PREREQUISITE REVIEW 10.1

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, find the limit.

1. $\lim_{x\to\infty} \frac{1}{x^3}$

2. $\lim_{x\to\infty} \frac{1}{x^2 - 4}$

3. $\lim_{x\to\infty} \frac{2x^2}{x^2 + 1}$

4. $\lim_{x\to\infty} \frac{x^3 - 1}{x^2 + 2}$

5. $\lim_{x\to\infty} e^{-2x}$

6. $\lim_{x\to\infty} \frac{1}{2^{x-1}}$

In Exercises 7–10, simplify the expression.

7. $\frac{n^2 - 4}{n^2 + 2n}$

8. $\frac{n^2 + n - 12}{n^2 - 16}$

9. $\frac{3}{n} + \frac{1}{n^3}$

10. $\frac{1}{n - 1} + \frac{1}{n + 2}, \quad n \ge 2$

EXERCISES 10.1

In Exercises 1–8, write out the first five terms of the specified sequence.

1. $a_n = 2^n$

2. $a_n = \left(-\frac{1}{2}\right)^n$

3. $a_n = \frac{n}{n + 1}$

4. $a_n = \frac{n - 1}{n^2 + 2}$

5. $a_n = \frac{3^n}{n!}$

6. $a_n = \frac{3n!}{(n - 1)!}$

7. $a_n = \frac{(-1)^n}{n^2}$

8. $a_n = 5 - \frac{1}{n} + \frac{1}{n^2}$

In Exercises 9–20, determine the convergence or divergence of the sequence. If the sequence converges, find its limit.

9. $a_n = \frac{5}{n}$

10. $a_n = \frac{n}{2}$

11. $a_n = \frac{n + 1}{n}$

12. $a_n = \frac{1}{n^{3/2}}$

13. $a_n = \frac{n^2 + 3n - 4}{2n^2 + n - 3}$

14. $a_n = \frac{\sqrt{n}}{\sqrt{n + 1}}$

15. $a_n = \frac{n^2 - 25}{n + 5}$

16. $a_n = \frac{n + 2}{n^2 + 1}$

17. $a_n = \frac{1 + (-1)^n}{n}$

18. $a_n = 1 + (-1)^n$

19. $a_n = \frac{n!}{n}$

20. $a_n = \frac{n!}{(n + 1)!}$

In Exercises 21–28, determine the convergence or divergence of the sequence. If the sequence converges, use a symbolic algebra utility to find its limit.

21. $a_n = 3 - \frac{1}{2^n}$

22. $a_n = \frac{n}{\sqrt{n^2 + 1}}$

23. $a_n = \frac{3^n}{4^n}$

24. $a_n = (0.5)^n$

25. $a_n = \frac{(n + 1)!}{n!}$

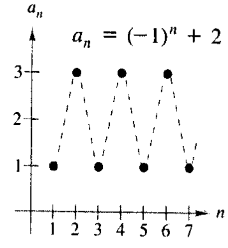
26. $a_n = \frac{(n - 2)!}{n!}$

27. $a_n = (-1)^n\left(\frac{n}{n + 1}\right)$

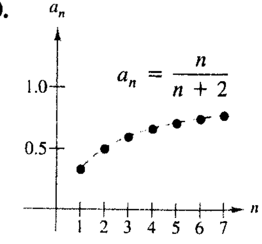
28. $a_n = \frac{n - 1}{n} - \frac{n}{n - 1}, \quad n \ge 2$

In Exercises 29 and 30, use the graph of the sequence to decide whether the sequence converges or diverges. Then verify your result analytically.

29.



30.



In Exercises 31–44, wı
sequence.

31. 1, 4, 7, 10, . . .

32. 3, 7, 11, 15, . . .

33. −1, 4, 9, 14, . .

34. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$

35. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$

36. $2, \frac{3}{3}, \frac{4}{5}, \frac{5}{7}, \frac{6}{9}, \ldots$

37. $2, -1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots$

38. $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \ldots$

39. $2, 1 + \frac{1}{2}, 1 + \frac{1}{3}, 1$

40. $1 + \frac{1}{2}, 1 + \frac{1}{4}, 1 +$

41. −2, 2, −2, 2, . .

42. 2, −4, 6, −8, 10,

43. $-x, \frac{x^2}{2}, -\frac{x^3}{3}, \frac{x^4}{4}, \ldots$

44. $1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x}{1}$

In Exercises 45–48, wr
sequence.

45. 2, 5, 8, 11, . . .

47. $1, \frac{5}{3}, \frac{7}{3}, 3, \ldots$

In Exercises 49–52, wr
sequence.

