PREREQUISITE REVIEW C.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, find the first and second derivatives of the function.

1. $y = 3x^2 + 2x + 1$

2. $y = -2x^3 - 8x + 4$

3. $y = -3e^{2x}$

4. $y = -3e^{x^2}$

In Exercises 5–8, use implicit differentiation to find dy/dx.

5. $x^2 + y^2 = 2x$

6. $2x - y^3 = 4y$

7. $xy^2 = 3$

8. $3xy + x^2y^2 = 10$

In Exercises 9 and 10, solve for k.

9. $0.5 = 9 - 9e^{-k}$

10. $14.75 = 25 - 25e^{-2k}$

EXERCISES C.1

In Exercises 1–10, verify that the function is a solution of the differential equation.

	Solution	*Differential Equation*
1.	$y = x^3 + 5$	$y' = 3x^2$
2.	$y = 2x^3 - x + 1$	$y' = 6x^2 - 1$
3.	$y = e^{-2x}$	$y' + 2y = 0$
4.	$y = 3e^{x^2}$	$y' - 2xy = 0$
5.	$y = 2x^3$	$y' - \frac{3}{x}y = 0$
6.	$y = 4x^2$	$y' - \frac{2}{x}y = 0$
7.	$y = x^2$	$x^2y'' - 2y = 0$
8.	$y = \frac{1}{x}$	$xy'' + 2y' = 0$
9.	$y = 2e^{2x}$	$y'' - y' - 2y = 0$
10.	$y = e^{x^3}$	$y'' - 3x^2y' - 6xy = 0$

In Exercises 11–28, verify that the function is a solution of the differential equation for any value of C.

	Solution	*Differential Equation*
11.	$y = \frac{1}{x} + C$	$\frac{dy}{dx} = -\frac{1}{x^2}$
12.	$y = \sqrt{4 - x^2} + C$	$\frac{dy}{dx} = -\frac{x}{\sqrt{4 - x^2}}$
13.	$y = Ce^{4x}$	$\frac{dy}{dx} = 4y$
14.	$y = Ce^{-4x}$	$\frac{dy}{dx} = -4y$
15.	$y = Ce^{-t/3} + 7$	$3\frac{dy}{dt} + y - 7 = 0$
16.	$y = Ce^{-t} + 10$	$y' + y - 10 = 0$
17.	$y = Cx^2 - 3x$	$xy' - 3x - 2y = 0$
18.	$y = x \ln x^2 + 2x^{3/2} + Cx$	$y' - \frac{y}{x} = 2 + \sqrt{x}$
19.	$y = x^2 + 2x + \frac{C}{x}$	$xy' + y = x(3x + 4)$
20.	$y = C_1 + C_2e^x$	$y'' - y' = 0$

Solution

21. $y = C_1e^{x/2} + C_2e^{-2x}$

22. $y = C_1e^{4x} + C_2e^{-x}$

23. $y = \frac{bx^4}{4 - a} + Cx^a$

24. $y = \frac{x^3}{5} - x + C\sqrt{x}$

25. $y = \frac{2}{1 + Ce^{x^2}}$

26. $y = Ce^{x - x^2}$

27. $y = x \ln x + Cx + 4$

28. $y = x(\ln x + C)$

In Exercises 29–32, use im equation is a solution of t of C.

Solution

29. $x^2 + y^2 = Cy$

30. $y^2 + 2xy - x^2 = C$

31. $x^2 + xy = C$

32. $x^2 - y^2 = C$

In Exercises 33–36, determi of the differential equation

33. $y = e^{-2x}$

34. $y = 5 \ln x$

35. $y = \frac{4}{x}$

36. $y = 4e^{2x}$

In Exercises 37–40, determi of the differential equation

37. $y = \frac{2}{9}xe^{-2x}$

38. $y = 4e^x + \frac{2}{9}xe^{-2x}$

39. $y = xe^x$

40. $y = x \ln x$

In Exercises 41–48, verify t differential equation. Then fies the initial condition.

41. General solution: $y =$
Differential equation:
Initial condition: $y =$

42. General solution: $2x^2$
Differential equation:
Initial condition: $y =$

r sections. You will

tion is a solution of the

Differential Equation

$\frac{dy}{dx} = -\frac{1}{x^2}$

$\frac{dy}{dx} = -\frac{x}{\sqrt{4 - x^2}}$

$\frac{dy}{dx} = 4y$

$\frac{dy}{dx} = -4y$

$3\frac{dy}{dt} + y - 7 = 0$

$y' + y - 10 = 0$

$xy' - 3x - 2y = 0$

$y' - \frac{y}{x} = 2 + \sqrt{x}$

$xy' + y = x(3x + 4)$

$y'' - y' = 0$

	Solution	*Differential Equation*
21.	$y = C_1e^{x/2} + C_2e^{-2x}$	$2y'' + 3y' - 2y = 0$
22.	$y = C_1e^{4x} + C_2e^{-x}$	$y'' - 3y' - 4y = 0$
23.	$y = \frac{bx^4}{4 - a} + Cx^a$	$y' - \frac{ay}{x} = bx^3$
24.	$y = \frac{x^3}{5} - x + C\sqrt{x}$	$2xy' - y = x^3 - x$
25.	$y = \frac{2}{1 + Ce^{x^2}}$	$y' + 2xy = xy^2$
26.	$y = Ce^{x-x^2}$	$y' + (2x - 1)y = 0$
27.	$y = x \ln x + Cx + 4$	$x(y' - 1) - (y - 4) = 0$
28.	$y = x(\ln x + C)$	$x + y - xy' = 0$

In Exercises 29–32, use implicit differentiation to verify that the equation is a solution of the differential equation for any value of C.

	Solution	*Differential Equation*
29.	$x^2 + y^2 = Cy$	$y' = \frac{2xy}{x^2 - y^2}$
30.	$y^2 + 2xy - x^2 = C$	$(x + y)y' - x + y = 0$
31.	$x^2 + xy = C$	$x^2y'' - 2(x + y) = 0$
32.	$x^2 - y^2 = C$	$y^3y'' + x^2 - y^2 = 0$

In Exercises 33–36, determine whether the function is a solution of the differential equation $y^{(4)} - 16y = 0$.

33. $y = e^{-2x}$

34. $y = 5 \ln x$

35. $y = \frac{4}{x}$

36. $y = 4e^{2x}$

In Exercises 37–40, determine whether the function is a solution of the differential equation $y''' - 3y' + 2y = 0$.

37. $y = \frac{2}{9}xe^{-2x}$

38. $y = 4e^x + \frac{2}{9}xe^{-2x}$

39. $y = xe^x$

40. $y = x \ln x$

In Exercises 41–48, verify that the general solution satisfies the differential equation. Then find the particular solution that satisfies the initial condition.

41. General solution: $y = Ce^{-2x}$
Differential equation: $y' + 2y = 0$
Initial condition: $y = 3$ when $x = 0$

42. General solution: $2x^2 + 3y^2 = C$
Differential equation: $2x + 3yy' = 0$
Initial condition: $y = 2$ when $x = 1$

43. General solution: $y = C_1 + C_2 \ln|x|, x > 0$
Differential equation: $xy'' + y' = 0$
Initial condition: $y = 5$ and $y' = 0.5$ when $x = 1$

44. General solution: $y = C_1x + C_2x^3$
Differential equation: $x^2y'' - 3xy' + 3y = 0$
Initial condition: $y = 0$ and $y' = 4$ when $x = 2$

45. General solution: $y = C_1e^{4x} + C_2e^{-3x}$
Differential equation: $y'' - y' - 12y = 0$
Initial condition: $y = 5$ and $y' = 6$ when $x = 0$

46. General solution: $y = Ce^{x-x^2}$
Differential equation: $y' + (2x - 1)y = 0$
Initial condition: $y = 2$ when $x = 1$

47. General solution: $y = e^{2x/3}(C_1 + C_2x)$
Differential equation: $9y'' - 12y' + 4y = 0$
Initial condition: $y = 4$ when $x = 0$
$y = 0$ when $x = 3$

48. General solution: $y = \left(C_1 + C_2x + \frac{1}{12}x^4\right)e^{2x}$
Differential equation: $y'' - 4y' + 4y = x^2e^{2x}$
Initial condition: $y = 2$ and $y' = 1$ when $x = 0$

In Exercises 49–52, the general solution of the differential equation is given. Use a graphing utility to graph the particular solutions that correspond to the indicated values of C.

