Math 17C
Kouba
Worksheet 7 1/2

1.) Let R be the region bounded by the graphs of $y = x^3$ and $y = 4x$ (in the first quadrant).
   a.) Describe R using vertical cross-sections.
   b.) Describe R using horizontal cross-sections.

2.) Let R be the region inside the circle of radius 2 centered at $(0,0)$ and above the y-axis.
   a.) Describe R using vertical cross-sections.
   b.) Describe R using horizontal cross-sections.

3.) Let R be the triangular region with vertices $(0,0), (-2,3)$, and $(2,3)$.
   a.) Describe R using vertical cross-sections.
   b.) Describe R using horizontal cross-sections.

4.) Let R be the region bounded by the graphs of $x = y^2$ and $x = 2 - y^2$.
   a.) Describe R using vertical cross-sections.
   b.) Describe R using horizontal cross-sections.

5.) Sketch each of the following regions described in two-dimensional space.
   a.) $0 \le x \le 1$, $3x \le y \le x + 2$ b.) $0 \le x \le \ln 4$, $e^x \le y \le 4$
   c.) $1 \le y \le 4$, $-\sqrt{y} \le x \le \sqrt{y}$ d.) $1 \le y \le \ln 5$, $\ln y \le x \le e^y$

6.) Evaluate the following double integrals.

a.) $\displaystyle\int_1^3 \int_0^1 (1 + 4xy)\, dx\, dy$ b.) $\displaystyle\int_0^1 \int_1^2 \frac{xe^x}{y}\, dy\, dx$

c.) $\displaystyle\int_0^2 \int_0^1 (2x + y)^8\, dx\, dy$ d.) $\displaystyle\int_0^1 \int_0^1 \frac{xy}{\sqrt{x^2 + y^2 + 1}}\, dy\, dx$

e.) $\displaystyle\int_0^1 \int_0^{x^2} (x + 2y)\, dy\, dx$ f.) $\displaystyle\int_0^1 \int_y^{e^y} \sqrt{x}\, dx\, dy$

g.) $\displaystyle\int_0^{\pi/2} \int_0^{\cos y} e^{\sin y}\, dx\, dy$ h.) $\displaystyle\int_0^{\pi/4} \int_0^{\pi/6} \cos 3x \sin 2y\, dy\, dx$

(Beware of the next two.)

i.) $\displaystyle\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy$ j.) $\displaystyle\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1}\, dx\, dy$

7.) Find the area of the region

a.) in problem 1. b.) in problem 4.

8.) Consider a mountain range above the grid $0 \le x \le 5,\ 0 \le y \le 8$, where distance is measured in miles. Elevation (miles) above sea level at the point (x, y) is given by $H(x, y) = (1/40)(10 - x^2 + y^2)$.

a.) Find the elevation at the points $(0, 0), (4, 2)$, and $(0, 8)$.

b.) Compute the average elevation of this mountain range.

9.) A flat plate lies in region R bounded by the graphs of $y = \sqrt{x}$ and $y = (1/2)x$. Temperature at point (x, y) is given by $T(x, y) = 50 + 2x + y\ (^oF)$.

a.) Find the area of the plate.

b.) Find the average width of the plate.

c.) Find the average height of the plate.

d.) Find the temperature at the points $(0, 0)$, $(2, 1.1)$, and $(4, 2)$.

e.) Find the average temperature of the plate.

10.) Sketch the solid in 3D-Space whose volume is given by the following double integral.

$$\int_0^1 \int_0^{y^{1/3}} (4 - x^2 - y^2)\, dx\, dy$$

11.) A flat plate lies in region R bounded by the graphs of $x = 0$, $y = x^3$, and $y = x^2 + 4$. Density at point (x, y) is given by $\delta(x, y) = 1 + x + 2y$ grams per square centimeter.

a.) Find the area of the plate.

b.) Find the average width of the plate.

c.) Find the average height of the plate.

d.) Find the density at the points $(0, 0)$, $(1, 3)$, and $(2, 8)$.

e.) Find the average density of the plate.

f.) Find the total mass of the plate.

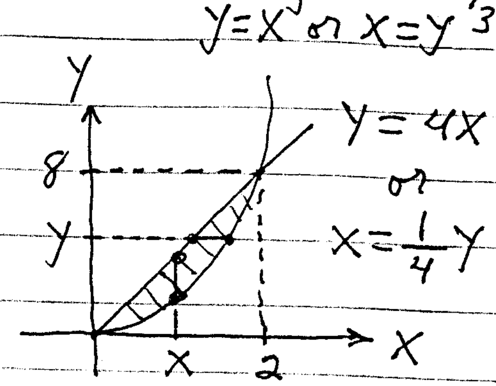
12.) Compute the volume of the solid lying above the region bounded by the graphs of $y = 2x$, $y = 2$, and $x = 0$ and below the paraboloid $z = x^2 + y^2$.

13.) Compute the volume of the solid which is above the region bounded by the graphs of $y = x^2$, $y = 0$, and $x = 2$ and between the planes $x + y + z = 6$ and $x - y + z = 12$.