49. $3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \ldots$

51. 2, 6, 18, 54, . . .

In Exercises 53–56, dete
or geometric, and write

53. $20, 10, 5, \frac{5}{2}, \ldots$

55. $\frac{8}{3}, \frac{10}{3}, 4, \frac{14}{3}, \ldots$

56. 378, −126, 42, −

In Exercises 57 and 58,
the given condition. (Tl

57. A sequence that c

58. A sequence that c

59. ***Compound Int***
whose *n*th term is

$A_n = P\left[1 + \frac{}{1}\right.$

where P is the pr
interest after n mo
Find the first 10 te
$r = 0.06$.

in earlier sections. You will

e convergence or divergence of
nverges, use a symbolic algebra

22. $a_n = \frac{n}{\sqrt{n^2 + 1}}$

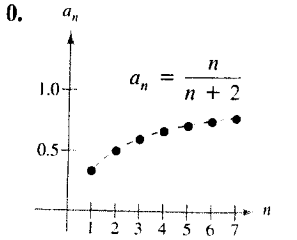
24. $a_n = (0.5)^n$

26. $a_n = \frac{(n - 2)!}{n!}$

2

iph of the sequence to decide
or diverges. Then verify your

0.



In Exercises 31–44, write an expression for the nth term of the sequence.

31. $1, 4, 7, 10, \ldots$

32. $3, 7, 11, 15, \ldots$

33. $-1, 4, 9, 14, \ldots$

34. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$

35. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \ldots$

36. $2, \frac{3}{3}, \frac{4}{5}, \frac{5}{7}, \frac{6}{9}, \ldots$

37. $2, -1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \ldots$

38. $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \ldots$

39. $2, 1 + \frac{1}{2}, 1 + \frac{1}{3}, 1 + \frac{1}{4}, \ldots$

40. $1 + \frac{1}{2}, 1 + \frac{1}{4}, 1 + \frac{1}{8}, 1 + \frac{1}{16}, \ldots$

41. $-2, 2, -2, 2, \ldots$

42. $2, -4, 6, -8, 10, \ldots$

43. $-x, \frac{x^2}{2}, -\frac{x^3}{3}, \frac{x^4}{4}, \ldots$

44. $1, x, \frac{x^2}{2}, \frac{x^3}{6}, \frac{x^4}{24}, \frac{x^5}{120}, \ldots$

In Exercises 45–48, write the next two terms of the arithmetic sequence.

45. $2, 5, 8, 11, \ldots$

46. $\frac{7}{2}, 4, \frac{9}{2}, 5, \ldots$

47. $1, \frac{5}{3}, \frac{7}{3}, 3, \ldots$

48. $\frac{1}{2}, \frac{5}{4}, 2, \frac{11}{4}, \ldots$

In Exercises 49–52, write the next two terms of the geometric sequence.

49. $3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \ldots$

50. $5, 10, 20, 40, \ldots$

51. $2, 6, 18, 54, \ldots$

52. $9, 6, 4, \frac{8}{3}, \ldots$

In Exercises 53–56, determine whether the sequence is arithmetic or geometric, and write the nth term of the sequence.

53. $20, 10, 5, \frac{5}{2}, \ldots$

54. $100, 92, 84, 76, \ldots$

55. $\frac{8}{3}, \frac{10}{3}, 4, \frac{14}{3}, \ldots$

56. $378, -126, 42, -14, \ldots$

In Exercises 57 and 58, give an example of a sequence satisfying the given condition. (There is more than one correct answer.)

57. A sequence that converges to $\frac{3}{4}$

58. A sequence that converges to 100

59. ***Compound Interest*** Consider the sequence $\{A_n\}$, whose nth term is given by

$$A_n = P\left[1 + \frac{r}{12}\right]^n$$

where P is the principal, A_n is the amount at compound interest after n months, and r is the annual percentage rate. Find the first 10 terms of the sequence for $P = \$9000$ and $r = 0.06$.

60. ***Compound Interest*** Consider the sequence $\{A$ whose nth term is given by

$$A_n = P(1 + r)^n$$

where P is the principal, A_n is the amount at compou interest after n years, and r is the annual percentage r Find the first 10 terms of the sequence for $P = \$5000$ $r = 0.10$.

61. ***Individual Retirement Account*** A deposit of \$200 made each year in an account that earns 11% interest co pounded annually. The balance after n years is given by

$$A_n = 2000(11)[(1.1)^n - 1].$$

(a) Compute the first six terms of the sequence.

(b) Find the balance after 20 years by finding the 20th te of the sequence.

