Math 16C

Exam 1 Solutions

1.) a.) $Y'' = \sec^2 x + 20x^3$

$\rightarrow Y' = \tan x + 5x^4 + c_1 \rightarrow \boxed{Y = \ln|\sec x| + x^5 + c_1 x + c_2}$

b.) $Y' = (\cos 3x + \sqrt{x})e^{4Y} \rightarrow$

$\int e^{-4Y}\,dy = \int (\cos 3x + x^{1/2})\,dx \rightarrow$

$$\boxed{-\frac{1}{4}e^{-4Y} = \frac{1}{3}\sin 3x + \frac{2}{3}x^{3/2} + c}$$

c.) $(Y+2)^5 \csc x\,dy = x\,dx \rightarrow$

$\int (Y+2)^5\,dy = \int \frac{x}{\csc x}\,dx \rightarrow$

$\int (Y+2)^5\,dy = \int x\sin x\,dx$ (Let $u = x$, $dv = \sin x\,dx$
$du = dx$, $v = -\cos x$)

$\rightarrow \frac{1}{6}(Y+2)^6 = -x\cos x + \int \cos x\,dx \rightarrow$

$$\boxed{\frac{1}{6}(Y+2)^6 = -x\cos x + \sin x + c}$$

d.) $Y' + (2x)Y = x^3 e^{-x^2}$ then

$\mu = e^{\int 2x\,dx} = e^{x^2 + \cancel{c}^{\,0}} = e^{x^2}$ so

$e^{x^2}Y' + 2xe^{x^2}Y = x^3 e^{x^2}e^{-x^2} = x^3 e^0 = x^3 \cdot 1 = x^3 \rightarrow$

$D(e^{x^2}Y) = x^3 \rightarrow \boxed{e^{x^2}Y = \frac{1}{4}x^4 + c}$

e.) $Y' + \left(\frac{-1}{x}\right)Y = (\ln x)^3$ then

$$\mu = e^{\int -\frac{1}{x}\,dx} = e^{-\ln x + \cancel{c}^{0}} = e^{\ln x^{-1}} = x^{-1} = \frac{1}{x}$$ so

$$\frac{1}{x}Y' - \frac{1}{x^2}Y = \frac{(\ln x)^3}{x} \longrightarrow$$

$$D\left(\frac{1}{x}Y\right) = \frac{(\ln x)^3}{x} \longrightarrow \frac{1}{x}Y = \int \frac{(\ln x)^3}{x}\,dx$$

(Let $u = \ln x \to du = \frac{1}{x}dx \to \ldots$) $\longrightarrow$

$$\boxed{\frac{1}{x}Y = \frac{(\ln x)^4}{4} + c}$$

2.) $$X^2 - Y^3 = c \xrightarrow{\text{diff.}} 2X - 3Y^2Y' = 0 \to Y' = \frac{2X}{3Y^2}$$

diff.

$$\to 2 - [3Y^2Y'' + 6YY' \cdot Y'] = 0$$

substitute

$$\to 2 - 3Y^2Y'' - 6Y(Y')^2 = 0$$

$$\to 2 - 3Y^2Y'' - 6Y\left(\frac{2X}{3Y^2}\right)^2 = 0$$

$$\to 2 - 3Y^2Y'' - 6Y \cdot \frac{4X^2}{9Y^4} = 0$$

$$\to 2 - 3Y^2Y'' - \frac{8X^2}{3Y^3} = 0$$ (mult. by $3Y^3$)

$$\to 6Y^3 - 9Y^5Y'' - 8X^2 = 0$$ ✓

3.) radius $r = \frac{1}{2}\sqrt{(1-0)^2+(2-0)^2+(-2-0)^2} = \frac{\sqrt{9}}{2} = \frac{3}{2}$

and center is

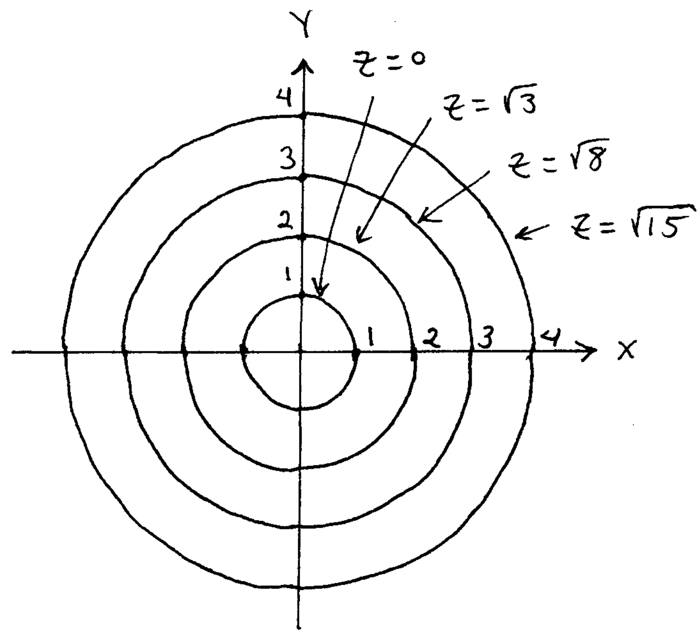
$$\left(\frac{0+1}{2}, \frac{0+2}{2}, \frac{0+-2}{2}\right) = \left(\frac{1}{2}, 1, -1\right) \text{ so}$$