	General Solution	*Differential Equation*	*C-Values*
49.	$y = Cx^2$	$xy' - 2y = 0$	$1, 2, 4$
50.	$4y^2 - x^2 = C$	$4yy' - x = 0$	$0, \pm 1, \pm 4$
51.	$y = C(x + 2)^2$	$(x + 2)y' - 2y = 0$	$0, \pm 1, \pm 2$
52.	$y = Ce^{-x}$	$y' + y = 0$	$0, \pm 1, \pm 2$

In Exercises 53–60, use integration to find the general solution of the differential equation.

53. $\frac{dy}{dx} = 3x^2$

54. $\frac{dy}{dx} = \frac{1}{1 + x}$

55. $\frac{dy}{dx} = \frac{x + 3}{x}$

56. $\frac{dy}{dx} = \frac{x - 2}{x}$

57. $\frac{dy}{dx} = \frac{1}{x^2 - 1}$

58. $\frac{dy}{dx} = \frac{x}{1 + x^2}$

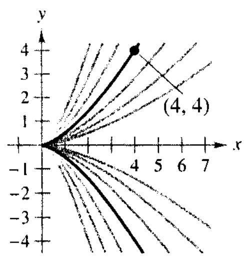
59. $\frac{dy}{dx} = x\sqrt{x - 3}$

60. $\frac{dy}{dx} = xe^x$

In Exercises 61–64, you are shown the graphs of some of the solutions of the differential equation. Find the particular solution whose graph passes through the indicated point.

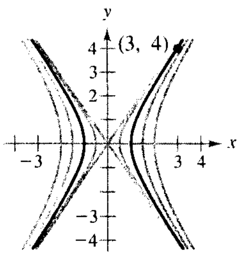
61. $y^2 = Cx^3$

$2xy' - 3y = 0$



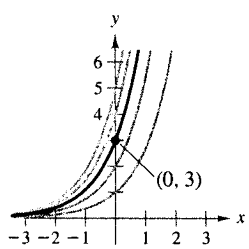
62. $2x^2 - y^2 = C$

$yy' - 2x = 0$



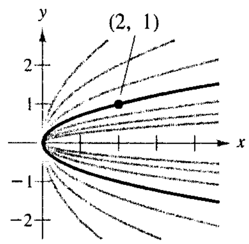
63. $y = Ce^x$

$y' - y = 0$



64. $y^2 = 2Cx$

$2xy' - y = 0$



65. ***Biology*** The limiting capacity of the habitat of a wildlife herd is 750. The growth rate dN/dt of the herd is proportional to the unutilized opportunity for growth, as described by the differential equation

$$\frac{dN}{dt} = k(750 - N).$$

The general solution of this differential equation is

$$N = 750 - Ce^{-kt}.$$

When $t = 0$, the population of the herd is 100. After 2 years, the population has grown to 160.

(a) Write the population function N as a function of t.

(b) Use a graphing utility to graph the population function.

(c) What is the population of the herd after 4 years?

66. ***Investment*** The rate of growth of an investment is proportional to the amount in the investment at any time t. That is,

$$\frac{dA}{dt} = kA.$$

The initial investment is \$1000, and after 10 years the balance is \$3320.12. The general solution is

$$A = Ce^{kt}.$$

What is the particular solution?

67. ***Marketing*** You are working in the marketing department of a computer software company. Your marketing team determines that a maximum of 30,000 units of a new product can be sold in a year. You hypothesize that the rate of growth of the sales x is proportional to the difference between the maximum sales and the current sales. That is,

$$\frac{dx}{dt} = k(30{,}000 - x).$$

The general solution of this differential equation is

$$x = 30{,}000 - Ce^{-kt}$$

where t is the time in years. During the first year, 2000 units are sold. Complete the table showing the numbers of units sold in subsequent years.

Year, t	2	4	6	8	10
Units, x					

68. ***Marketing*** In Exercise 67, suppose that the maximum annual sales are 50,000 units. How does this change the sales shown in the table?

69. ***Safety*** Assume that the rate of change in the number of miles s of road cleared per hour by a snowplow is inversely proportional to the depth h of the snow. This rate of change is described by the differential equation

$$\frac{ds}{dh} = \frac{k}{h}.$$

Show that

$$s = 25 - \frac{13}{\ln 3}\ln\frac{h}{2}$$

is a solution of this differential equation.

70. Show that $y = a + Ce^{k(1-b)t}$ is a solution of the differential equation

$$y = a + b(y - a) + \left(\frac{1}{k}\right)\left(\frac{dy}{dt}\right)$$

where k is a constant.

71. The function $y = Ce^{kx}$ is a solution of the differential equation

$$\frac{dy}{dx} = 0.07y.$$

Is it possible to determine C or k from the information given? If so, find its value.

True or False? In Exercises 72 and 73, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

72. A differential equation can have more than one solution.

73. If $y = f(x)$ is a solution of a differential equation, then $y = f(x) + C$ is also a solution.

C.2 SEPAR

Use separati
problems.

Separation of

The simplest type of
that this type of equa

$y = \int f(x)\,dx.$

In this section, you
family of differential
This technique is call

Separation of Va

If f and g are con

$\frac{dy}{dx} = f(x)g(y$

has a general solu

$\int \frac{1}{g(y)}\,dy =$

Essentially, the t
implies. For a differer
to one side and the y
each side to obtain th

EXAMPLE 1

Find the general solut

$\frac{dy}{dx} = \frac{x}{y^2 + 1}.$

SOLUTION Begin by

$\frac{dy}{dx} =$

$(y^2 + 1)\,dy =$

$\int (y^2 + 1)\,dy =$

$\frac{y^3}{3} + y =$

PREREQUISITE REVIEW C.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, find the indefinite integral and check your result by differentiating.

1. $\int x^{3/2}\,dx$

2. $\int (t^3 - t^{1/3})\,dt$

3. $\int \frac{2}{x-5}\,dx$

4. $\int \frac{y}{2y^2+1}\,dy$

5. $\int e^{2y}\,dy$

6. $\int xe^{1-x^2}\,dx$

In Exercises 7–10, solve the equation for C or k.

7. $(3)^2 - 6(3) = 1 + C$

8. $(-1)^2 + (-2)^2 = C$

9. $10 = 2e^{2k}$

10. $(6)^2 - 3(6) = e^{-k}$

EXERCISES C.2

In Exercises 1–6, decide whether the variables in the differential equation can be separated.

1. $\frac{dy}{dx} = \frac{x}{y+3}$

2. $\frac{dy}{dx} = \frac{x+1}{x}$

3. $\frac{dy}{dx} = \frac{1}{x} + 1$

4. $\frac{dy}{dx} = \frac{x}{x+y}$

5. $\frac{dy}{dx} = x - y$

6. $x\frac{dy}{dx} = \frac{1}{y}$

In Exercises 7–26, use separation of variables to find the general solution of the differential equation.