(c) Use a symbolic algebra utility to find the balance a 40 years by finding the 40th term of the sequence.

62. ***Investment*** A deposit of \$100 is made each month ir account that earns 6% interest, compounded monthly. balance in the account after n months is given by

$$A_n = 100(201)[(1.005)^n - 1].$$

(a) Compute the first six terms of this sequence.

(b) Find the balance after 5 years by computing the 6 term of the sequence.

(c) Find the balance after 20 years by computing the 24 term of the sequence.

63. ***Population Growth*** Consider an idealized populat with the characteristic that each population member p duces one offspring at the end of every time period. If e population member has a lifespan of three time periods the population begins with 10 newborn members, then table shown below gives the population during the first time periods.

	Time period				
Age bracket	1	2	3	4	5
0–1	10	10	20	40	70
1–2		10	10	20	40
2–3			10	10	20
Total	10	20	40	70	130

The sequence for the total population has the property

$$S_n = S_{n-1} + S_{n-2} + S_{n-3}, \qquad n > 3.$$

Find the total population during the next five time peri

64. ***Cost*** The average costs per day for a hospital room from 1995 through 2001 are shown in the table, where a_n is the average cost in dollars and n is the year, with $n = 1$ corresponding to 1995. *(Source: Health Forum)*

n	1	2	3	4	5	6	7
a_n	968	1006	1033	1067	1103	1149	1217

(a) Use the *regression* feature of a graphing utility to find a model of the form

$$a_n = kn + b, \quad n = 1, 2, 3, 4, 5, 6, 7$$

for the data. Use a graphing utility to plot the points and graph the model.

(b) Use the model to predict the cost in the year 2010.

65. ***Federal Debt*** It took more than 200 years for the United States to accumulate a \$1 trillion debt. Then it took just 8 years to get to \$3 trillion. The federal debt during the years 1987 through 2002 is approximated by the model

$$a_n = \frac{2.4 + 0.16n^2}{1 + 0.024n^2}, \quad n = 0, 1, 2, 3, \ldots, 15$$

where a_n is the debt in trillions and n is the year, with $n = 0$ corresponding to 1987. *(Source: U.S. Office of Management and Budget)*

(a) Find the terms of this finite sequence.

(b) Construct a bar graph that represents the sequence.

66. The sum of the first n positive integers is given by

$$S_n = \frac{n(n+1)}{2}, \quad n = 1, 2, 3, \ldots .$$

(a) Compute the first five terms of this sequence and verify that each term gives the correct sum.

(b) Find the sum of the first 50 positive integers.

67. The sum of the squares of the first n positive integers is

$$S_n = \frac{n(n+1)(2n+1)}{6}, \quad n = 1, 2, 3, \ldots .$$

(a) Compute the first five terms of this sequence and verify that each term gives the correct sum.

(b) Find the sum of the squares of the first 20 positive integers.

68. ***Physical Science*** A ball is dropped from a height of 12 feet, and on each rebound it rises to $\frac{2}{3}$ its preceding height.

(a) Write an expression for the height of the nth rebound.

(b) Determine the convergence or divergence of this sequence. If it converges, find the limit.

69. ***Budget Analysis*** A government program that currently costs taxpayers \$1.3 billion per year is to be cut back by 15% per year.

(a) Write an expression for the amount budgeted for this program after n years.

(b) Compute the budget amounts for the first 4 years.

(c) Determine the convergence or divergence of the sequence of reduced budgets. If the sequence converges, find its limit.

70. ***Inflation Rate*** If the average price of a new car increases $5\frac{1}{2}$% per year and the average price is currently \$13,000, then the average price after n years is

$$P_n = \$13{,}000(1.055)^n.$$

Compute the average prices for the first 5 years of increases.

71. ***Cost*** A well-drilling company charges \$16 for drilling the first foot of a well, \$16.10 for drilling the second foot, \$16.20 for the third foot, and so on. Determine the cost of drilling a 100-foot well.

72. ***Salary*** A person accepts a position with a company at a salary of \$32,800 for the first year. The person is guaranteed a raise of 5% per year for the first 4 years. Determine the person's salary during the fourth year of employment.

73. ***Sales*** The sales a_n (in billions of dollars) of Wal-Mart from 1994 through 2003 are shown below as ordered pairs of the form (n, a_n), where n is the year, with $n = 1$ corresponding to 1994. *(Source: Wal-Mart Stores, Inc.)*

(1, 82.494), (2, 93.627), (3, 104.859), (4, 117.958), (5, 137.634), (6, 165.013), (7, 191.329), (8, 217.799), (9, 244.524), (10, 256.329)

(a) Use the *regression* feature of a graphing utility to find a model of the form

$$a_n = bn^3 + cn^2 + dn + f, \quad n = 1, 2, 3, \ldots, 10$$

for the data. Graphically compare the points and the model.

