Math 16C
Kouba
Systems of Equations

Solve each of the following systems of equations. Realize that some of these are nonstandard sytems and require some mathematical common sense and creativity.

1.) $\begin{cases} 3x + 4y = 0 \\ x - 7y = 1 \end{cases}$

2.) $\begin{cases} \beta - \mu = 3 \\ 2\beta + 5\mu = 2 \end{cases}$

3.) $\begin{cases} x + y + z = 6 \\ 2x - y - z = -3 \\ x + 3y + z = 10 \end{cases}$

4.) $\begin{cases} x - z = 3 \\ x - y = -1 \\ 2y + z = 0 \end{cases}$

5.) $\begin{cases} x + y = 3 \\ x - y + 2\beta = -2 \\ 2y + \beta = 4 \end{cases}$

6.) $\begin{cases} x^2 + (y-1)^2 = 1 \\ xy - 2x = 0 \end{cases}$

7.) $\begin{cases} x + y - z = 0 \\ xyz = 0 \\ 2x + y - 3z = 1 \end{cases}$

8.) $\begin{cases} \beta x + \beta y + x = -2 \\ \beta(y-1)x = 0 \\ \beta + x - y = 3 \end{cases}$

9.) $\begin{cases} x + y - z - w = 0 \\ x - y + z + 2w = 1 \\ -x + y - z + 3w = -1 \\ -x + 3y + z + w = 2 \end{cases}$

10.) $\begin{cases} x + y - \beta + \mu = 0 \\ y - z + \beta + \mu = 1 \\ x + 2z - \mu = -1 \\ x + 2y = 3 \\ y = z \end{cases}$

11.) $\begin{cases} 3 = Ce^{2k} \\ 5C = 3e^{k} \end{cases}$

12.) $\begin{cases} x^3 + 3y = 0 \\ 3x + y = 0 \end{cases}$

Systems of Equations

1.) $3X + 4Y = 0 \longrightarrow 3(1+7Y) + 4Y = 0$

$X - 7Y = 1 \rightarrow X = 1 + 7Y$

$3 + 21Y + 4Y = 0 \rightarrow 25Y = -3$

$\rightarrow Y = \frac{-3}{25}$ and $X = 1 + 7\left(\frac{-3}{25}\right) = \frac{4}{25}$

2.) $\left.\begin{array}{l} \beta - \mu = 3 \\ 2\beta + 5\mu = 2 \end{array}\right\}$ $\left.\begin{array}{l} 2\beta - 2\mu = 6 \\ 2\beta + 5\mu = 2 \end{array}\right\}$ $-7\mu = 4 \longrightarrow$

$\mu = \frac{-4}{7}$ and $\beta - \left(\frac{-4}{7}\right) = 3 \rightarrow \beta = 3 - \frac{4}{7} = \frac{17}{7}$

3.) $\left.\begin{array}{l} X + Y + Z = 6 \\ 2X - Y - Z = -3 \\ X + 3Y + Z = 10 \end{array}\right\}$ $\left.\begin{array}{l} -3Y - 3Z = -15 \\ 2Y = 4 \end{array}\right\}$ $Y + Z = 5$

$Y = 2, Z = 3, X = 1$

4.) $\left.\begin{array}{l} X - Z = 3 \\ X - Y = -1 \end{array}\right\}$ $Z - Y = -4 \rightarrow Z = Y - 4$

$2Y + Z = 0 \longrightarrow 2Y + (Y - 4) = 0$

$\rightarrow 3Y = 4 \rightarrow Y = 4/3, \ Z = \frac{-8}{3}, \ X = \frac{1}{3}$

5.) $\left.\begin{array}{l} X + Y = 3 \\ X - Y + 2\beta = -2 \end{array}\right\}$ $\left.\begin{array}{l} -2Y + 2\beta = -5 \\ 2Y + \beta = 4 \end{array}\right\}$ $3\beta = -1 \rightarrow$

$2Y + \beta = 4 \longrightarrow 2Y + \beta = 4$

$\beta = \frac{-1}{3}, \ Y = \frac{13}{6}, \ X = \frac{5}{6}$

6.) $X^2 + (Y-1)^2 = 1$

$XY - 2X = 0 \longrightarrow X(Y-2) = 0 \rightarrow X = 0$ or $Y = 2$;

if $X = 0$ then $(0)^2 + (Y-1)^2 = 1 \longrightarrow Y - 1 = 1$ or -1

$\rightarrow$ $Y = 2$ or 0 so $X=0, Y=0$ and
$X=0, Y=2$ are solutions ;
if $Y=2$ then $X^2+(2-1)^2 = 1 \rightarrow X^2+1=1 \rightarrow$
$X^2=0 \rightarrow X=0$ (this is not a new solution!)

7.) $X+Y-Z=0$
$XYZ=0 \longrightarrow X=0, Y=0,$ or $Z=0$;
$2X+Y-3Z=1$

if $X=0$ then $\left.\begin{array}{l} Y-Z=0 \\ Y-3Z=1 \end{array}\right\} 2Z=-1 \rightarrow Z=\frac{-1}{2}, Y=\frac{-1}{2}$;

if $Y=0$ then $\left.\begin{array}{l} X-Z=0 \\ 2X-3Z=1 \end{array}\right\} -Z=1 \rightarrow Z=-1, X=-1$;

if $Z=0$ then $\left.\begin{array}{l} X+Y=0 \\ 2X+Y=1 \end{array}\right\} X=1, Y=-1$.