7. $\frac{dy}{dx} = 2x$

8. $\frac{dy}{dx} = \frac{1}{x}$

9. $3y^2\frac{dy}{dx} = 1$

10. $\frac{dy}{dx} = x^2y$

11. $(y+1)\frac{dy}{dx} = 2x$

12. $(1+y)\frac{dy}{dx} - 4x = 0$

13. $y' - xy = 0$

14. $y' - y = 5$

15. $\frac{dy}{dt} = \frac{e^t}{4y}$

16. $e^y\frac{dy}{dt} = 3t^2 + 1$

17. $\frac{dy}{dx} = \sqrt{1-y}$

18. $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$

19. $(2+x)y' = 2y$

20. $y' = (2x-1)(y+3)$

21. $xy' = y$

22. $y' - y(x+1) = 0$

23. $y' = \frac{x}{y} - \frac{x}{1+y}$

24. $\frac{dy}{dx} = \frac{x^2+2}{3y^2}$

25. $e^x(y'+1) = 1$

26. $yy' - 2xe^x = 0$

In Exercises 27–32, use the initial condition to find the particular solution of the differential equation.

Differential Equation	*Initial Condition*
27. $yy' - e^x = 0$	$y = 4$ when $x = 0$
28. $\sqrt{x} + \sqrt{y}\,y' = 0$	$y = 4$ when $x = 1$
29. $x(y+4) + y' = 0$	$y = -5$ when $x = 0$
30. $\frac{dy}{dx} = x^2(1+y)$	$y = 3$ when $x = 0$
31. $dP - 6P\,dt = 0$	$P = 5$ when $t = 0$
32. $dT + k(T-70)\,dt = 0$	$T = 140$ when $t = 0$

In Exercises 33 and 34, fir
through the point and h
equation.

33. Point: $(-1, 1)$

Slope: $y' = \frac{6x}{5y}$

34. Point: $(8, 2)$

Slope: $y' = \frac{2y}{3x}$

Velocity In Exercises 3
to find velocity v as a fun
differential equation mc
toboggan after consider
air resistance.

35. $12.5\frac{dv}{dt} = 43.2 - 1$

36. $12.5\frac{dv}{dt} = 43.2 - 1$

Chemistry: Newton's
use Newton's Law of Coo
in the temperature T of a
between the temperatur
of the surrounding envir
ential equation $dT/dt =$

37. A steel ingot whos
room whose temper
the temperature of t
temperature 5 hours

38. A room is kept at a
placed in the room
utes. How long wi
temperature of 80°F

39. Food at a temperatu
set at 0°F. After 1 h

(a) Find the temper
freezer 6 hours.

(b) How long will i
of 10°F?

40. ***Biology: Cell Gr***
cal cell with volume
For a sphere, the s
$S = kV^{2/3}$. So, a m

$$\frac{dV}{dt} = kV^{2/3}.$$

Solve this differenti

'arlier sections. You will

$\frac{dy}{dx} = \sqrt{\frac{x}{y}}$

. $y' = (2x - 1)(y + 3)$

. $y' - y(x + 1) = 0$

. $\frac{dy}{dx} = \frac{x^2 + 2}{3y^2}$

. $yy' - 2xe^x = 0$

dition to find the particular

tial Condition

= 4 when $x = 0$

= 4 when $x = 1$

= −5 when $x = 0$

= 3 when $x = 0$

= 5 when $t = 0$

= 140 when $t = 0$

In Exercises 33 and 34, find an equation for the graph that passes through the point and has the specified slope. Then graph the equation.

33. Point: $(-1, 1)$

 Slope: $y' = \frac{6x}{5y}$

34. Point: $(8, 2)$

 Slope: $y' = \frac{2y}{3x}$

Velocity In Exercises 35 and 36, solve the differential equation to find velocity v as a function of time t if $v = 0$ when $t = 0$. The differential equation models the motion of two people on a toboggan after consideration of the force of gravity, friction, and air resistance.

35. $12.5\frac{dv}{dt} = 43.2 - 1.25v$

36. $12.5\frac{dv}{dt} = 43.2 - 1.75v$

Chemistry: Newton's Law of Cooling In Exercises 37–39, use Newton's Law of Cooling, which states that the rate of change in the temperature T of an object is proportional to the difference between the temperature T of the object and the temperature T_0 of the surrounding environment. This is described by the differential equation $dT/dt = k(T - T_0)$.

37. A steel ingot whose temperature is 1500°F is placed in a room whose temperature is a constant 90°F. One hour later, the temperature of the ingot is 1120°F. What is the ingot's temperature 5 hours after it is placed in the room?

38. A room is kept at a constant temperature of 70°F. An object placed in the room cools from 350°F to 150°F in 45 minutes. How long will it take for the object to cool to a temperature of 80°F?

39. Food at a temperature of 70°F is placed in a freezer that is set at 0°F. After 1 hour, the temperature of the food is 48°F.
 (a) Find the temperature of the food after it has been in the freezer 6 hours.
 (b) How long will it take the food to cool to a temperature of 10°F?

40. ***Biology: Cell Growth*** The rate of growth of a spherical cell with volume V is proportional to its surface area S. For a sphere, the surface area and volume are related by $S = kV^{2/3}$. So, a model for the cell's growth is

$$\frac{dV}{dt} = kV^{2/3}.$$

 Solve this differential equation.

41. ***Learning Theory*** The management of a factory has found that a worker can produce at most 30 units per day. The number of units N per day produced by a new employee will increase at a rate proportional to the difference between 30 and N. This is described by the differential equation

$$\frac{dN}{dt} = k(30 - N)$$

 where t is the time in days. Solve this differential equation.

42. ***Sales*** The rate of increase in sales S (in thousands of units) of a product is proportional to the current level of sales and inversely proportional to the square of the time t. This is described by the differential equation

$$\frac{dS}{dt} = \frac{kS}{t^2}$$

 where t is the time in years. The saturation point for the market is 50,000 units. That is, the limit of S as $t \to \infty$ is 50. After 1 year, 10,000 units have been sold. Find S as a function of the time t.

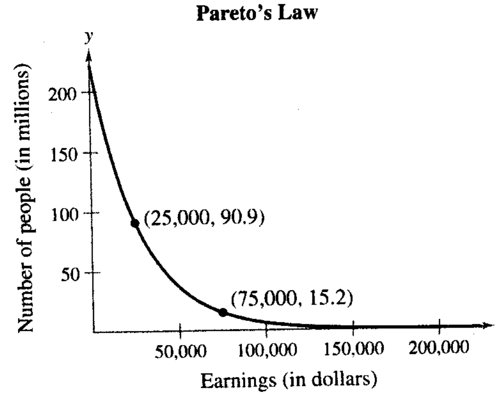
43. ***Economics: Pareto's Law*** According to the economist Vilfredo Pareto (1848–1923), the rate of decrease of the number of people y in a stable economy having an income of at least x dollars is directly proportional to the number of such people and inversely proportional to their income x. This is modeled by the differential equation

$$\frac{dy}{dx} = -k\frac{y}{x}.$$

 Solve this differential equation.

44. ***Economics: Pareto's Law*** In 2001, 15.2 million people in the United States earned more than \$75,000 and 90.9 million people earned more than \$25,000 (see figure). Assume that Pareto's Law holds and use the result of Exercise 43 to determine the number of people (in millions) who earned (a) more than \$20,000 and (b) more than \$100,000. *(Source: U.S. Census Bureau)*

Pareto's Law



or 20 years, money is
e trust fund at the rate
rns 8% interest, com-
after 20 years?

e balance increases in
he rate at which the

neral solution is

,500. So, the revenue

PREREQUISITE REVIEW C.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, simplify the expression.

1. $e^{-x}(e^{2x} + e^x)$

2. $\dfrac{1}{e^{-x}}(e^{-x} + e^{2x})$

3. $e^{-\ln x^3}$

4. $e^{2\ln x + x}$

In Exercises 5–10, find the indefinite integral.

5. $\displaystyle\int e^x(2 + e^{-2x})\,dx$

6. $\displaystyle\int e^{2x}(xe^x + 1)\,dx$

7. $\displaystyle\int \frac{1}{2x + 5}\,dx$

8. $\displaystyle\int \frac{x + 1}{x^2 + 2x + 3}\,dx$

9. $\displaystyle\int (4x - 3)^2\,dx$

10. $\displaystyle\int x(1 - x^2)^2\,dx$

EXERCISES C.3

In Exercises 1–6, write the linear differential equation in standard form.