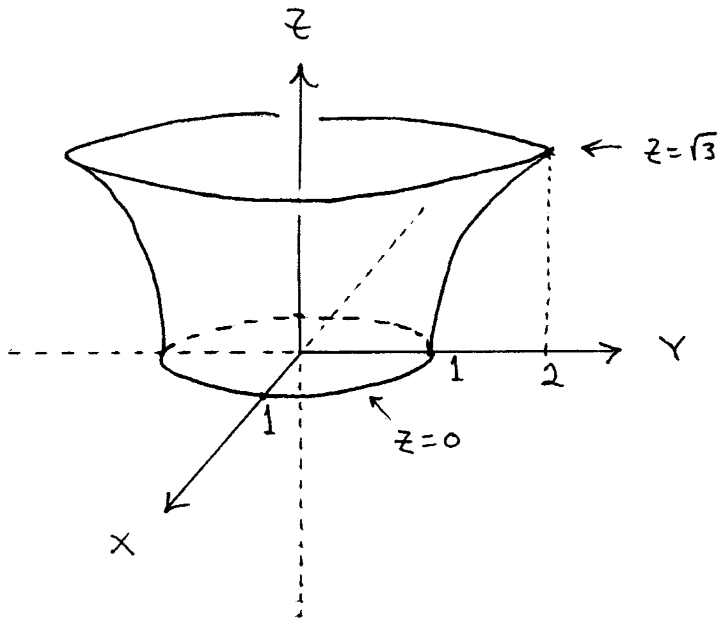
sphere is given by

$$\boxed{\left(x-\frac{1}{2}\right)^2 + (y-1)^2 + (z+1)^2 = \left(\frac{3}{2}\right)^2}$$

4.)

z-value	level curve
0	$x^2+y^2=1$
$\sqrt{3}$	$x^2+y^2=4$
$\sqrt{8}$	$x^2+y^2=9$
$\sqrt{15}$	$x^2+y^2=16$





5.) $\frac{dA}{dt} = k \cdot \sqrt{A+100} \rightarrow \int (A+100)^{-1/2} dA = \int k\, dt$

$\rightarrow 2(A+100)^{1/2} = kt + c \rightarrow (A+100)^{1/2} = \frac{k}{2}t + c_1$

$\rightarrow A + 100 = \left(\frac{k}{2}t + c_1\right)^2 \rightarrow A = \left(\frac{k}{2}t + c_1\right)^2 - 100$;

$t=0, A=44$ so $44 = c_1^2 - 100 \rightarrow c_1 = 12$ and

$t=3, A=125$ so $125 = \left(\frac{3k}{2} + 12\right)^2 - 100 \rightarrow k = 2$ so

$\boxed{A = (t+12)^2 - 100}$; if $\underline{t=4}$ then $A = 156$ caribou.

6.) a.) $\frac{dS}{dt} = \left(\frac{0 \text{ lbs.}}{\text{gal.}}\right)\left(\frac{3 \text{ gal.}}{\text{min.}}\right) - \left(\frac{S \text{ lbs.}}{200-t \text{ gal.}}\right)\left(\frac{4 \text{ gal.}}{\text{min.}}\right) \rightarrow$

$\frac{dS}{dt} = \frac{4S}{t-200}$

b.) $\int \frac{1}{S}\, dS = \int \frac{4}{t-200}\, dt \rightarrow \ln S = 4 \ln(t-200) + c \rightarrow$

$e^{\ln S} = e^{4\ln(t-200)+c} = e^{\ln(t-200)^4} \cdot \cancel{e^c}^{\,c_1} \rightarrow$

$S = c_1 (t-200)^4$; $t=0, S = 100$ lbs $\rightarrow$

$100 = c_1 (200)^4 \rightarrow c_1 = \frac{100}{(200)^4} = \frac{1}{2(200)^3}$ then

$\boxed{S = \frac{1}{2(200)^3}(t-200)^4}$ .. OR

$S' + \frac{-4}{t-200} S = 0$ so $\mu = e^{\int \frac{-4}{t-200} dt}$

$= e^{-4\ln(t-200)} = e^{\ln(t-200)^{-4}} = (t-200)^{-4}$

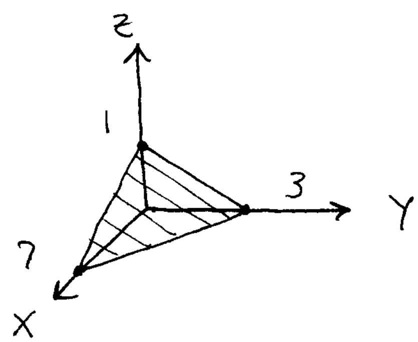
then $(t-200)^{-4} S' - 4(t-200)^{-5} S = 0 \rightarrow$

$$D\left\{(t-200)^{-4}S\right\}=0 \rightarrow (t-200)^{-4}S = c \rightarrow$$

$$S = c\,(t-200)^4 \rightarrow S = \frac{1}{2(200)^3}(t-200)^4 .$$

EXTRA CREDIT

1.)



$$\frac{x}{7} + \frac{y}{3} + \frac{z}{1} = 1$$

2.) $S = \left(\frac{1}{32}\right)(t-5)^2 + 50 + \ln(t+1) \rightarrow$

$S' = \frac{1}{16}(t-5) + \frac{1}{t+1} \rightarrow$

$S'' = \frac{1}{16} - \frac{1}{(t+1)^2} = 0 \rightarrow t = 3$ days