(b) Use the model to predict the sales in the year 2010.

74. ***Biology*** Suppose that you have a single bacterium able to divide to form two new cells every half hour. At the end of the first half hour there are two individuals, at the end of the first hour there are four individuals, and so on.

(a) Write an expression for the nth term of the sequence.

(b) How many bacteria will there be after 10 hours? After 20 hours? *(Source: Adapted from Levine/Miller, Biology: Discovering Life, Second Edition)*

75. ***Think About It*** Consider the sequence whose nth term a_n is given by

$$a_n = \left(1 + \frac{1}{n}\right)^n.$$

Demonstrate that the terms of this sequence approach e by finding $a_1, a_{10}, a_{100}, a_{1000}$, and $a_{10,000}$.

10.

Sigma Notatio

In this section, you v

$\frac{1}{3}$ is a simple exampl

$\frac{1}{3} = 0.33333\ldots$

$= 0.3 + 0.03$

$= \frac{3}{10} + \frac{3}{10^2}$

$= \sum_{n=1}^{\infty} \frac{3}{10^n}$

The last notation is

Sigma Notation

The finite sum a

$\sum_{i=1}^{N} a_i.$

The letter i is the

upper limits of s

EXAMPLE 1

Sum

(a) $1 + 2 + 3 + 4$

(b) $3(1) + 3\left(\frac{1}{2}\right) +$

TRY IT 1

Use sigma notatio

$4\left(-\frac{1}{2}\right) + 4($

PREREQUISITE REVIEW 10.2

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1 and 2, add the fractions.

1. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

2. $1 + \frac{3}{4} + \frac{4}{6} + \frac{5}{8}$

In Exercises 3–6, evaluate the expression.

3. $\dfrac{1 - (\frac{1}{2})^5}{1 - \frac{1}{2}}$

4. $\dfrac{3[1 - (\frac{1}{3})^4]}{1 - \frac{1}{3}}$

5. $\dfrac{2[1 - (\frac{1}{4})^3]}{1 - \frac{1}{4}}$

6. $\dfrac{\frac{1}{2}[1 - (\frac{1}{2})^5]}{1 - \frac{1}{2}}$

In Exercises 7–10, find the limit.

7. $\lim_{n\to\infty} \dfrac{3n}{4n + 1}$

8. $\lim_{n\to\infty} \dfrac{3n}{n^2 + 1}$

9. $\lim_{n\to\infty} \dfrac{n!}{n! - 3}$

10. $\lim_{n\to\infty} \dfrac{2n! + 1}{4n! - 1}$

EXERCISES 10.2

In Exercises 1–4, find the first five terms of the sequence of partial sums.

1. $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$

2. $\sum_{n=1}^{\infty} (-1)^{n+1}\frac{3^n}{2^{n-1}} = 3 - \frac{9}{2} + \frac{27}{4} - \frac{81}{8} + \frac{243}{16} - \cdots$

3. $\sum_{n=1}^{\infty} \frac{3}{2^{n-1}} = 3 + \frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \cdots$

4. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} = 1 - \frac{1}{2} + \frac{1}{6} - \frac{1}{24} + \frac{1}{120} - \cdots$

In Exercises 5–12, verify that the infinite series diverges.

5. $\sum_{n=1}^{\infty} \frac{n}{n+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots$

6. $\sum_{n=1}^{\infty} \frac{n}{2n+3} = \frac{1}{5} + \frac{2}{7} + \frac{3}{9} + \frac{4}{11} + \cdots$

7. $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1} = \frac{1}{2} + \frac{4}{5} + \frac{9}{10} + \frac{16}{17} + \cdots$

8. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}} = \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{5}} + \frac{3}{\sqrt{10}} + \frac{4}{\sqrt{17}} + \cdots$

9. $\sum_{n=0}^{\infty} 3\left(\frac{3}{2}\right)^n = 3 + \frac{9}{2} + \frac{27}{4} + \frac{81}{8} + \cdots$

10. $\sum_{n=0}^{\infty} 5\left(-\frac{3}{2}\right)^n = 5 - \frac{15}{2} + \frac{45}{4} - \frac{135}{8} + \cdots$

11. $\sum_{n=0}^{\infty} 1000(1.055)^n = 1000 + 1055 + 1113.025 + \cdots$

12. $\sum_{n=0}^{\infty} 2(-1.03)^n = 2 - 2.06 + 2.1218 - \cdots$

In Exercises 13–16, verify that the geometric series converges.