1. $x^3 - 2x^2y' + 3y = 0$

2. $y' - 5(2x - y) = 0$

3. $xy' + y = xe^x$

4. $xy' + y = x^3y$

5. $y + 1 = (x - 1)y'$

6. $x = x^2(y' + y)$

In Exercises 7–18, solve the differential equation.

7. $\dfrac{dy}{dx} + 3y = 6$

8. $\dfrac{dy}{dx} + 5y = 15$

9. $\dfrac{dy}{dx} + y = e^{-x}$

10. $\dfrac{dy}{dx} + 3y = e^{-3x}$

11. $\dfrac{dy}{dx} + \dfrac{y}{x} = 3x + 4$

12. $\dfrac{dy}{dx} + \dfrac{2y}{x} = 3x + 1$

13. $y' + 5xy = x$

14. $y' + 5y = e^{5x}$

15. $(x - 1)y' + y = x^2 - 1$

16. $xy' + y = x^2 + 1$

17. $x^3y' + 2y = e^{1/x^2}$

18. $xy' + y = x^2 \ln x$

In Exercises 19–22, solve for y in two ways.

19. $y' + y = 4$

20. $y' + 10y = 5$

21. $y' - 2xy = 2x$

22. $y' + 4xy = x$

In Exercises 23–26, match the differential equation with its solution.

	Differential Equation	*Solution*
23.	$y' - 2x = 0$	(a) $y = Ce^{x^2}$
24.	$y' - 2y = 0$	(b) $y = -\frac{1}{2} + Ce^{x^2}$
25.	$y' - 2xy = 0$	(c) $y = x^2 + C$
26.	$y' - 2xy = x$	(d) $y = Ce^{2x}$

In Exercises 27–34, find the particular solution that satisfies the initial condition.

	Differential Equation	*Initial Condition*
27.	$y' + y = 6e^x$	$y = 3$ when $x = 0$
28.	$y' + 2y = e^{-2x}$	$y = 4$ when $x = 1$
29.	$xy' + y = 0$	$y = 2$ when $x = 2$
30.	$y' + y = x$	$y = 4$ when $x = 0$
31.	$y' + 3x^2y = 3x^2$	$y = 6$ when $x = 0$
32.	$y' + (2x - 1)y = 0$	$y = 2$ when $x = 1$
33.	$xy' - 2y = -x^2$	$y = 5$ when $x = 1$
34.	$x^2y' - 4xy = 10$	$y = 10$ when $x = 1$

35. *Sales* The rate of change (in thousands of units) in sales S is modeled by

$$\frac{dS}{dt} = 0.2(100 - S) + 0.2t$$

where t is the time in years. Solve this differential equation and use the result to complete the table.

t	0	1	2	3	4	5	6	7	8	9	10
S	0										

36. *Sales* The rate of change in sales S is modeled by

$$\frac{dS}{dt} = k_1(L - S) + k_2t$$

where t is the time in years and $S = 0$ when $t = 0$. Solve this differential equation for S as a function of t.

Elasticity of Demand In Exercises 37 and 38, find the demand function $p = f(x)$. Recall from Section 3.5 that the price elasticity of demand was defined as $\eta = (p/x)/(dp/dx)$.

37. $\eta = 1 - \dfrac{400}{3x}$, $p = 340$ when $x = 20$

38. $\eta = 1 - \dfrac{500}{3x}$, $p = 2$ when $x = 100$

Supply and Demand In Exercises 39 and 40, use the demand and supply functions to find the price p as a function of time t. Begin by setting $D(t)$ equal to $S(t)$ and solving the resulting differential equation. Find the general solution, and then use the initial condition to find the particular solution.

39. $D(t) = 480 + 5p(t) - 2p'(t)$ Demand function

$S(t) = 300 + 8p(t) + p'(t)$ Supply function

$p(0) = \$75.00.$ Initial condition

40. $D(t) = 4000 + 5p(t) - 4p'(t)$ Demand function

$S(t) = 2800 + 7p(t) + 2p'(t)$ Supply function

$p(0) = \$1000.00$ Initial condition

41. *Investment* A brokerage firm opens a new real estate investment plan for which the earnings are equivalent to continuous compounding at the rate of r. The firm estimates that deposits from investors will create a net cash flow of Pt dollars, where t is the time in years. The rate of change in the total investment A is modeled by

$$\frac{dA}{dt} = rA + Pt.$$

(a) Solve the differential equation and find the total investment A as a function of t. Assume that $A = 0$ when $t = 0$.

(b) Find the total investment A after 10 years given that $P = \$500{,}000$ and $r = 9\%$.

42. *Investment* Let $A(t)$ be the amount in a fund earning interest at the annual rate of r, compounded continuously. If a continuous cash flow of P dollars per year is withdrawn from the fund, then the rate of decrease of A is given by the differential equation

$$\frac{dA}{dt} = rA - P$$

where $A = A_0$ when $t = 0$.

(a) Solve this equation for A as a function of t.

(b) Use the result of part (a) to find A when $A_0 = \$2{,}000{,}000$, $r = 7\%$, $P = \$250{,}000$, and $t = 5$ years.

(c) Find A_0 if a retired person wants a continuous cash flow of \$40,000 per year for 20 years. Assume that the person's investment will earn 8%, compounded continuously.

43. *Velocity* A booster rocket carrying an observation satellite is launched into space. The rocket and satellite have mass m and are subject to air resistance proportional to the velocity v at any time t. A differential equation that models the velocity of the rocket and satellite is

$$m\frac{dv}{dt} = -mg - kv$$

where g is the acceleration due to gravity. Solve the differential equation for v as a function of t.

44. *Health* An infectious disease spreads through a large population according to the model

$$\frac{dy}{dt} = \frac{1 - y}{4}$$

where y is the percent of the population exposed to the disease, and t is the time in years.

(a) Solve this differential equation, assuming $y(0) = 0$.

(b) Find the number of years it takes for half of the population to have been exposed to the disease.

(c) Find the percentage of the population that has been exposed to the disease after 4 years.

45. *Research Project* Use your school's library, the Internet, or some other reference source to find an article in a scientific or business journal that uses a differential equation to model a real-life situation. Write a short paper describing the situation. If possible, describe the solution of the differential equation.

C.4 APPLICA

Use differentia

EXAMPLE 1 I

The new cereal produc advertising campaign t which the population h number of people who of the population has h end of 2 years?

SOLUTION Let y be t of the product. This n heard of it, and dy/dt i From the given assum

$$\frac{dy}{dt} = k(1 -$$

Rate of change of y | is proportional to

Using separation of va general solution to be

$y = 1 - Ce^{-kt}.$

To solve for the const $y = 0$ when $t = 0$, yc when $t = 1$, it follows

$k = \ln 2 \approx 0.693.$

So, the particular solut

$y = 1 - e^{-0.693t}.$

This model is shown g mine that the number

$y = 1 - e^{-0.693(2)}$

≈ 0.75 or 750,0

PREREQUISITE REVIEW C.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, use separation of variables to find the general solution of the differential equation.

1. $\dfrac{dy}{dx} = 3x$
2. $2y\dfrac{dy}{dx} = 3$
3. $\dfrac{dy}{dx} = 2xy$
4. $\dfrac{dy}{dx} = \dfrac{x-4}{4y^3}$

In Exercises 5–8, use an integrating factor to solve the first-order linear differential equation.

5. $y' + 2y = 4$
6. $y' + 2y = e^{-2x}$
7. $y' + xy = x$
8. $xy' + 2y = x^2$

In Exercises 9 and 10, write the equation that models the statement.

9. The rate of change of y with respect to x is proportional to the square of x.
10. The rate of change of x with respect to t is proportional to the difference of x and t.

EXERCISES C.4

In Exercises 1–6, assume that the rate of change of y is proportional to y. Solve the resulting differential equation $dy/dx = ky$ and find the particular solution that passes through the points.

