Math 16C

Exam 2 Solutions

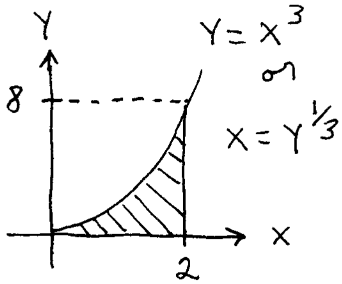
1.) $z = xy^3 + \tan(x-y) \rightarrow$

$z_x = y^3 + \sec^2(x-y)$ , $z_y = 3xy^2 - \sec^2(x-y)$ ,

$z_{xy} = 3y^2 + 2\sec(x-y)\cdot\sec(x-y)\tan(x-y)\cdot(-1)$

2.) $\int_0^1 \int_0^{\sqrt{x}} (2x^2y - 3)\,dy\,dx = \int_0^1 (x^2y^2 - 3y)\Big|_{y=0}^{y=\sqrt{x}}\,dx$

$= \int_0^1 (x^3 - 3\sqrt{x})\,dx = \frac{1}{4}x^4 - 2x^{3/2}\Big|_0^1 = \frac{1}{4} - 2 = \frac{-7}{4}$

3.) 


$\int_0^8 \int_{y^{1/3}}^2 \frac{6y}{\sqrt{1+x^7}}\,dx\,dy$

$= \int_0^2 \int_0^{x^3} \frac{6y}{\sqrt{1+x^7}}\,dy\,dx$

$= \int_0^2 \frac{3y^2}{\sqrt{1+x^7}}\Big|_{y=0}^{y=x^3}\,dx = \int_0^2 \frac{3x^6}{\sqrt{1+x^7}}\,dx$ (Let $u = 1+x^7 \rightarrow \cdots$)

$= 3\cdot\frac{1}{7}\cdot 2(1+x^7)^{1/2}\Big|_0^2 = \frac{6}{7}\sqrt{129} - \frac{6}{7}$

4.) area of $R = \frac{1}{2}(1)(3) = \frac{3}{2}$ so

$AVE = \frac{1}{\text{area } R}\int_0^1 \int_0^{3x} \cos(3x-y)\,dy\,dx$

$= \frac{2}{3}\int_0^1 -\sin(3x-y)\Big|_{y=0}^{y=3x}\,dx$

$= \frac{2}{3}\int_0^1 [-\overset{0}{\sin(0)} + \sin(3x)]\,dx = \frac{2}{3}\cdot\frac{-1}{3}\cos(3x)\Big|_0^1$

$= \frac{-2}{9}(\cos(3) - \overset{1}{\cos(0)}) = \boxed{\frac{2}{9}(1-\cos(3))}$ .

5.) a.) $z = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ then

$z_x = 3x^2 + 3y^2 - 6x = 3(x^2 + y^2 - 2x) = 0 \rightarrow \boxed{x^2 + y^2 - 2x = 0}$,

$z_y = 6xy - 6y = 6y(x-1) = 0 \rightarrow \boxed{x = 1 \text{ or } y = 0}$;

if $x = 1$ then $1^2 + y^2 - 2(1) = 0 \rightarrow y^2 = 1 \rightarrow y = \pm 1$ so $\boxed{(1,1)}$ and $\boxed{(1,-1)}$ are critical points;

if $y = 0$ then $x^2 + (0)^2 - 2x = 0 \rightarrow x(x-2) = 0$ so $x = 0$ or $x = 2$ so $\boxed{(0,0)}$ and $\boxed{(2,0)}$ are critical points.

b.) $z_{xx} = 6x - 6$, $z_{yy} = 6x - 6$, $z_{xy} = 6y$ then

For (0,0): $d = z_{xx} z_{yy} - (z_{xy})^2 = (-6)(-6) - (0)^2 = 36 > 0$ and $z_{xx} = -6 < 0$ so $(0,0)$ determines a maximum value at $z = 4$.

For (2,0): $d = z_{xx} z_{yy} - (z_{xy})^2 = (6)(6) - (0)^2 = 36 > 0$ and $z_{xx} = 6 > 0$ so $(2,0)$ determines a minimum value at $z = 0$.

For (1,1): $d = z_{xx} z_{yy} - (z_{xy})^2 = (0)(0) - (6)^2 = -36 < 0$ so $(1,1)$ determines a saddle point at $z = 2$.

For (1,-1): $d = z_{xx} z_{yy} - (z_{xy})^2 = (0)(0) - (-6)^2 = -36 < 0$
so (1,-1) determines a saddle point
at $z = 2$ .