13. $\sum_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^n = 2 + \frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \frac{81}{128} + \cdots$

14. $\sum_{n=0}^{\infty} 2\left(-\frac{1}{2}\right)^n = 2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \cdots$

15. $\sum_{n=0}^{\infty} (0.9)^n = 1 + 0.9 + 0.81 + 0.729 + \cdots$

16. $\sum_{n=0}^{\infty} (-0.6)^n = 1 - 0.6 + 0.36 - 0.216 + \cdots$

In Exercises 17–20, use a s
of the convergent series.

17. $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} +$

18. $\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^n = 2 + \frac{4}{3}$

19. $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n = 1 - \frac{1}{2}$

20. $\sum_{n=0}^{\infty} 2\left(-\frac{2}{3}\right)^n = 2 -$

In Exercises 21–30, find th

21. $\sum_{n=0}^{\infty} 2\left(\frac{1}{\sqrt{2}}\right)^n = 2 +$

22. $\sum_{n=0}^{\infty} 4\left(\frac{1}{4}\right)^n = 4 + 1 \cdot$

23. $1 + 0.1 + 0.01 + 0.$

24. $8 + 6 + \frac{9}{2} + \frac{27}{8} + \cdot$

25. $2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} + \cdot$

26. $4 - 2 + 1 - \frac{1}{2} + \cdot \cdot$

27. $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n}\right)$

28. $\sum_{n=0}^{\infty} [(0.7)^n + (0.9)^n]$

29. $\sum_{n=0}^{\infty} \left(\frac{1}{3^n} + \frac{1}{4^n}\right)$

30. $\sum_{n=0}^{\infty} [(0.4)^n - (0.8)^n]$

In Exercises 31–40, determ
the series. Use a symbolic a

31. $\sum_{n=1}^{\infty} \frac{n+10}{10n+1}$

33. $\sum_{n=1}^{\infty} \frac{n!+1}{n!}$

35. $\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$

36. $\sum_{n=0}^{\infty} \frac{1}{4^n}$

37. $\sum_{n=0}^{\infty} (1.075)^n$

38. $\sum_{n=1}^{\infty} \frac{2^n}{100}$

39. $\sum_{n=0}^{\infty} \frac{3}{4^n}$

40. $\sum_{n=0}^{\infty} n!$

earlier sections. You will

$\frac{81}{8} + \cdots$

$-\frac{135}{8} + \cdots$

055 + 1113.025 + · · ·

2.1218 − · · ·

eometric series converges.

$\frac{7}{2} + \frac{81}{128} + \cdots$

$\frac{1}{4} + \frac{1}{8} - \cdots$

+ 0.729 + · · ·

5 − 0.216 + · · ·

In Exercises 17–20, use a symbolic algebra utility to find the sum of the convergent series.

17. $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

18. $\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^n = 2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \cdots$

19. $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$

20. $\sum_{n=0}^{\infty} 2\left(-\frac{2}{3}\right)^n = 2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27} + \cdots$

In Exercises 21–30, find the sum of the convergent series.

21. $\sum_{n=0}^{\infty} 2\left(\frac{1}{\sqrt{2}}\right)^n = 2 + \sqrt{2} + 1 + \frac{1}{\sqrt{2}} + \cdots$

22. $\sum_{n=0}^{\infty} 4\left(\frac{1}{4}\right)^n = 4 + 1 + \frac{1}{4} + \frac{1}{16} + \cdots$

23. $1 + 0.1 + 0.01 + 0.001 + \cdots$

24. $8 + 6 + \frac{9}{2} + \frac{27}{8} + \cdots$

25. $2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} + \cdots$

26. $4 - 2 + 1 - \frac{1}{2} + \cdots$

27. $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} - \frac{1}{3^n}\right)$

28. $\sum_{n=0}^{\infty} [(0.7)^n + (0.9)^n]$

29. $\sum_{n=0}^{\infty} \left(\frac{1}{3^n} + \frac{1}{4^n}\right)$

30. $\sum_{n=0}^{\infty} [(0.4)^n - (0.8)^n]$

In Exercises 31–40, determine the convergence or divergence of the series. Use a symbolic algebra utility to verify your result.