Worksheet 7 1/2

1.) a.) $0 \le x \le 2$, $x^3 \le y \le 4x$

b.) $0 \le y \le 8$, $\frac{1}{4}y \le x \le y^{1/3}$



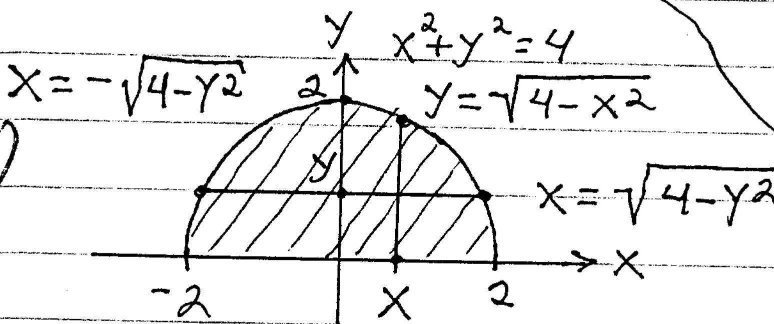
$x^3 = 4x \longrightarrow$

$x^3 - 4x = 0 \longrightarrow$

$x(x-2)(x+2) = 0 \longrightarrow$

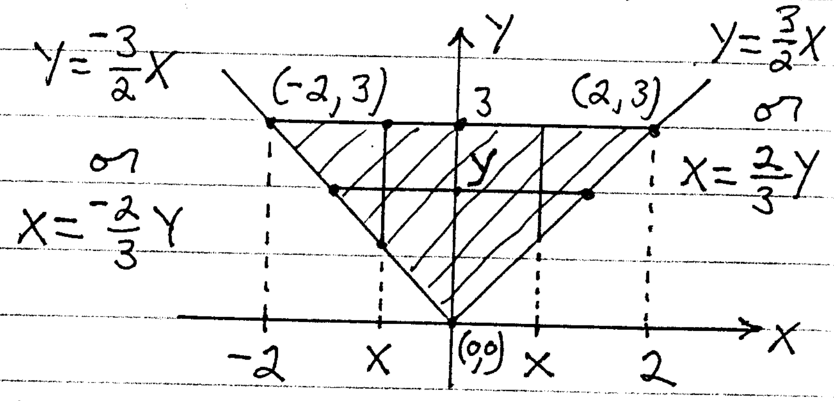
$x = 0,\ x = 2,\ x = -2$

2.)



a.) $-2 \le x \le 2$, $0 \le y \le \sqrt{4-x^2}$ b.) $0 \le y \le 2$, $-\sqrt{4-y^2} \le x \le \sqrt{4-y^2}$

3.) a.) $-2 \le x \le 0$, $-\frac{3}{2}x \le y \le 3$ and $0 \le x \le 2$, $\frac{3}{2}x \le y \le 3$

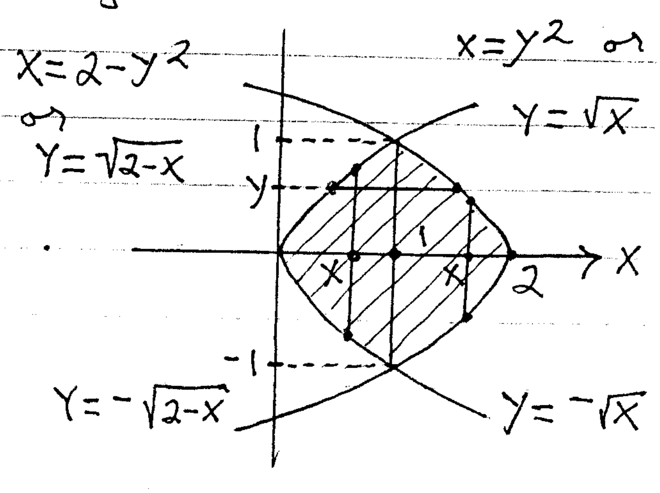


b.) $0 \le y \le 3$, $-\frac{2}{3}y \le x \le \frac{2}{3}y$

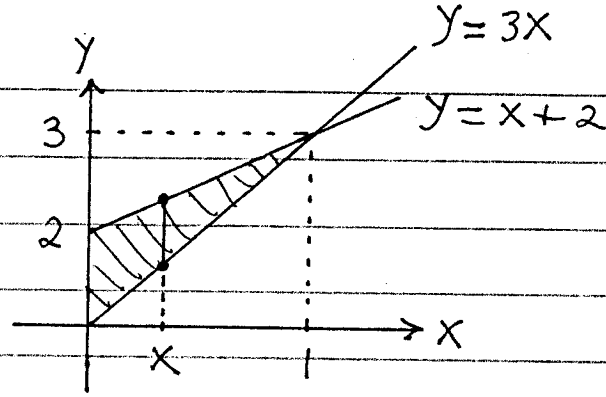
4.) $y^2 = 2 - y^2 \longrightarrow 2y^2 = 2$
$\longrightarrow y^2 = 1 \longrightarrow y = \pm 1$;

a.) $0 \le x \le 1$, $-\sqrt{x} \le y \le +\sqrt{x}$;
$1 \le x \le 2$, $-\sqrt{2-x} \le y \le \sqrt{2-x}$