1. (0, 1), (3, 2)
2. (0, 4), (1, 6)
3. (0, 4), (4, 1)
4. (0, 60), (5, 30)
5. (2, 2), (3, 4)
6. (1, 4), (2, 1)

7. ***Investment*** The rate of growth of an investment is proportional to the amount A of the investment at any time t. An investment of \$2000 increases to a value of \$2983.65 in 5 years. Find its value after 10 years.

8. ***Population Growth*** The rate of change of the population of a city is proportional to the population P at any time t. In 1998, the population was 400,000, and the constant of proportionality was 0.015. Estimate the population of the city in the year 2005.

9. ***Sales Growth*** The rate of change in sales S (in thousands of units) of a new product is proportional to the difference between L and S (in thousands of units) at any time t. When $t = 0$, $S = 0$. Write and solve the differential equation for this sales model.

10. ***Sales Growth*** Use the result of Exercise 9 to write S as a function of t if (a) $L = 100$, $S = 25$ when $t = 2$, and (b) $L = 500$, $S = 50$ when $t = 1$.

In Exercises 11–14, the rate of change of y is proportional to the product of y and the difference of L and y. Solve the resulting differential equation $dy/dx = ky(L - y)$ and find the particular solution that passes through the points for the given value of L.

11. $L = 20$; (0, 1), (5, 10)
12. $L = 100$; (0, 10), (5, 30)
13. $L = 5000$; (0, 250), (25, 2000)
14. $L = 1000$; (0, 100), (4, 750)

15. ***Biology*** At any t
tion N of deer in a
of N and $L - N$, w
of deer the park ca
when $t = 4$, $N = 2$

16. ***Sales Growth*** T
sands of units) of
product of S and L
mated maximum l
Write and solve t
model.

Learning Theory I
rate of change in the p
trials is proportional to
the limiting proportion

17. Write and solve th
theory model.

18. Use the solution o
n, and then use a g
(a) $L = 1.00$
$P = 0.50$ whe
$P = 0.85$ whe
(b) $L = 0.80$
$P = 0.25$ whe
$P = 0.60$ whe

Chemical Reaction
reaction model in Exan
of t, and use a graphing

19. $y = 45$ grams whe
20. $y = 75$ grams whe

In Exercises 21 and
described in Example 3
its graph.

21. $L = 500$; $y = 100$
22. $L = 5000$; $y = 50$

23. ***Biology*** A popu
duced into a new w
maximum popula
beavers. After 3 y
population follow
beavers will be in

24. ***Biology*** A popu
into a new region.
lation the region c
population is esti
follows a Gomper
be present after 3

15. ***Biology*** At any time t, the rate of growth of the population N of deer in a state park is proportional to the product of N and $L - N$, where $L = 500$ is the maximum number of deer the park can maintain. When $t = 0$, $N = 100$, and when $t = 4$, $N = 200$. Write N as a function of t.

16. ***Sales Growth*** The rate of change in sales S (in thousands of units) of a new product is proportional to the product of S and $L - S$. L (in thousands of units) is the estimated maximum level of sales, and $S = 10$ when $t = 0$. Write and solve the differential equation for this sales model.

Learning Theory In Exercises 17 and 18, assume that the rate of change in the proportion P of correct responses after n trials is proportional to the product of P and $L - P$, where L is the limiting proportion of correct responses.

17. Write and solve the differential equation for this learning theory model.

18. Use the solution of Exercise 17 to write P as a function of n, and then use a graphing utility to graph the solution.

(a) $L = 1.00$

$P = 0.50$ when $n = 0$

$P = 0.85$ when $n = 4$

(b) $L = 0.80$

$P = 0.25$ when $n = 0$

$P = 0.60$ when $n = 10$

Chemical Reaction In Exercises 19 and 20, use the chemical reaction model in Example 2 to find the amount y as a function of t, and use a graphing utility to graph the function.

19. $y = 45$ grams when $t = 0$; $y = 4$ grams when $t = 2$

20. $y = 75$ grams when $t = 0$; $y = 12$ grams when $t = 1$

In Exercises 21 and 22, use the Gompertz growth model described in Example 3 to find the growth function, and sketch its graph.

21. $L = 500$; $y = 100$ when $t = 0$; $y = 150$ when $t = 2$

22. $L = 5000$; $y = 500$ when $t = 0$; $y = 625$ when $t = 1$

23. ***Biology*** A population of eight beavers has been introduced into a new wetlands area. Biologists estimate that the maximum population the wetlands can sustain is 60 beavers. After 3 years, the population is 15 beavers. If the population follows a Gompertz growth model, how many beavers will be in the wetlands after 10 years?

24. ***Biology*** A population of 30 rabbits has been introduced into a new region. It is estimated that the maximum population the region can sustain is 400 rabbits. After 1 year, the population is estimated to be 90 rabbits. If the population follows a Gompertz growth model, how many rabbits will be present after 3 years?

Biology In Exercises 25 and 26, use the hybrid selection model in Example 4 to find the percent of the population that has the indicated characteristic.

25. You are studying a population of mayflies to determine how quickly characteristic A will pass from one generation to the next. At the start of the study, half the population has characteristic A. After four generations, 75% of the population has characteristic A. Find the percent of the population that will have characteristic A after 10 generations. (Assume $a = 2$ and $b = 1$.)

26. A research team is studying a population of snails to determine how quickly characteristic B will pass from one generation to the next. At the start of the study, 40% of the snails have characteristic B. After five generations, 80% of the population has characteristic B. Find the percent of the population that will have characteristic B after eight generations. (Assume $a = 2$ and $b = 1$.)

27. ***Chemical Reaction*** In a chemical reaction, a compound changes into another compound at a rate proportional to the unchanged amount, according to the model

$$\frac{dy}{dt} = ky.$$

(a) Solve the differential equation.

(b) If the initial amount of the original compound is 20 grams, and the amount remaining after 1 hour is 16 grams, when will 75% of the compound have been changed?

28. ***Chemical Mixture*** A 100-gallon tank is full of a solution containing 25 pounds of a concentrate. Starting at time $t = 0$, distilled water is admitted to the tank at the rate of 5 gallons per minute, and the well-stirred solution is withdrawn at the same rate.

(a) Find the amount Q of the concentrate in the solution as a function of t. (*Hint:* $Q' + Q/20 = 0$)

(b) Find the time when the amount of concentrate in the tank reaches 15 pounds.

29. ***Chemical Mixture*** A 200-gallon tank is half full of distilled water. At time $t = 0$, a solution containing 0.5 pound of concentrate per gallon enters the tank at the rate of 5 gallons per minute, and the well-stirred mixture is withdrawn at the same rate. Find the amount Q of concentrate in the tank after 30 minutes. (*Hint:* $Q' + Q/20 = \frac{5}{2}$)

30. ***Safety*** Assume that the rate of change in the number of miles s of road cleared per hour by a snowplow is inversely proportional to the depth h of snow. That is,

$$\frac{ds}{dh} = \frac{k}{h}.$$

Find s as a function of h if $s = 25$ miles when $h = 2$ inches and $s = 12$ miles when $h = 6$ inches $(2 \le h \le 15)$.

31. ***Chemistry*** A wet towel hung from a clothesline to dry loses moisture through evaporation at a rate proportional to its moisture content. If after 1 hour the towel has lost 40% of its original moisture content, after how long will it have lost 80%?

32. ***Biology*** Let x and y be the sizes of two internal organs of a particular mammal at time t. Empirical data indicate that the relative growth rates of these two organs are equal, and can be modeled by

$$\frac{1}{x}\frac{dx}{dt} = \frac{1}{y}\frac{dy}{dt}.$$

Use this differential equation to write y as a function of x.

