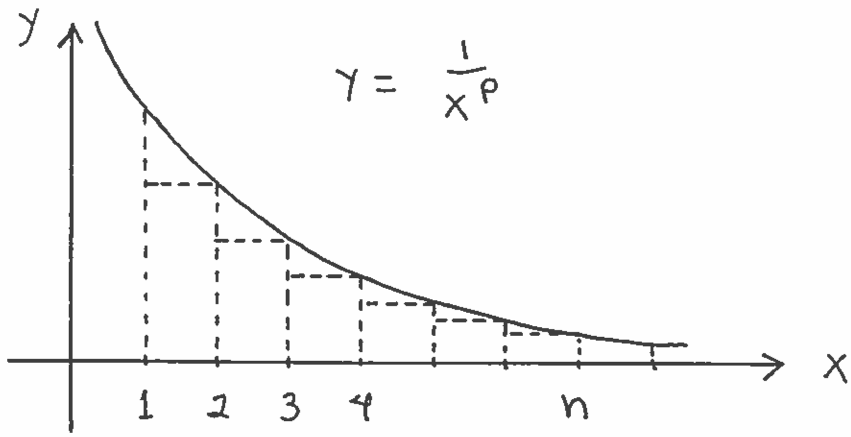
p-Series Test: The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$

i.) converges if $p > 1$.
ii.) diverges if $p \leq 1$.

Math 16C
Kouba
p- series Handout



(A) $\int_1^{n+1} \frac{1}{x^p}\,dx < \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p}$ and



$\frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots \frac{1}{n^p} < \int_1^n \frac{1}{x^p}\,dx$ so that

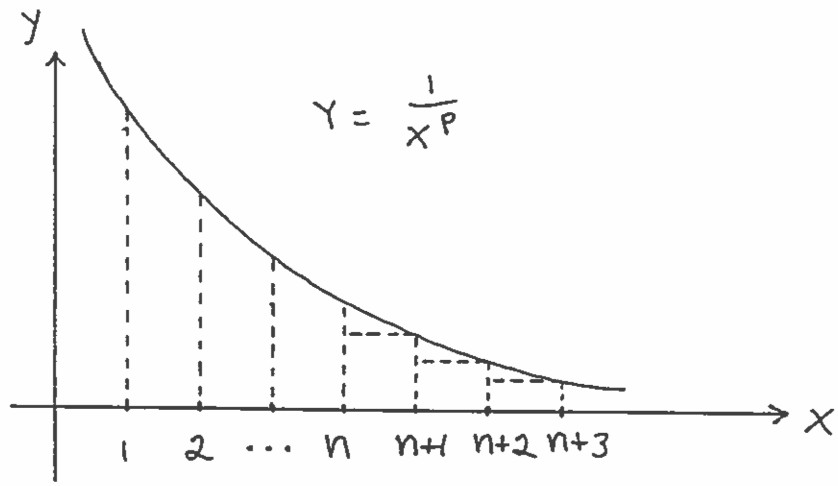
(B) $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} < 1 + \int_1^n \frac{1}{x^p}\,dx$.

Letting $n \to \infty$ in (A) and (B) results in

$$\int_1^\infty \frac{1}{x^p}\,dx < \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots < 1 + \int_1^\infty \frac{1}{x^p}\,dx .$$

The results of the p-series test follow.

<u>Estimating Convergent p-Series</u>:



⊛ $$\underbrace{\frac{1}{(n+1)^p} + \frac{1}{(n+2)^p} + \frac{1}{(n+3)^p} + \cdots}_{\text{Error}} < \int_n^\infty \frac{1}{x^p}\,dx = \frac{n^{1-p}}{p-1} .$$