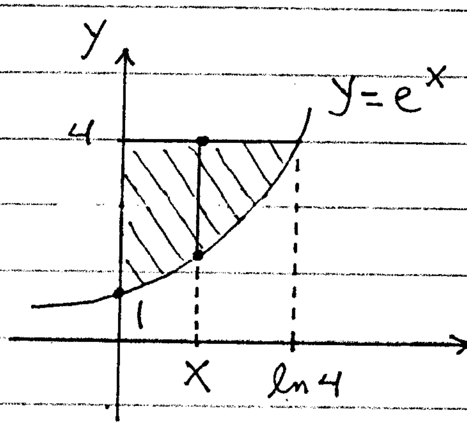
b.) $-1 \le y \le 1$, $y^2 \le x \le 2 - y^2$



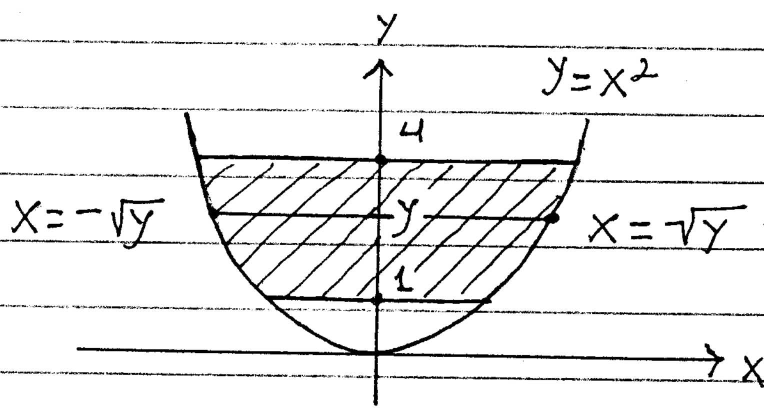
5.) a.)



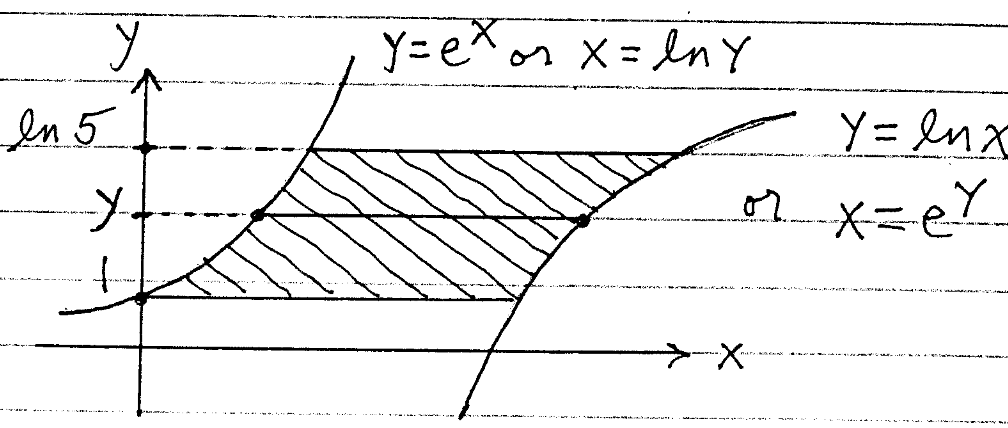
b.)



c.)



d.)



6.) a.) $\int_1^3 \int_0^1 (1+4xy)\,dx\,dy$

$= \int_1^3 (x + 2x^2 y)\Big|_{x=0}^{x=1}\,dy$

$$= \int_1^3 \left((1+2y) - (0+0)\right) dy = \int_1^3 (1+2y)\, dy$$

$$= (y + y^2)\Big|_1^3 = (3+9) - (1+1) = 10$$

b.) $$\int_0^1 \int_1^2 \frac{xe^x}{y}\, dy\, dx = \int_0^1 \left(xe^x \ln y \Big|_{y=1}^{y=2}\right) dx$$

$$= \int_0^1 xe^x (\ln 2 - \cancel{\ln 1}^{\,0})\, dx$$ (Let $u = x$, $dv = e^x dx$ $\rightarrow du = dx$, $v = e^x$)

$$= \ln 2 \left[xe^x \Big|_0^1 - \int_0^1 e^x\, dx \right]$$

$$= \ln 2 \left[e - 0 - e^x \Big|_0^1 \right]$$

$$= \ln 2 \left[e - (e - \cancel{e^0}^{\,1}) \right] = \ln 2$$

c.) $$\int_0^2 \int_0^1 (2x+y)^8\, dx\, dy = \int_0^2 \frac{1}{9} \cdot \frac{1}{2} (2x+y)^9 \Big|_{x=0}^{x=1} dy$$

$$= \int_0^2 \left[\frac{1}{18}(2+y)^9 - \frac{1}{18} y^9 \right] dy$$

$$= \left(\frac{1}{18} \cdot \frac{1}{10} (2+y)^{10} - \frac{1}{18} \cdot \frac{1}{10} y^{10} \right) \Big|_0^2$$

$$= \left(\frac{1}{180} 4^{10} - \frac{1}{180} 2^{10} \right) - \left(\frac{1}{180} 2^{10} - 0 \right)$$