33. ***Population Growth*** When predicting population growth, demographers must consider birth and death rates as well as the net change caused by the difference between the rates of immigration and emigration. Let P be the population at time t and let N be the net increase per unit time due to the difference between immigration and emigration. So, the rate of growth of the population is given by

$$\frac{dP}{dt} = kP + N, \quad N \text{ is constant.}$$

Solve this differential equation to find P as a function of time.

34. ***Meteorology*** The barometric pressure y (in inches of mercury) at an altitude of x miles above sea level decreases at a rate proportional to the current pressure according to the model

$$\frac{dy}{dx} = -0.2y$$

where $y = 29.92$ inches when $x = 0$. Find the barometric pressure (a) at the top of Mt. St. Helens (8364 feet) and (b) at the top of Mt. McKinley (20,320 feet).

35. ***Investment*** A large corporation starts at time $t = 0$ to invest part of its receipts at a rate of P dollars per year in a fund for future corporate expansion. Assume that the fund earns r percent interest per year compounded continuously. So, the rate of growth of the amount A in the fund is given by

$$\frac{dA}{dt} = rA + P$$

where $A = 0$ when $t = 0$. Solve this differential equation for A as a function of t.

Investment In Exercises 36–38, use the result of Exercise 35.

36. Find A for each situation.

(a) $P = \$100{,}000$, $r = 12\%$, and $t = 5$ years

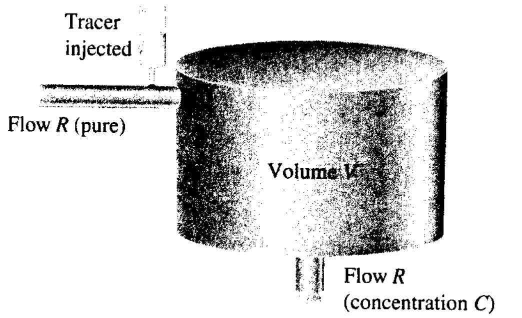
(b) $P = \$250{,}000$, $r = 15\%$, and $t = 10$ years

37. Find P if the corporation needs \$120,000,000 in 8 years and the fund earns $16\frac{1}{4}\%$ interest compounded continuously.

38. Find t if the corporation needs \$800,000 and it can invest \$75,000 per year in a fund earning 13% interest compounded continuously.

Medical Science In Exercises 39–41, a medical researcher wants to determine the concentration C (in moles per liter) of a tracer drug injected into a moving fluid. Solve this problem by considering a single-compartment dilution model (see figure). Assume that the fluid is continuously mixed and that the volume of fluid in the compartment is constant.

Figure for 39–41



39. If the tracer is injected instantaneously at time $t = 0$, then the concentration of the fluid in the compartment begins diluting according to the differential equation

$$\frac{dC}{dt} = \left(-\frac{R}{V}\right)C, \quad C = C_0 \text{ when } t = 0.$$

(a) Solve this differential equation to find the concentration as a function of time.

(b) Find the limit of C as $t \to \infty$.

40. Use the solution of the differential equation in Exercise 39 to find the concentration as a function of time, and use a graphing utility to graph the function.

(a) $V = 2$ liters, $R = 0.5$ L/min, and $C_0 = 0.6$ mol/L

(b) $V = 2$ liters, $R = 1.5$ L/min, and $C_0 = 0.6$ mol/L

41. In Exercises 39 and 40, it was assumed that there was a single initial injection of the tracer drug into the compartment. Now consider the case in which the tracer is continuously injected (beginning at $t = 0$) at the rate of Q mol/min. Considering Q to be negligible compared with R, use the differential equation

$$\frac{dC}{dt} = \frac{Q}{V} - \left(\frac{R}{V}\right)C, \quad C = 0 \text{ when } t = 0.$$

(a) Solve this differential equation to find the concentration as a function of time.

(b) Find the limit of C as $t \to \infty$.

PREREQUISITE REVIEW 7.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, find the distance between the points.

1. $(5, 1), (3, 5)$
2. $(2, 3), (-1, -1)$
3. $(-5, 4), (-5, -4)$
4. $(-3, 6), (-3, -2)$

In Exercises 5–8, find the midpoint of the line segment connecting the points.

5. $(2, 5), (6, 9)$
6. $(-1, -2), (3, 2)$
7. $(-6, 0), (6, 6)$
8. $(-4, 3), (2, -1)$

In Exercises 9 and 10, write the standard equation of the circle.

9. Center: $(2, 3)$; radius: 2
10. Endpoints of a diameter: $(4, 0), (-2, 8)$

EXERCISES 7.1

In Exercises 1–4, plot the points on the same three-dimensional coordinate system.

1. (a) $(2, 1, 3)$
   (b) $(-1, 2, 1)$
2. (a) $(3, -2, 5)$
   (b) $\left(\frac{3}{2}, 4, -2\right)$
3. (a) $(5, -2, 2)$
   (b) $(5, -2, -2)$
4. (a) $(0, 4, -5)$
   (b) $(4, 0, 5)$

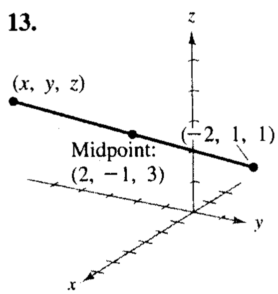
In Exercises 5–8, find the distance between the two points.

5. $(4, 1, 5), (8, 2, 6)$
6. $(-4, -1, 1), (2, -1, 5)$
7. $(-1, -5, 7), (-3, 4, -4)$
8. $(8, -2, 2), (8, -2, 4)$

In Exercises 9–12, find the coordinates of the midpoint of the line segment joining the two points.

9. $(6, -9, 1), (-2, -1, 5)$
10. $(4, 0, -6), (8, 8, 20)$
11. $(-5, -2, 5), (6, 3, -7)$
12. $(0, -2, 5), (4, 2, 7)$

In Exercises 13–16, find (x, y, z).

13.


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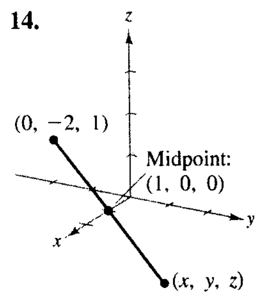

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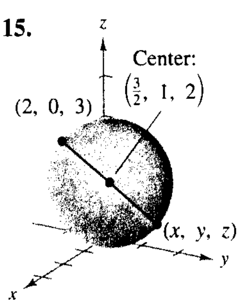

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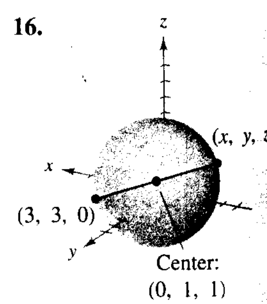
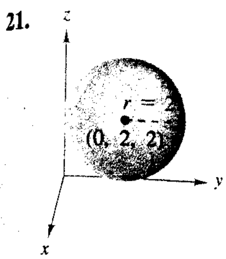
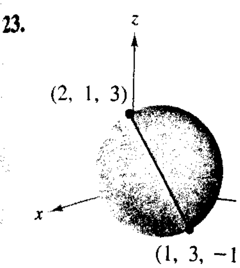


In Exercises 17–20, find
with the given vertices, …
right triangle, an isoscele…

17. (0, 0, 0), (2, 2, 1), (…
18. (5, 3, 4), (7, 1, 3), (…
19. (−2, 2, 4), (−2, 2, …
20. (5, 0, 0), (0, 2, 0), (…

In Exercises 21–30, find th…
sphere.

21.


23.


