

Math 16C
Kouba
Exam 1

Print your name here.

Your HW #

1. Please do not turn this page until told to do so.
2. No notes, books, or classmates may be used as resources for this exam. You may *NOT* use a calculator.
3. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will *not* receive full credit. *What* you write down and *how* you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.
4. Stay calm and put forth your best effort on this exam.
5. Don't be overly alarmed by problems that you cannot immediately solve. Just maintain your composure and work at a steady rate.
6. Make sure that you have eight (8) pages, including the cover page.
7. You have until 8 : 50 o'clock sharp to finish the exam.
8. **WHEN SOLVING FIRST-ORDER LINEAR DIFFERENTIAL EQUATIONS, MAKE SURE THAT YOU SHOW ALL NECESSARY STEPS. DO NOT USE SHORTCUTS FROM THE BOOK.**

1.) (9 pts. each) Solve the following differential equations. You need not solve explicitly for y .

a.) $y'' = \sec^2 x + 20x^3$

b.) $y' = (\cos 3x + \sqrt{x}) e^{4y}$

c.) $(y + 2)^5 \csc x \, dy = x \, dx$

d.) $y' + 2xy = x^3 e^{-x^2}$

e.) $x y' - y = x (\ln x)^3$

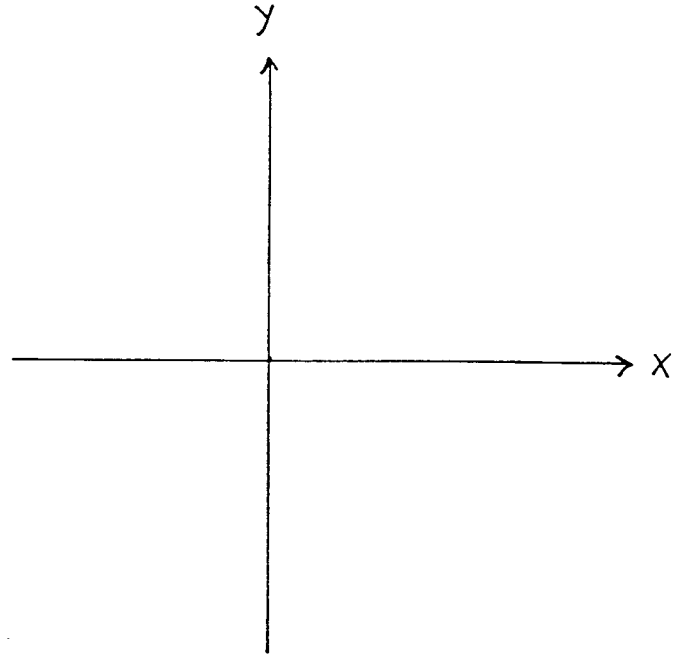
2.) (10 pts.) Show that the equation $x^2 - y^3 = C$ solves the differential equation

$$6y^3 - 9y^5 y'' - 8x^2 = 0 .$$

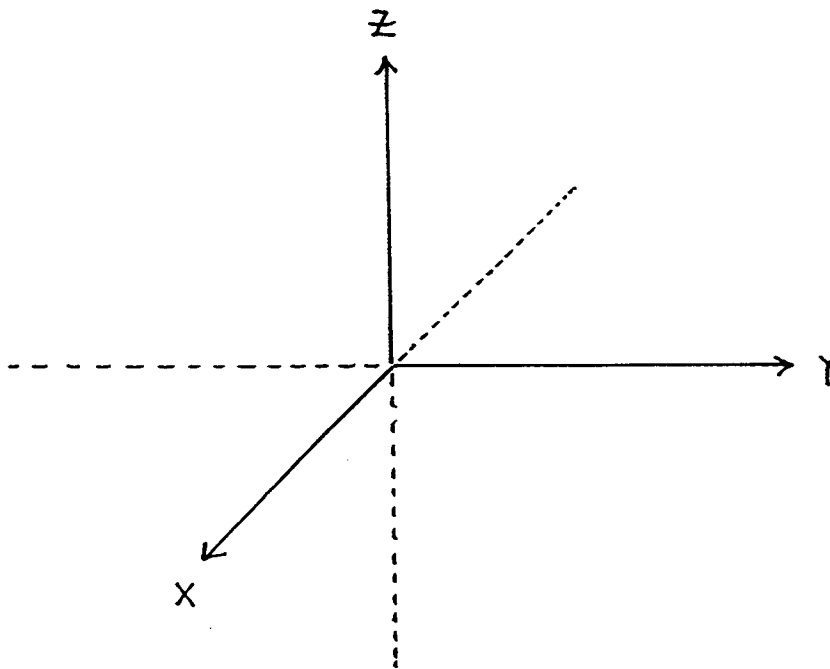
3.) (8 pts.) Determine an equation for the sphere which has a diameter with endpoints $(0, 0, 0)$ and $(1, 2, -2)$.

4.) Consider the equation $z = \sqrt{x^2 + y^2 - 1}$.

a.) (7 pts.) Neatly sketch the level curves in the xy -plane using the following values of z : $0, \sqrt{3}, \sqrt{8}, \sqrt{15}$. Clearly label level curves and intercepts.



b.) (3 pts.) Do your best to sketch a portion of the related surface in 3-space.



5.) (12 pts.) The rate at which the number of caribou A in a population changes at time t (in years) is proportional to the square root of $\{ A + 100 \}$. Initially, there are 44 caribou. After three (3) years there are 125 caribou. How many caribou do you expect there to be after four (4) years?

6.) Pure water (containing no salt) begins to flow into a tank at the rate of 3 gallons per minute, and the well-stirred mixture flows out of the tank at the rate of 4 gallons per minute. The tank initially holds 200 gallons of water containing 100 pounds of salt.

Let S represent the number of pounds of salt in the tank at time t minutes.

a.) (5 pts.) Set up a differential equation which describes the rate at which the amount of salt in the tank changes at time t .

b.) (10 pts.) Solve the differential equation in part a.). Determine all unknown constants and solve explicitly for S .

EXTRA CREDIT PROBLEM -- Each of the following problems is worth 10 extra credit points and is *optional* .

1.) Determine an equation of the plane which passes through the points $(0, 0, 1)$, $(0, 3, 0)$, $(7, 0, 0)$.

2.) Assume that the amount of salt S (in grams) in a tank at time t (in days) is given by the formula

$$S = (1/32) (t-5)^2 + 50 + \ln(t+1) \text{ for } t \geq 0 .$$

At what time t is the amount of salt in the tank changing most rapidly ?