$$= \frac{1}{180} 4^{10} - \frac{2}{180} \cdot 2^{10} = \frac{1}{180} (1046528) = \frac{261,632}{45}$$

d.) $$\int_0^1 \int_0^1 xy (x^2 + y^2 + 1)^{-1/2}\, dy\, dx$$

$$= \int_0^1 x \cdot \frac{1}{\cancel{2}} \cdot \cancel{2} (x^2 + y^2 + 1)^{1/2} \Big|_{y=0}^{y=1} dx$$

$$= \int_0^1 \left(x(x^2+2)^{1/2} - x(x^2+1)^{1/2} \right) dx$$

$$= \left(\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3}(x^2+2)^{3/2} - \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3}(x^2+1)^{3/2} \right) \Big|_0^1$$

$$= \left(\frac{1}{3} 3^{3/2} - \frac{1}{3} 2^{3/2} \right) - \left(\frac{1}{3} 2^{3/2} - \frac{1}{3} 1^{3/2} \right)$$

$$= 3^{1/2} - \frac{2}{3} 2^{3/2} + \frac{1}{3} = 3^{1/2} - \frac{1}{3} 2^{5/2} + \frac{1}{3}$$

e.) $$\int_0^1 \int_0^{x^2} (x+2y)\, dy\, dx = \int_0^1 (xy+y^2) \Big|_{y=0}^{y=x^2} dx$$

$$= \int_0^1 \left((x^3+x^4) - (0+0) \right) dx = \int_0^1 (x^3+x^4)\, dx$$

$$= \left(\frac{1}{4}x^4 + \frac{1}{5}x^5 \right) \Big|_0^1 = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}$$

f.) $$\int_0^1 \int_y^{e^y} \sqrt{x}\, dx\, dy = \int_0^1 \left(\frac{2}{3} x^{3/2} \Big|_{x=y}^{x=e^y} \right) dy$$

$$= \int_0^1 \left(\frac{2}{3} e^{\frac{3}{2}y} - \frac{2}{3} y^{3/2} \right) dy$$

$$= \left(\frac{2}{3} \cdot \frac{2}{3} e^{\frac{3}{2}y} - \frac{2}{3} \cdot \frac{2}{5} y^{5/2} \right) \Big|_0^1$$

$$= \left(\frac{4}{9} e^{3/2} - \frac{4}{15} \right) - \left(\frac{4}{9} - 0 \right) = \frac{4}{9} e^{3/2} - \frac{32}{45}$$

g.) $$\int_0^{\frac{\pi}{2}} \int_0^{\cos y} e^{\sin y}\, dx\, dy = \int_0^{\frac{\pi}{2}} \left(x e^{\sin y} \Big|_{x=0}^{x=\cos y} \right) dy$$

$$= \int_0^{\frac{\pi}{2}} \cos y\, e^{\sin y}\, dy = e^{\sin y} \Big|_0^{\frac{\pi}{2}}$$

$$= e^{\sin \frac{\pi}{2}} - e^{\sin 0} = e^1 - e^0 = e - 1$$

h.) $$\int_0^{\pi/4} \int_0^{\frac{\pi}{6}} \cos 3x \sin 2y\, dy\, dx = \int_0^{\frac{\pi}{4}} \left(\cos 3x \cdot \frac{-1}{2} \cos 2y \right) \Big|_{y=0}^{y=\frac{\pi}{6}} dx$$

$$= \int_0^{\pi/4} -\frac{1}{2}\cos 3x \cdot \left(\overset{1/2}{\cos\frac{\pi}{3}} - \overset{1}{\cos 0}\right) dx$$

$$= \int_0^{\pi/4} \frac{1}{4}\cos 3x \, dx = \frac{1}{4}\cdot\frac{1}{3}\sin 3x \Big|_0^{\pi/4}$$

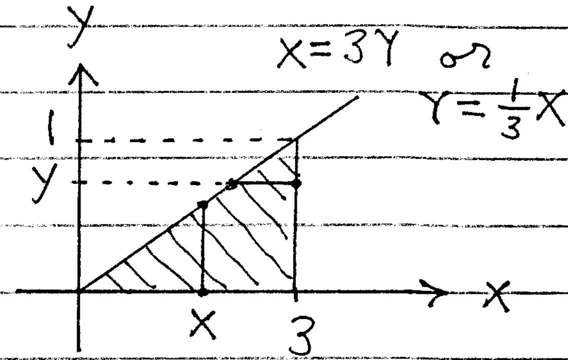
$$= \frac{1}{12}\sin\frac{3}{4}\pi - \frac{1}{12}\overset{0}{\sin 0}$$

$$= \frac{1}{12}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{24}$$

i.) $\int_0^1 \int_{3y}^3 e^{x^2}\,dx\,dy \rightarrow$

$(0 \le y \le 1,\ 3y \le x \le 3$

or $0 \le x \le 3,\ 0 \le y \le \frac{1}{3}x\ )$



(SWITCH ORDER of INTEGRATION)

$$= \int_0^3 \int_0^{\frac{1}{3}x} e^{x^2}\,dy\,dx = \int_0^3 \left(e^{x^2}\cdot y \Big|_{y=0}^{y=\frac{1}{3}x} \right) dx$$