25. Center: (1, 1, 5); radi…
26. Center: (4, −1, 1); ra…
27. Endpoints of diameter…
28. Endpoints of diameter…
29. Center: (−2, 1, 1); ta…
30. Center: (1, 2, 0); tang…

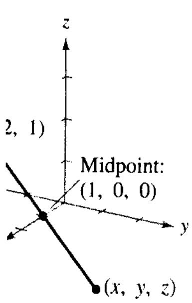
In Exercises 31–36, find the…

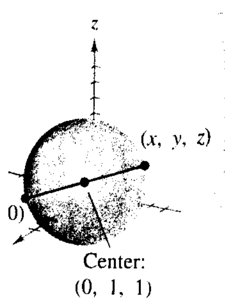
31. $x^2 + y^2 + z^2 - 5x =$ …
32. $x^2 + y^2 + z^2 - 8y =$ …
33. $x^2 + y^2 + z^2 - 2x +$ …
34. $x^2 + y^2 + z^2 - 4y +$ …
35. $2x^2 + 2y^2 + 2z^2 - 4x$ …
36. $4x^2 + 4y^2 + 4z^2 - 8x$ …

In Exercises 37–40, sketch th…

37. $(x - 1)^2 + (y - 3)^2 +$ …
38. $(x + 1)^2 + (y + 2)^2 +$ …
39. $x^2 + y^2 + z^2 - 6x - 1$ …
40. $x^2 + y^2 + z^2 - 4y + 2$ …

ɿr sections. You will





In Exercises 17–20, find the lengths of the sides of the triangle with the given vertices, and determine whether the triangle is a right triangle, an isosceles triangle, or neither of these.

17. (0, 0, 0), (2, 2, 1), (2, −4, 4)

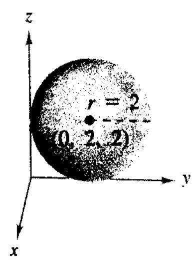
18. (5, 3, 4), (7, 1, 3), (3, 5, 3)

19. (−2, 2, 4), (−2, 2, 6), (−2, 4, 8)

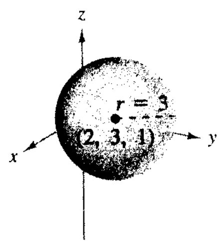
20. (5, 0, 0), (0, 2, 0), (0, 0, −3)

In Exercises 21–30, find the standard form of the equation of the sphere.

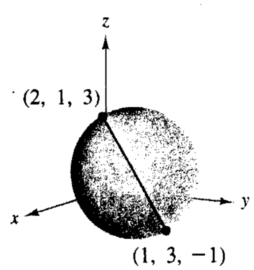
21.



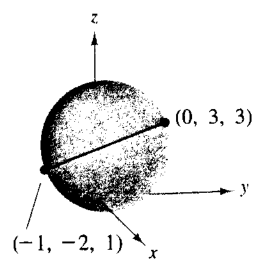
22.



23.



24.



25. Center: (1, 1, 5); radius: 3

26. Center: (4, −1, 1); radius: 5

27. Endpoints of diameter: (2, 0, 0), (0, 6, 0)

28. Endpoints of diameter: (1, 0, 0), (0, 5, 0)

29. Center: (−2, 1, 1); tangent to the *xy*-coordinate plane

30. Center: (1, 2, 0); tangent to the *yz*-coordinate plane

In Exercises 31–36, find the sphere's center and radius.

31. $x^2 + y^2 + z^2 - 5x = 0$

32. $x^2 + y^2 + z^2 - 8y = 0$

33. $x^2 + y^2 + z^2 - 2x + 6y + 8z + 1 = 0$

34. $x^2 + y^2 + z^2 - 4y + 6z + 4 = 0$

35. $2x^2 + 2y^2 + 2z^2 - 4x - 12y - 8z + 3 = 0$

36. $4x^2 + 4y^2 + 4z^2 - 8x + 16y + 11 = 0$

In Exercises 37–40, sketch the *xy*-trace of the sphere.

37. $(x - 1)^2 + (y - 3)^2 + (z - 2)^2 = 25$

38. $(x + 1)^2 + (y + 2)^2 + (z - 2)^2 = 16$

39. $x^2 + y^2 + z^2 - 6x - 10y + 6z + 30 = 0$

40. $x^2 + y^2 + z^2 - 4y + 2z - 60 = 0$

In Exercises 41 and 42, sketch the *yz*-trace of the sphere.

41. $x^2 + y^2 + z^2 - 4x - 4y - 6z - 12 = 0$

42. $x^2 + y^2 + z^2 - 6x - 10y + 6z + 30 = 0$

In Exercises 43–46, sketch the trace of the intersection of each plane with the given sphere.

43. $x^2 + y^2 + z^2 = 25$

(a) $z = 3$ (b) $x = 4$

44. $x^2 + y^2 + z^2 = 169$

(a) $x = 5$ (b) $y = 12$

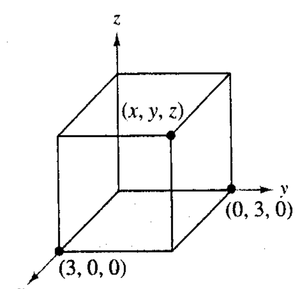
45. $x^2 + y^2 + z^2 - 4x - 6y + 9 = 0$

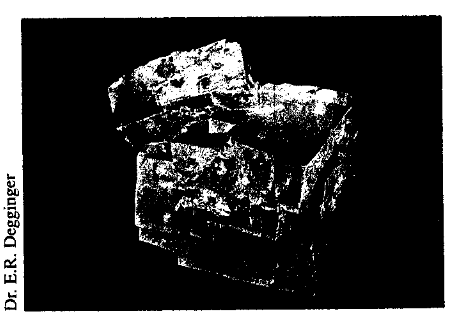
(a) $x = 2$ (b) $y = 3$

46. $x^2 + y^2 + z^2 - 8x - 6z + 16 = 0$

(a) $x = 4$ (b) $z = 3$

47. ***Geology*** Crystals are classified according to their symmetry. Crystals shaped like cubes are classified as isometric. Suppose you have mapped the vertices of a crystal onto a three-dimensional coordinate system. Determine (x, y, z) if the crystal is isometric.





Dr. E.R. Degginger

Halite crystals (rock salt) are classified as isometric.

48. ***Physical Science*** Assume that Earth is a sphere with a radius of 3963 miles. If the center of Earth is placed at the origin of a three-dimensional coordinate system, what is the equation of the sphere? Lines of longitude that run north-south could be represented by what trace(s)? What shape would each of these traces form? Why? Lines of latitude that run east-west could be represented by what trace(s)? Why? What shape would each of these traces form? Why?

PREREQUISITE REVIEW 7.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, find the *x*- and *y*-intercepts of the function.

1. $3x + 4y = 12$
2. $6x + y = -8$
3. $-2x + y = -2$
4. $-x - y = 5$

In Exercises 5–8, rewrite the expression by completing the square.

5. $x^2 + y^2 + z^2 - 2x - 4y - 6z + 15 = 0$
6. $x^2 + y^2 - z^2 - 8x + 4y - 6z + 11 = 0$
7. $z - 2 = x^2 + y^2 + 2x - 2y$
8. $x^2 + y^2 + z^2 - 6x + 10y + 26z = -202$

In Exercises 9 and 10, write the expression in standard form.

9. $16x^2 - 16y^2 + 16z^2 = 4$
10. $9x^2 - 9y^2 + 9z^2 = 36$

EXERCISES 7.2

In Exercises 1–12, find the intercepts and sketch the graph of the plane.

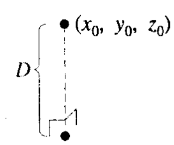
1. $4x + 2y + 6z = 12$
2. $3x + 6y + 2z = 6$
3. $3x + 3y + 5z = 15$
4. $x + y + z = 3$
5. $2x - y + 3z = 8$
6. $2x - y + z = 4$
7. $z = 3$
8. $y = -4$
9. $y + z = 5$
10. $x + 2y = 8$
11. $x + y - z = 0$
12. $x - 3z = 3$

In Exercises 13–22, determine whether the planes $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ are parallel, perpendicular, or neither. The planes are parallel if there exists a nonzero constant k such that $a_1 = ka_2$, $b_1 = kb_2$, $c_1 = kc_2$, and perpendicular if $a_1a_2 + b_1b_2 + c_1c_2 = 0$.