6.) $F(x,y,z,\lambda) = (x^2 + 2y^2 + 3z^2) - \lambda(3x - 2y + z - 6)$ then

$F_x = 2x - 3\lambda = 0 \rightarrow \lambda = \frac{2}{3}x$

$F_y = 4y + 2\lambda = 0 \rightarrow \lambda = -2y$

$\frac{2}{3}x = -2y \rightarrow x = -3y$

$F_z = 6z - \lambda = 0 \rightarrow \lambda = 6z$

$-2y = 6z \rightarrow z = \frac{-1}{3}y$

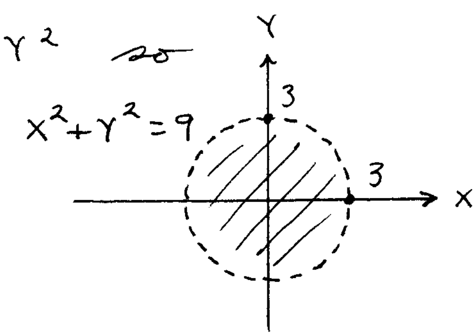
$F_\lambda = 6 - 3x + 2y - z = 0$

$6 - 3(-3y) + 2y - (\frac{-1}{3}y) = 0 \rightarrow 6 + 9y + 2y + \frac{1}{3}y = 0 \rightarrow$

$\frac{34}{3}y = -6 \rightarrow y = \frac{-9}{17}$, $x = \frac{27}{17}$, $z = \frac{3}{17}$ and

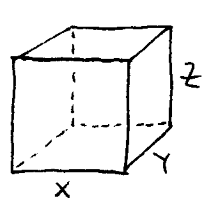
min. $S = (\frac{27}{17})^2 + 2(\frac{-9}{17})^2 + 3(\frac{3}{17})^2 = \frac{918}{289} \rightarrow S = \frac{54}{17}$

7.) a.) $9 - x^2 - y^2 > 0 \rightarrow 9 > x^2 + y^2$ so
domain is all points
$(x,y)$ lying on the inside
of circle $x^2 + y^2 = 3^2$




b.) $z = \ln(9 - (x^2 + y^2))$ so range is
all values $z \leq \ln 9$

8.)




Volume $xyz = 36 \rightarrow z = \frac{36}{xy}$,
minimize cost

$C = \underset{\text{top}}{2(xy)} + \underset{\text{bottom}}{6(xy)} + \underset{\text{sides}}{3(2xz + 2yz)}$

$$C = 8XY + 6X\left(\frac{36}{XY}\right) + 6Y\left(\frac{36}{XY}\right) = 8XY + \frac{216}{Y} + \frac{216}{X} \quad \text{then}$$

$$\left.\begin{array}{l} C_X = 8Y - \dfrac{216}{X^2} = 0 \rightarrow \quad Y = \dfrac{27}{X^2} \\ C_Y = 8X - \dfrac{216}{Y^2} = 0 \rightarrow \quad X = \dfrac{27}{Y^2} \end{array}\right\} \quad Y = \frac{27}{\left(\frac{27}{Y^2}\right)^2} = \frac{1}{27}Y^4 \rightarrow$$

$$27Y - Y^4 = Y(27 - Y^3) = 0$$

$$\downarrow \qquad \downarrow$$

$Y = 0$ No!

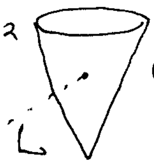
$Y = 3$ ft. $\quad X = 3$ ft.

$Z = 4$ ft. and

minimum cost is $\boxed{C = \$216}$.

EXTRA CREDIT :

$$Z^2 = (X-1)^2 + (Y-2)^2$$



$(X, Y, Z)$

$(0,0,0)$ $L$

Minimize distance

$$L = \sqrt{(X-0)^2 + (Y-0)^2 + (Z-0)^2} = \sqrt{X^2 + Y^2 + (X-1)^2 + (Y-2)^2} \quad \text{then}$$

$$L_X = \tfrac{1}{2}(\sim)^{-1/2}\left[2X + 2(X-1)\right] = 0 \rightarrow 2X - 1 = 0 \rightarrow X = \tfrac{1}{2},$$

$$L_Y = \tfrac{1}{2}(\sim)^{-1/2}\left[2Y + 2(Y-2)\right] = 0 \rightarrow 2Y - 2 = 0 \rightarrow Y = 1$$

and minimum distance is

$$L = \sqrt{\tfrac{1}{4} + 1 + \tfrac{1}{4} + 1} = \sqrt{\tfrac{5}{2}}$$