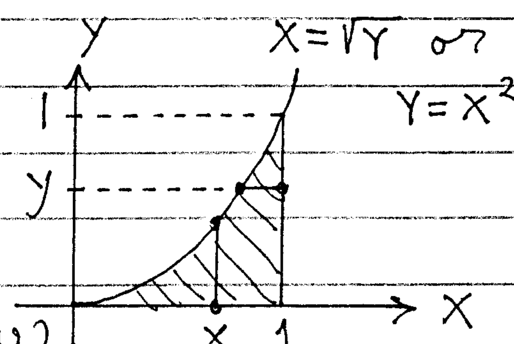
$$= \int_0^3 \frac{1}{3}x e^{x^2}\,dx = \frac{1}{3}\cdot\frac{1}{2}e^{x^2}\Big|_0^3$$

$$= \frac{1}{6}e^9 - \frac{1}{6}e^0 = \frac{1}{6}e^9 - \frac{1}{6}$$

j.) $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dx\,dy$

$(0 \le y \le 1,\ \sqrt{y} \le x \le 1$

or $0 \le x \le 1,\ 0 \le y \le x^2)$



(SWITCH ORDER of INTEGRATION)

$$= \int_0^1 \int_0^{x^2} \sqrt{x^3+1}\,dy\,dx = \int_0^1 \left(\sqrt{x^3+1}\cdot y \Big|_{y=0}^{y=x^2} \right) dx$$

$$= \int_0^1 x^2\sqrt{x^3+1}\,dx = \frac{1}{3}\cdot\frac{2}{3}(x^3+1)^{3/2}\Big|_0^1$$

$$= \frac{2}{9}(2)^{3/2} - \frac{2}{9}(1)^{3/2} = \frac{1}{9}2^{5/2} - \frac{2}{9}$$

7.) a.) Area $= \int_0^2 \int_{x^3}^{4x} 1 \, dy \, dx$

$= \int_0^2 \left(y \Big|_{y=x^3}^{y=4x} \right) dx = \int_0^2 (4x - x^3)\, dx$

$= \left(2x^2 - \frac{1}{4}x^4\right)\Big|_0^2 = (8-4)-(0-0) = 4$

b.) Area $= \int_{-1}^{1} \int_{y^2}^{2-y^2} 1 \, dx \, dy$

$= \int_{-1}^{1} \left(x \Big|_{y^2}^{2-y^2} \right) dy = \int_{-1}^{1} \left((2-y^2) - y^2\right) dy$

$= \int_{-1}^{1} (2-2y^2)\, dy = \left(2y - \frac{2}{3}y^3\right)\Big|_{-1}^{1}$

$= \left(2 - \frac{2}{3}\right) - \left(-2 + \frac{2}{3}\right) = 4 - \frac{4}{3}$

$= \frac{12}{3} - \frac{4}{3} = \frac{8}{3}$

8.) $H(x,y) = \frac{1}{40}(10 - x^2 + y^2)$

a.) $H(0,0) = \frac{1}{40}(10 - 0 + 0) = \frac{1}{4}$ mi.,

~~$H(4,2) = \frac{1}{40}(10 - 16 + 4) = \frac{-2}{40} = \frac{-1}{20}$ mi.,~~

$H(0,8) = \frac{1}{40}(10 - 0 + 64) = \frac{74}{40} = 1.85$ mi.

b.) $R: 0 \le x \le 5,\ 0 \le y \le 8$ so

Area of $R = 40$, then

$AVE = \frac{1}{\text{area}R} \int_0^5 \int_0^8 \frac{1}{40}(10 - x^2 + y^2)\, dy\, dx$

$= \frac{1}{40} \cdot \frac{1}{40} \int_0^5 \int_0^8 (10 - x^2 + y^2)\, dy\, dx$

$$= \frac{1}{1600}\int_0^5 \left(10y - x^2y + \frac{1}{3}y^3\right)\Big|_{y=0}^{y=8} dx$$

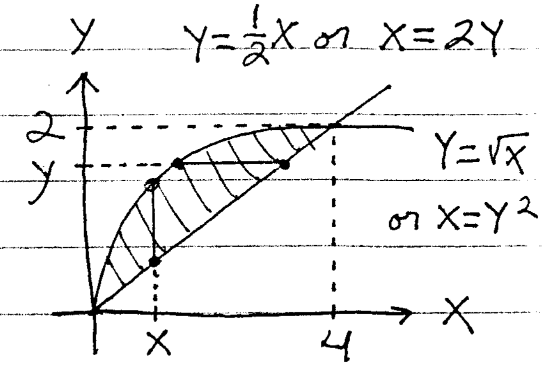
$$= \frac{1}{1600}\int_0^5 \left(80 - 8x^2 + \frac{1}{3}512\right) dx$$

$$= \frac{1}{1600}\left(80x - \frac{8}{3}x^3 + \frac{512}{3}x\right)\Big|_0^5$$

$$= \frac{1}{1600}\left(400 - \frac{1000}{3} + \frac{2560}{3}\right)$$

$$= \frac{1}{1600}\left(\frac{1200}{3} + \frac{1560}{3}\right) = \frac{1}{1600}\cdot\frac{2760}{3}$$