13. $5x - 3y + z = 4$, $x + 4y + 7z = 1$
14. $3x + y - 4z = 3$, $-9x - 3y + 12z = 4$
15. $x - 5y - z = 1$, $5x - 25y - 5z = -3$
16. $x + 3y + 2z = 6$, $4x - 12y + 8z = 24$
17. $x + 2y = 3$, $4x + 8y = 5$
18. $x + 3y + z = 7$, $x - 5z = 0$
19. $2x + y = 3$, $3x - 5z = 0$
20. $2x - z = 1$, $4x + y + 8z = 10$
21. $x = 6$, $y = -1$
22. $x = -2$, $y = 4$

In Exercises 23–30, find the distance between the point and the plane (see figure). The distance D between a point (x_0, y_0, z_0) and the plane $ax + by + cz + d = 0$ is

$$D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$



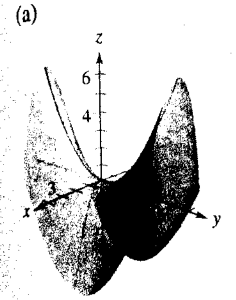
Plane:
$ax + by + cz + d = 0$

23. $(0, 0, 0)$, $2x + 3y + z = 12$
24. $(0, 0, 0)$, $x - 3y + 4z = 6$
25. $(1, 5, -4)$, $3x - y + 2z = 6$
26. $(1, 2, 3)$, $2x - y + z = 4$
27. $(1, 0, -1)$, $2x - 4y + 3z = 12$
28. $(2, -1, 0)$, $3x + 3y + 2z = 6$
29. $(3, 2, -1)$, $2x - 3y + 4z = 24$
30. $(-2, 1, 0)$, $2x + 5y - z = 20$

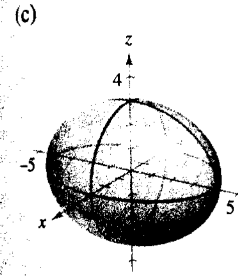
In Exercises 31–38, matc
graph. [The graphs are lal

31. $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{9} = 1$
33. $4x^2 + 4y^2 - z^2 = 4$
34. $y^2 = 4x^2 + 9z^2$
35. $4x^2 - 4y + z^2 = 0$
36. $12z = -3y^2 + 4x^2$
37. $4x^2 - y^2 + 4z = 0$
38. $x^2 + y^2 + z^2 = 9$

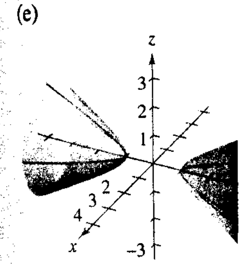
(a)



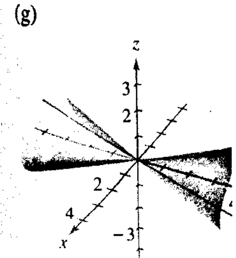
(c)



(e)



(g)



arlier sections. You will

between the point and the
:ween a point (x_0, y_0, z_0) and

In Exercises 31–38, match the given equation with the correct graph. [The graphs are labeled (a)–(h).]

31. $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{9} = 1$

32. $x^2 - \frac{4y^2}{15} + z^2 = -\frac{4}{15}$

33. $4x^2 + 4y^2 - z^2 = 4$

34. $y^2 = 4x^2 + 9z^2$

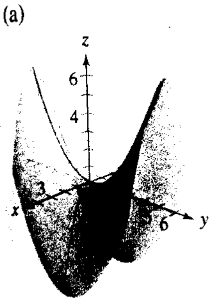
35. $4x^2 - 4y + z^2 = 0$

36. $12z = -3y^2 + 4x^2$

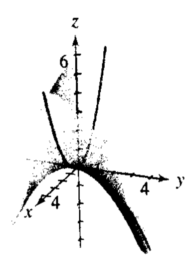
37. $4x^2 - y^2 + 4z = 0$

38. $x^2 + y^2 + z^2 = 9$

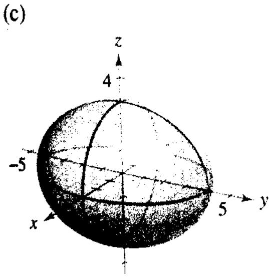
(a)



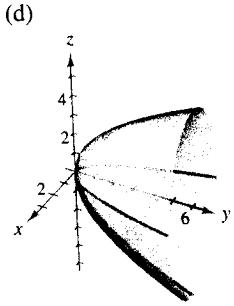
(b)



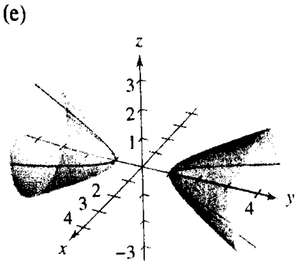
(c)



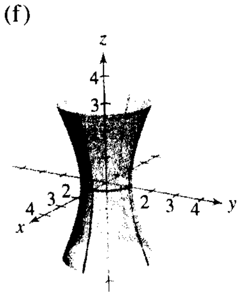
(d)



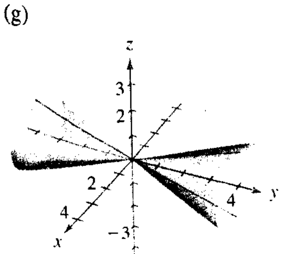
(e)



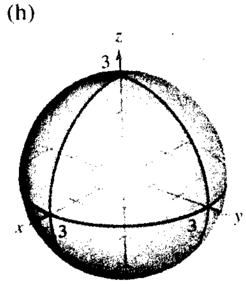
(f)



(g)



(h)



In Exercises 39–42, describe the traces of the surface in the given planes.

	Surface	*Planes*
39.	$x^2 - y - z^2 = 0$	xy-plane, $y = 1$, yz-plane
40.	$y = x^2 + z^2$	xy-plane, $y = 1$, yz-plane
41.	$\frac{x^2}{4} + y^2 + z^2 = 1$	xy-plane, xz-plane, yz-plane
42.	$y^2 + z^2 - x^2 = 1$	xy-plane, xz-plane, yz-plane

In Exercises 43–56, identify the quadric surface.

43. $x^2 + \frac{y^2}{4} + z^2 = 1$

44. $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{16} = 1$

45. $25x^2 + 25y^2 - z^2 = 5$

46. $9x^2 + 4y^2 - 8z^2 = 72$

47. $x^2 - y + z^2 = 0$

48. $z = 4x^2 + y^2$

49. $x^2 - y^2 + z = 0$

50. $z^2 - x^2 - \frac{y^2}{4} = 1$

51. $4x^2 - y^2 + 4z^2 = -16$

52. $z^2 = x^2 + \frac{y^2}{4}$

53. $z^2 = 9x^2 + y^2$

54. $4y = x^2 + z^2$

55. $3z = -y^2 + x^2$

56. $z^2 = 2x^2 + 2y^2$

In Exercises 57–60, use a three-dimensional graphing utility to graph the function.

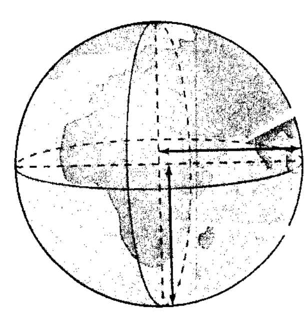
57. $z = y^2 - x^2 + 1$

58. $z = x^2 + y^2 + 1$

59. $z = \frac{x^2}{2} + \frac{y^2}{4}$

60. $z = \frac{1}{12}\sqrt{144 - 16x^2 - 9y^2}$

61. ***Physical Science*** Because of the forces caused by its rotation, Earth is actually an oblate ellipsoid rather than a sphere. The equatorial radius is 3963 miles and the polar radius is 3950 miles. Find an equation of the ellipsoid. Assume that the center of Earth is at the origin and the xy-trace ($z = 0$) corresponds to the equator.



Equatorial radius = 3963 mi

Polar radius = 3950 mi