31. $\sum_{n=1}^{\infty} \frac{n + 10}{10n + 1}$

32. $\sum_{n=0}^{\infty} \frac{4}{2^n}$

33. $\sum_{n=1}^{\infty} \frac{n! + 1}{n!}$

34. $\sum_{n=1}^{\infty} \frac{n + 1}{2n - 1}$

35. $\sum_{n=1}^{\infty} \frac{3n - 1}{2n + 1}$

36. $\sum_{n=0}^{\infty} \frac{1}{4^n}$

37. $\sum_{n=0}^{\infty} (1.075)^n$

38. $\sum_{n=1}^{\infty} \frac{2^n}{100}$

39. $\sum_{n=0}^{\infty} \frac{3}{4^n}$

40. $\sum_{n=0}^{\infty} n!$

In Exercises 41–44, the repeating decimal is expressed as a geometric series. Find the sum of the geometric series and write the decimal as the ratio of two integers.

41. $0.\overline{6} = 0.6 + 0.06 + 0.006 + 0.0006 + \cdots$

42. $0.\overline{23} = 0.23 + 0.0023 + 0.000023 + \cdots$

43. $0.\overline{81} = 0.81 + 0.0081 + 0.000081 + \cdots$

44. $0.\overline{21} = 0.21 + 0.0021 + 0.000021 + \cdots$

45. ***Sales*** A company produces a new product for which it estimates the annual sales to be 8000 units. Suppose that in any given year 10% of the units (regardless of age) will become inoperative.

(a) How many units will be in use after n years?

(b) Find the market stabilization level of the product.

46. ***Sales*** Repeat Exercise 45 with the assumption that 25% of the units will become inoperative each year.

47. ***Physical Science*** A ball is dropped from a height of 16 feet. Each time it drops h feet, it rebounds $0.64h$ feet. Find the total vertical distance traveled by the ball.

48. ***Physical Science*** The ball in Exercise 47 takes the times listed below for each fall. (t is measured in seconds.)

$s_1 = -16t^2 + 16$ $\qquad s_1 = 0$ if $t = 1$

$s_2 = -16t^2 + 16(0.64)$ $\qquad s_2 = 0$ if $t = 0.8$

$s_3 = -16t^2 + 16(0.64)^2$ $\qquad s_3 = 0$ if $t = (0.8)^2$

$s_4 = -16t^2 + 16(0.64)^3$ $\qquad s_4 = 0$ if $t = (0.8)^3$

$\vdots \qquad\qquad \vdots$

$s_n = -16t^2 + 16(0.64)^{n-1}$ $\qquad s_n = 0$ if $t = (0.8)^{n-1}$

Beginning with s_2, the ball takes the same amount of time to bounce up as to fall, and so the total elapsed time before the ball comes to rest is

$$t = 1 + 2\sum_{n=1}^{\infty} (0.8)^n.$$

Find the total time it takes for the ball to come to rest.

49. ***Annuity*** A deposit of \$100 is made at the beginning of each month for 5 years in an account that pays 10% interest, compounded monthly. Use a symbolic algebra utility to find the balance A in the account at the end of the 5 years.

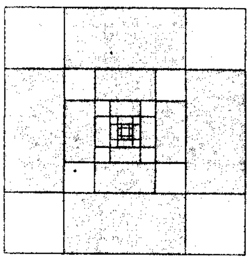
$$A = 100\left(1 + \frac{0.10}{12}\right) + \cdots + 100\left(1 + \frac{0.10}{12}\right)^{60}$$

50. ***Annuity*** A deposit of P dollars is made every month for t years in an account that pays an annual interest rate of r%, compounded monthly. Let $N = 12t$ be the total number of deposits. Show that the balance in the account after t years is

$$A = P\left[\left(1 + \frac{r}{12}\right)^N - 1\right]\left(1 + \frac{12}{r}\right), \quad t > 0.$$

51. ***Consumer Trends: Multiplier Effect*** The annual spending by tourists in a resort city is 100 million dollars. Approximately 75% of that revenue is again spent in the resort city, and of that amount approximately 75% is again spent in the resort city. If this pattern continues, write the geometric series that gives the total amount of spending generated by the 100 million dollars and find the sum of the series.

52. ***Area*** Find the fraction of the total area of the square that is eventually shaded blue if the pattern of shading shown in the accompanying figure is continued. (Note that the lengths of the sides of the shaded corner squares are one-fourth the lengths of the sides of the squares in which they are placed.)



53. ***Wages*** Suppose that an employer offered to pay you 1¢ the first day, and then double your wages each day thereafter. Find your total wages for working 20 days.

54. ***Probability: Coin Toss*** A fair coin is tossed until a head appears. The probability that the first head appears on the nth toss is given by $P = \left(\frac{1}{2}\right)^n$, where $n \geq 1$. Show that

$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = 1.$$

55. ***Probability: Coin Toss*** Use a symbolic algebra utility to estimate the expected number of tosses required until the first head occurs in the experiment in Exercise 54.