$$= \frac{2760}{4800} = 0.575 \text{ mi.}$$



9.) a.) Area $= \int_0^4 \int_{\frac{1}{2}x}^{\sqrt{x}} 1 \, dy \, dx$

$$= \int_0^4 \left(y\Big|_{y=\frac{1}{2}x}^{y=\sqrt{x}}\right) dx$$

$$= \int_0^4 \left(\sqrt{x} - \frac{1}{2}x\right) dx$$

$$= \left(\frac{2}{3}x^{3/2} - \frac{1}{2}\cdot\frac{1}{2}x^2\right)\Big|_0^4$$

$$= \frac{2}{3}(4)^{3/2} - 4 = \frac{16}{3} - \frac{12}{3} = \frac{4}{3}$$

$2y = y^2 \rightarrow$

$y^2 - 2y = y(y-2) = 0 \rightarrow$

$y = 0,\ y = 2$

b.) width $w(y) = 2y - y^2$, $0 \le y \le 2$;

$$AVE = \frac{1}{2-0}\int_0^2 (2y - y^2)\, dy$$

$$= \frac{1}{2}\left(y^2 - \frac{1}{3}y^3\right)\Big|_0^2 = \frac{1}{2}\left(4 - \frac{8}{3}\right) = \frac{1}{2}\cdot\frac{4}{3} = \frac{2}{3}$$

c.) height $h(x) = \sqrt{x} - \frac{1}{2}x$, $0 \le x \le 4$;

$$AVE = \frac{1}{4-0}\int_0^4 (\sqrt{x} - \frac{1}{2}x)\,dx$$

$$= \frac{1}{4}\left(\frac{2}{3}x^{3/2} - \frac{1}{2}\cdot\frac{1}{2}x^2\right)\Big|_0^4$$

$$= \frac{1}{6}(4)^{3/2} - \frac{1}{16}(16) = \frac{8}{6} - 1$$

$$= \frac{4}{3} - \frac{3}{3} = \frac{1}{3}$$

d.) $T(x,y) = 50 + 2x + y \longrightarrow$

$T(0,0) = 50\ ^\circ F$,

$T(2, 1.1) = 50 + 4 + 1.1 = 55.1\ ^\circ F$,

$T(4,2) = 50 + 8 + 2 = 60\ ^\circ F$

e.) $$AVE = \frac{1}{\text{area } R}\int_0^4 \int_{\frac{1}{2}x}^{\sqrt{x}} (50 + 2x + y)\,dy\,dx$$

$$= \frac{1}{4/3}\int_0^4 \left(50y + 2xy + \frac{1}{2}y^2\right)\Big|_{y=\frac{1}{2}x}^{y=\sqrt{x}}\,dx$$

$$= \frac{3}{4}\int_0^4 \left[\left(50\sqrt{x} + 2x^{3/2} + \frac{1}{2}x\right) - \left(25x + x^2 + \frac{1}{8}x^2\right)\right]dx$$

$$= \frac{3}{4}\int_0^4 \left[50\sqrt{x} + 2x^{3/2} - \frac{49}{2}x - \frac{9}{8}x^2\right]dx$$

$$= \frac{3}{4}\left(50\cdot\frac{2}{3}x^{3/2} + 2\cdot\frac{2}{5}x^{5/2} - \frac{49}{4}x^2 - \frac{3}{8}x^3\right)\Big|_0^4$$

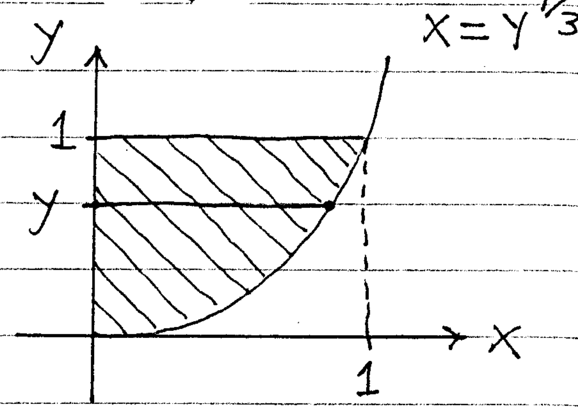
$$= 25(4)^{3/2} + \frac{3}{5}(4)^{5/2} - \frac{147}{16}(16) - \frac{9}{32}(64)$$

$$= 25(8) + \frac{3}{5}(32) - 147 - 18$$

$$= 35 + \frac{96}{5} = \frac{175}{5} + \frac{96}{5} = \frac{271}{5} = 54.2\ ^\circ F$$

10.) $\int_0^1 \int_0^{y^{1/3}} (4 - x^2 - y^2)\, dx\, dy$:

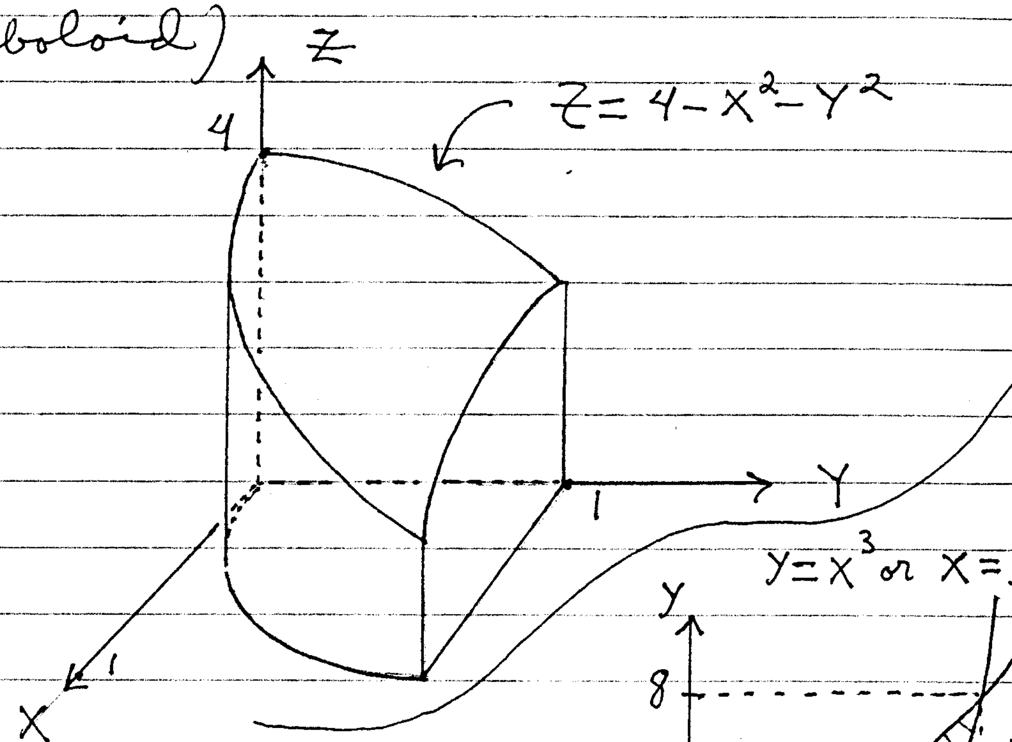
$$\begin{cases} 0 \le y \le 1 \\ 0 \le x \le y^{1/3} \end{cases}$$

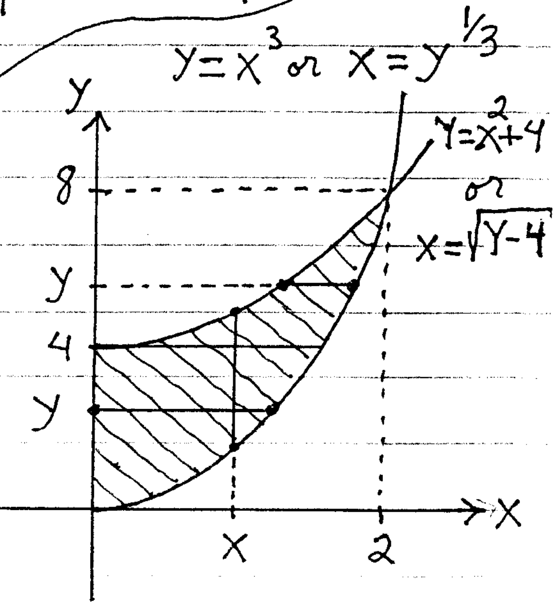


surface

$z = 4 - x^2 - y^2$

(paraboloid)





11.) a.) Area $= \int_0^2 ((x^2+4) - x^3)\, dx$

$= \left(\frac{1}{3}x^3 + 4x - \frac{1}{4}x^4\right)\Big|_0^2$

$= \frac{8}{3} + 8 - 4 = \frac{8}{3} + \frac{12}{3} = \frac{20}{3}$

b.) $w(y) = \begin{cases} y^{1/3}, & 0 \le y \le 4 \\ y^{1/3} - \sqrt{y-4}, & 4 \le y \le 8 \end{cases}$;

$$AVE = \frac{1}{8-0} \int_0^8 w(y)\,dy$$

$$= \frac{1}{8}\left[\int_0^4 w(y)\,dy + \int_4^8 w(y)\,dy\right]$$

$$= \frac{1}{8}\left[\int_0^4 y^{1/3}\,dy + \int_4^8 (y^{1/3} - \sqrt{y-4})\,dy\right]$$

$$= \frac{1}{8}\left[\frac{3}{4}y^{4/3}\Big|_0^4 + \left(\frac{3}{4}y^{4/3} - \frac{2}{3}(y-4)^{3/2}\right)\Big|_4^8\right]$$

$$= \frac{1}{8}\left[\frac{3}{4}(4)^{4/3} + \left(\frac{3}{4}(8)^{4/3} - \frac{2}{3}(4)^{3/2}\right) - \left(\frac{3}{4}(4)^{4/3} - 0\right)\right]$$

$$= \cancel{\frac{3}{32}(4)^{4/3}} + \frac{3}{32}(16) - \frac{1}{12}(4)^{3/2} - \cancel{\frac{3}{32}(4)^{4/3}}$$

$$= \frac{3}{2} - \frac{1}{12}(4)^{3/2} \approx 0.833$$

c.) $h(x) = (x^2+4) - x^3$ for $0 \le x \le 2$;

$$AVE = \frac{1}{2-0}\int_0^2 h(x)\,dx = \frac{1}{2}\int_0^2 (x^2+4-x^3)\,dx$$

$$= \frac{1}{2}\left(\frac{1}{3}x^3 + 4x - \frac{1}{4}x^4\right)\Big|_0^2 = \frac{1}{2}\left(\frac{8}{3} + 8 - 4\right)$$

$$= \frac{1}{2}\left(\frac{8}{3} + \frac{12}{3}\right) = \frac{10}{3}$$

d.) $\delta(x,y) = 1 + x + 2y$:

$\delta(0,0) = 1 + 0 + 2(0) = 1$ gm./cm^2 ,

$\delta(1,3) = 1+1+6 = 8$ gm./cm.^{2}

$\delta(2,8) = 1+2+16 = 19$ gm/cm.^{2},

e.) $AVE = \frac{1}{\text{area } R} \int_0^2 \int_{x^3}^{x^2+4} (1+x+2y)\,dy\,dx$

$= \frac{1}{\frac{20}{3}} \int_0^2 (y+xy+y^2) \Big|_{y=x^3}^{y=x^2+4} dx$

$= \frac{3}{20} \int_0^2 \Big[(x^2+4) + x(x^2+4) + (x^2+4)^2$
$- (x^3+x^4+x^6)\Big] dx$

$= \frac{3}{20} \int_0^2 \Big[x^2+4+\cancel{x^3}+4x+\cancel{x^4}+8x^2+16$
$-\cancel{x^3}-\cancel{x^4}-x^6 \Big] dx$

$= \frac{3}{20} \int_0^2 [9x^2+4x+20-x^6]\,dx$

$= \frac{3}{20}\left(3x^3+2x^2+20x-\frac{1}{7}x^7\right)\Big|_0^2$

$= \frac{3}{20}\left(24+8+40-\frac{128}{7}\right)$

$= \frac{3}{20}\left(72-\frac{128}{7}\right) = \frac{3}{20}\cdot\frac{376}{7} = \frac{3}{5}\cdot\frac{94}{7} = \frac{282}{35}$ $\frac{\text{gm.}}{\text{cm.}^2}$

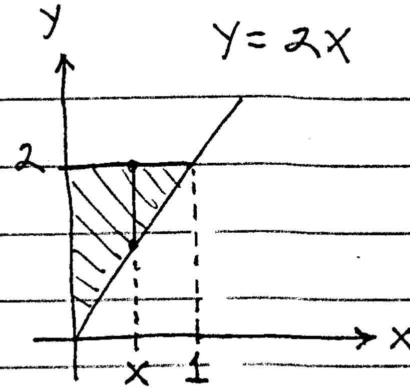
≈ 8.057 gm./cm.^{2}

f.) $MASS = \int_0^2 \int_{x^3}^{x^2+4} \delta(x,y)\,dy\,dx$

$= \int_0^2 \int_{x^3}^{x^2+4} (1+x+2y)\,dy\,dx = \frac{376}{7}$ gm.

≈ 53.714 gm.

12.) $\text{Vol} = \int_0^1 \int_{2x}^{2} (x^2+y^2)\,dy\,dx$



$= \int_0^1 \left(x^2 y + \frac{1}{3}y^3\right)\Big|_{y=2x}^{y=2} dx$

$= \int_0^1 \left[\left(2x^2+\frac{8}{3}\right) - \left(2x^3+\frac{8}{3}x^3\right)\right] dx$

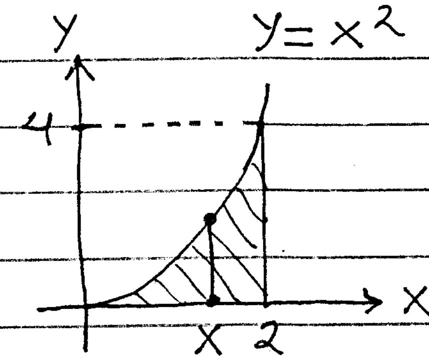
$= \int_0^1 \left(2x^2 + \frac{8}{3} - \frac{14}{3}x^3\right) dx$

$= \left(\frac{2}{3}x^3 + \frac{8}{3}x - \frac{14}{3}\cdot\frac{1}{4}x^4\right)\Big|_0^1$

$= \frac{2}{3} + \frac{8}{3} - \frac{7}{6} = \frac{4}{6} + \frac{16}{6} - \frac{7}{6} = \frac{13}{6}$

13.) $\text{Vol} = \int_0^2 \int_0^{x^2} (12 - x + y)\,dy\,dx$

$- \int_0^2 \int_0^{x^2} (6 - x - y)\,dy\,dx$



$= \int_0^2 \int_0^{x^2} \left[(12-x+y) - (6-x-y)\right] dy\,dx$

$= \int_0^2 \int_0^{x^2} \left[6 + 2y\right] dy\,dx$

$= \int_0^2 \left(6y + y^2\right)\Big|_{y=0}^{y=x^2} dx$

$= \int_0^2 \left(6x^2 + x^4\right) dx$

$= \left(2x^3 + \frac{1}{5}x^5\right)\Big|_0^2$

$= 16 + \frac{1}{5}(32)$

$= \frac{80}{5} + \frac{32}{5} = \frac{112}{5}$