Math 16C
Exam 3 Solutions

1.) a.) $4^2, 5^2, 6^2, 7^2, 8^2, \ldots$
$16, 25, 36, 49, 64, \ldots$ $\boxed{(n+3)^2}$
position: 1 2 3 4 5 ... n

b.) $1, 3, 5, 7, 9, \ldots$ $\boxed{2n-1}$
position: 1 2 3 4 5 ... n

c.) $3/1, 6/2, 9/6, 12/24, \frac{15}{120} \ldots$
$\frac{1\cdot 3}{1}, \frac{2\cdot 3}{2\cdot 1}, \frac{3\cdot 3}{3\cdot 2\cdot 1}, \frac{3\cdot 4}{4\cdot 3\cdot 2\cdot 1}, \frac{3\cdot 5}{5\cdot 4\cdot 3\cdot 2\cdot 1}, \ldots$ $\boxed{\frac{3n}{n!}}$
position: 1 2 3 4 5 ... n

2.) a.) Series converges by p-series test since $p = 3 > 1$.

b.) Series converges by ratio test since

$$\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{5^{n+1}}{(2(n+1))!} \cdot \frac{(2n)!}{5^n}$$

$$= \lim_{n\to\infty} \frac{5^{n+1}}{5^n} \cdot \frac{\cancel{(2n)(2n-1)\cdots(3)(2)(1)}}{(2n+2)(2n+1)\cancel{(2n)(2n-1)\cdots(3)(2)(1)}}$$

$$= \lim_{n\to\infty} \frac{5}{(2n+2)(2n+1)} = 0 < 1.$$

c.) Series diverges by nth term test since

$$\lim_{n\to\infty} \frac{n^2+1}{\sqrt{5n^4+3n+2}} = \lim_{n\to\infty} \frac{\sqrt{(n^2+1)^2}}{\sqrt{5n^4+3n+2}}$$

$$= \lim_{n\to\infty} \sqrt{\frac{n^4+2n^2+1}{5n^4+3n+2}} = \lim_{n\to\infty} \sqrt{\frac{1+2/n^2+1/n^4}{5+3/n^3+2/n^4}} = \sqrt{\frac{1}{5}} \neq 0$$

d.) Series <u>diverges</u> by geometric series test

since $\frac{1}{5} - \frac{3}{10} + \frac{9}{20} - \frac{27}{40} + \frac{81}{80} - \cdots$

$$= \frac{1}{5}\left(1 - \frac{3}{2} + \frac{9}{4} - \frac{27}{8} + \frac{81}{16} - \cdots\right)$$

$$= \frac{1}{5}\left(1 + \left(\frac{-3}{2}\right) + \left(\frac{-3}{2}\right)^2 + \left(\frac{-3}{2}\right)^3 + \cdots\right) \quad \text{and } r = -3/2 .$$

3.) $\text{Total} = 1 + 2 + 2^2 + 2^3 + \cdots + 2^{63} = \frac{1 - 2^{64}}{1 - 2}$

(square) 1 2 3 4 ... 64

$$= 2^{64} - 1 \ \text{¢} = \$18,446,744,0,700,000,000!$$

4.) $\frac{n^{1-p}}{p-1} = \frac{n^{1-6}}{6-1} = \frac{n^{-5}}{5} \le 0.001 \rightarrow$

$n^{-5} \le 0.005 \rightarrow 200 \le n^5 \rightarrow$

$(200)^{1/5} \le n \rightarrow n = 3$ works!

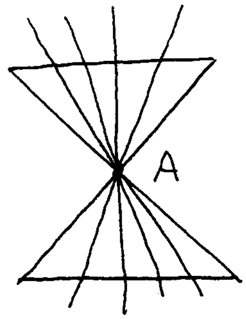
5.) ratio test: $\lim_{n\to\infty} \frac{|5x-3|^{n+1}}{2(n+1)+1} \cdot \frac{2n+1}{|5x-3|^n}$

$$= \lim_{n\to\infty} |5x-3| \cdot \frac{2n+1}{2n+3} = \lim_{n\to\infty} |5x-3| \cdot \frac{2 + \cancelto{0}{1/n}}{2 + \cancelto{0}{3/n}}$$

$= |5x-3| < 1 \rightarrow -1 < 5x - 3 < +1 \rightarrow$

$2 < 5x < 4 \rightarrow \frac{2}{5} < x < 4/5$ and $R = \frac{1}{5}$

6.)



# lines	# Δ's	OR
1	$6 = 2\cdot 3 = 2(1+2)$	$2\cdot 3$
2	$6+6=12=2\cdot 6 = 2(1+2+3)$	$3\cdot 4$
3	$12+8=20=2\cdot 10 = 2(1+2+3+4)$	$4\cdot 5$
4	$20+10=30=2\cdot 15 = 2(1+2+3+4+5)$	$5\cdot 6$
$\vdots$	$\vdots$	$\vdots$
n	$2(1+2+3+\cdots+(n+1))$ $= \dfrac{\cancel{2}(n+1)(n+2)}{\cancel{2}}$	

$\rightarrow n = 200$ lines $\rightarrow (201)(202) = 40{,}602$ Δ's

7.) $$\int_0^{\frac{\pi}{6}}\int_0^{\frac{\pi}{4}} \cos^2(x+y)\,dy\,dx = \int_0^{\frac{\pi}{6}}\int_0^{\frac{\pi}{4}} \tfrac{1}{2}\left(1+\cos 2(x+y)\right)dy\,dx$$

$$= \int_0^{\frac{\pi}{6}}\int_0^{\frac{\pi}{4}} \tfrac{1}{2}\left(1+\cos(2x+2y)\right)dy\,dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} \left(y + \tfrac{1}{2}\sin(2x+2y)\right)\Big|_{y=0}^{y=\frac{\pi}{4}} dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} \left[\frac{\pi}{4} + \frac{1}{2}\sin\left(2x+\frac{\pi}{2}\right) - \frac{1}{2}\sin(2x)\right]dx$$

$$= \frac{1}{2}\left[\frac{\pi}{4}x + \frac{-1}{4}\cos\left(2x+\frac{\pi}{2}\right) + \frac{1}{4}\cos(2x)\right]\Big|_0^{\pi/6}$$

$$= \frac{1}{2}\left[\frac{\pi^2}{24} - \frac{1}{4}\cos\left(\frac{5\pi}{6}\right) + \frac{1}{4}\cos\left(\frac{\pi}{3}\right)\right]$$

$$- \frac{1}{2}\left[-\frac{1}{4}\cos\left(\frac{\pi}{2}\right) + \frac{1}{4}\cos(0)\right]$$

$$= \frac{\pi^2}{48} - \frac{1}{8}\left(-\frac{\sqrt{3}}{2}\right) + \frac{1}{8}\left(\frac{1}{2}\right) + \frac{1}{8}(0) - \frac{1}{8}(1)$$

$$= \frac{\pi^2}{48} + \frac{\sqrt{3}}{16} - \frac{1}{16}$$

Extra Credit :

1.) $$\frac{3 + (-1)^n \cdot 2}{n\,(n+2)}$$

2.)