56. ***Profit*** The annual profits for eBay from 1998 through 2003 can be approximated by the model

$$a_n = 5.981e^{0.722n}, \qquad n = 1, 2, 3, \ldots, 6$$

where a_n is the annual net profit (in millions of dollars) and n is the year, with $n = 1$ corresponding to 1998. Use the formula for the sum of a geometric sequence to approximate the total net profits earned during this 6-year period. *(Source: eBay, Inc.)*

57. ***Environment*** A factory is polluting a river such that at every mile down river from the factory an environmental expert finds 15% less pollutant than at the preceding mile. If the pollutant's concentration is 500 ppm at the factory, what is its concentration 12 miles down river?

58. ***Physical Science*** In a certain brand of CD player, after the STOP function is activated, the disc, during each second after the first second, makes 85% fewer revolutions than it made during the preceding second. In coming to rest, how many revolutions does the disc make if it makes 5.5 revolutions during the first second after the STOP function is activated?

59. ***Finance: Annuity*** The simplest kind of annuity is a straight-line annuity, which pays a fixed amount per month until the annuitant dies. Suppose that, when he turns 65, Bob wants to purchase a straight-line annuity that has a premium of \$100,000 and pays \$880 per month. Use sigma notation to represent each scenario below, and give the numerical amount that the summation represents. *(Source: Adapted from Garman/Forgue, Personal Finance, Fifth Edition)*

(a) Suppose Bob dies 10 months after he takes out the annuity. How much will he have collected up to that point?

(b) Suppose Bob lives the average number of months beyond age 65 for a man (168 months). How much more or less than the \$100,000 will he have collected?

60. ***Area*** A news reporter is being televised in such a way that an inset of the same camera view is shown in the corner of the screen above the reporter's left shoulder. Another inset view appears in this inset view, and so forth for all subsequent inset views. If in the principal camera view, the reporter occupies approximately 80 square inches of the screen, and if each repeating corner view is $\frac{1}{8}$ of the area of the preceding view, what is the total area of the screen occupied by the news reporter?

In Exercises 61–66, use a symbolic algebra utility to evaluate the summation.

61. $\displaystyle\sum_{n=1}^{\infty} n^2 \left(\frac{1}{2}\right)^n$

62. $\displaystyle\sum_{n=1}^{\infty} 2n^3 \left(\frac{1}{5}\right)^n$

63. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n)!}$

64. $\displaystyle\sum_{n=1}^{\infty} n \left(\frac{4}{11}\right)^n$

65. $\displaystyle\sum_{n=1}^{\infty} e^2 \left(\frac{1}{e}\right)^n$

66. $\displaystyle\sum_{n=1}^{\infty} \ln 2 \left(\frac{1}{8}\right)^{2n}$

True or False? In Exercises 67 and 68, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

67. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

68. If $|r| < 1$, then $\sum_{n=1}^{\infty} ar^n = \dfrac{a}{1-r}$.

10.

p-Series

In Section 10.2, yc
another common ty|

Definition of p-

Let p be a positi

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^{\iota}}$$

is called a ***p*-seri**

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 -$$

is called the **har**

EXAMPLE 1

Classify each infinit

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n}$

SOLUTION

(a) The infinite serie

$$\sum_{n=1}^{\infty} \frac{1}{n^3} = \frac{1}{1^3}$$

is a p-series with

(b) The infinite serie

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \frac{}{1}$$

is a p-series with

(c) The infinite serie

$$\sum_{n=1}^{\infty} \frac{1}{3^n} = \frac{1}{3^1}$$

is *not* a p-series.

PREREQUISITE REVIEW 10.3

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, simplify the expression.

1. $\dfrac{n!}{(n+1)!}$

2. $\dfrac{(n+1)!}{n!}$

3. $\dfrac{3^{n+1}}{n+1} \cdot \dfrac{n}{3^n}$

4. $\dfrac{(n+1)^2}{(n+1)!} \cdot \dfrac{n!}{n^2}$

In Exercises 5–8, find the limit.

5. $\lim\limits_{n\to\infty} \dfrac{(n+1)^2}{n^2}$

6. $\lim\limits_{n\to\infty} \dfrac{5^{n+1}}{5^n}$

7. $\lim\limits_{n\to\infty} \left(\dfrac{5}{n+1} \div \dfrac{5}{n}\right)$

8. $\lim\limits_{n\to\infty} \left(\dfrac{(n+1)^3}{3^{n+1}} \div \dfrac{n^3}{3^n}\right)$

In Exercises 9 and 10, decide whether the series is geometric.