8.) $\beta X+\beta Y+X=-2$
$\beta(Y-1)X = 0 \longrightarrow \beta=0, Y=1,$ or $X=0$;
$\beta+X-Y=3$

if $\beta=0$ then $\left.\begin{array}{l} X=-2 \\ X-Y=3 \end{array}\right\} Y=X-3=-5$;

if $Y=1$ then $\left.\begin{array}{l} \beta X+\beta+X=-2 \\ \beta+X-1=3 \end{array}\right\} \begin{array}{l} \beta X+\beta+X=-2 \\ X=4-\beta \end{array}$

$\rightarrow \beta(4-\beta)+\beta+(4-\beta)=-2$
$\rightarrow 4\beta-\beta^2+\beta+4-\beta=-2$
$\rightarrow 0=\beta^2-4\beta-6 \rightarrow \beta=\dfrac{4\pm\sqrt{16-4(1)(-6)}}{2}$

$$= \frac{4 \pm \sqrt{40}}{2} = 2 \pm \sqrt{10} \;;$$

if $\beta = 2+\sqrt{10}$ then $(2+\sqrt{10}) + x - (1) = 3 \rightarrow$
$x = 2-\sqrt{10}$;

if $\beta = 2-\sqrt{10}$ then $(2-\sqrt{10}) + x - (1) = 3 \rightarrow$
$x = 2+\sqrt{10}$;

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if $x=0$ then $\left.\begin{array}{l}\beta\gamma = -2 \\ \beta-\gamma = 3\end{array}\right\}$ $\begin{array}{l}\beta\gamma = -2 \\ \gamma = \beta - 3\end{array}$

$\rightarrow \beta(\beta-3) = -2 \rightarrow \beta^2 - 3\beta + 2 = 0 \rightarrow$
$(\beta-2)(\beta-1) = 0 \rightarrow \beta = 2$ or $\beta = 1$;

if $\beta = 2$ then $(2)+(0)-\gamma = 3 \rightarrow \gamma = -1$;
if $\beta = 1$ then $(1)+(0)-\gamma = 3 \rightarrow \gamma = -2$.

9.) $\left.\begin{array}{l} x+y-z-w=0 \\ x-y+z+2w=1 \\ -x+y-z+3w=-1 \\ -x+3y+z+w=2 \end{array}\right\}$ $\begin{array}{l} \left.\begin{array}{l} 2y-2z-3w=-1 \\ 2y-2z+2w=-1 \end{array}\right\} \rightarrow \\ 4y = 2 \rightarrow y = \frac{1}{2} \end{array}$

$\left.\begin{array}{l} 1-2z-3w=-1 \\ 1-2z+2w=-1 \end{array}\right\} -5w = 0 \rightarrow w = 0 \rightarrow$

$1-2z-3(0) = -1 \rightarrow 2z = 2 \rightarrow z = 1 \rightarrow$
$x + (\frac{1}{2}) - (1) - (0) = 0 \rightarrow x = \frac{1}{2}$

10.) $$\left.\begin{aligned} X+Y-\beta+\mu&=0\\ Y-Z+\beta+\mu&=1\\ X+2Z-\mu&=-1\\ X+2Y&=3\\ Y&=Z \end{aligned}\right\} \quad \left.\begin{aligned} Y-Z+\beta+\mu&=1\\ Y-2Z-\beta+2\mu&=1\\ -Y-\beta+\mu&=-3\\ Y-Z&=0 \end{aligned}\right\}$$

$$\left.\begin{aligned} Z+2\beta-\mu&=0\\ -Z+2\mu&=-2\\ \beta+\mu&=1 \end{aligned}\right\} \quad \left.\begin{aligned} 2\beta+\mu&=-2\\ \beta+\mu&=1 \end{aligned}\right\} \quad \beta=-3 \rightarrow$$

$$\mu=4 \rightarrow Z=10 \rightarrow Y=10 \rightarrow X=-17$$

11.) $$\left.\begin{aligned} 3&=Ce^{2k}\\ 5C&=3e^{k} \end{aligned}\right\} \quad C=3/e^{2k} \rightarrow 5(3/e^{2k})=3e^{k} \rightarrow$$

$$15/e^{2k}=3e^{k} \rightarrow 15=3e^{2k}e^{k} \rightarrow$$

$$5=e^{3k} \rightarrow \ln 5=\ln e^{3k} \rightarrow$$

$$\ln 5=3k \rightarrow k=\frac{1}{3}\ln 5 \quad \text{and}$$

$$C=3e^{-2k}=3e^{-2(\frac{1}{3}\ln 5)}=3e^{-\frac{2}{3}\ln 5}$$

$$=3(e^{\ln 5})^{(-2/3)}=3(5)^{-2/3}.$$

12.) $$\left.\begin{aligned} x^3+3y&=0\\ 3x+y&=0 \end{aligned}\right\} \quad y=-3x \quad x^3+3(-3x)=0 \rightarrow$$

$$x^3-9x=0 \rightarrow x(x-3)(x+3)=0 \rightarrow$$

$x=0$, $y=0$; $x=3$, $y=-9$; $x=-3$, $y=9$