9. $\sum\limits_{n=1}^{\infty} \dfrac{1}{4^n}$

10. $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^4}$

EXERCISES 10.3

In Exercises 1–6, determine whether the series is a *p*-series.

1. $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^2}$

2. $\sum\limits_{n=1}^{\infty} \dfrac{1}{\sqrt{n}}$

3. $\sum\limits_{n=1}^{\infty} \dfrac{1}{3^n}$

4. $\sum\limits_{n=1}^{\infty} n^{-3/4}$

5. $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^n}$

6. $\sum\limits_{n=1}^{\infty} \dfrac{1}{n+1}$

In Exercises 7–16, determine the convergence or divergence of the *p*-series.

7. $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^3}$

8. $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^{1/3}}$

9. $\sum\limits_{n=1}^{\infty} \dfrac{1}{\sqrt[3]{n}}$

10. $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^{4/3}}$

11. $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^{1.03}}$

12. $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^{\pi}}$

13. $1 + \dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{3}} + \dfrac{1}{\sqrt{4}} + \cdots$

14. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$

15. $1 + \dfrac{1}{2\sqrt{2}} + \dfrac{1}{3\sqrt{3}} + \dfrac{1}{4\sqrt{4}} + \cdots$

16. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

In Exercises 17–30, use the Ratio Test to determine the convergence or divergence of the series.

17. $\sum\limits_{n=0}^{\infty} \dfrac{3^n}{n!}$

18. $\sum\limits_{n=1}^{\infty} n\left(\dfrac{2}{3}\right)^n$

19. $\sum\limits_{n=0}^{\infty} \dfrac{n!}{3^n}$

20. $\sum\limits_{n=1}^{\infty} n\left(\dfrac{3}{2}\right)^n$

21. $\sum\limits_{n=1}^{\infty} \dfrac{n}{4^n}$

22. $\sum\limits_{n=1}^{\infty} \dfrac{n^2}{2^n}$

23. $\sum\limits_{n=1}^{\infty} \dfrac{2^n}{n^5}$

24. $\sum\limits_{n=0}^{\infty} (-1)^n e^{-n}$

25. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!}$

26. $\displaystyle\sum_{n=0}^{\infty} \frac{4n}{n!}$

27. $\displaystyle\sum_{n=0}^{\infty} \frac{4^n}{3^n + 1}$

28. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n + 1}$

29. $\displaystyle\sum_{n=0}^{\infty} \frac{n5^n}{n!}$

30. $\displaystyle\sum_{n=1}^{\infty} \frac{2n!}{n^5}$

In Exercises 31–34, approximate the sum of the convergent series using the indicated number of terms. Estimate the maximum error of your approximation.

31. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$, four terms

32. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$, four terms

33. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$, 10 terms

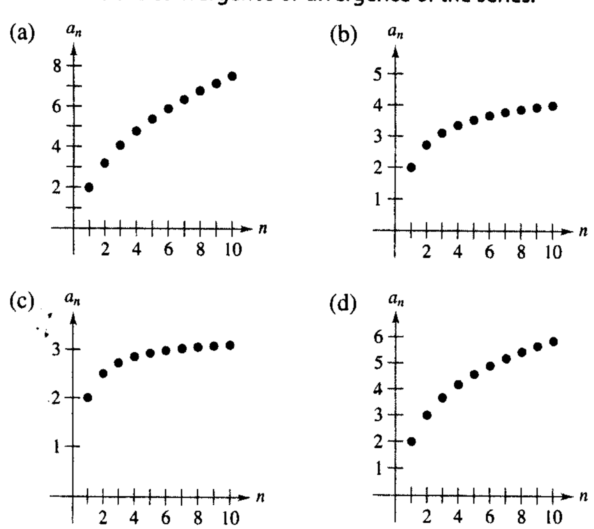
34. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{10}}$, three terms

In Exercises 35 and 36, verify that the Ratio Test is inconclusive for the *p*-series.

35. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$

In Exercises 37–40, match the series with the graph of its sequence of partial sums. [The graphs are labeled (a)–(d).] Determine the convergence or divergence of the series.



37. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{\sqrt[4]{n^3}}$

38. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n}$

39. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$

40. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n^2}$

In Exercises 41–56, test the series for convergence or divergence using any appropriate test from this chapter. Identify the test used and explain your reasoning. If the series is convergent, find the sum whenever possible.

41. $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{n + 1}$

42. $\displaystyle\sum_{n=1}^{\infty} \frac{10}{3\sqrt[3]{n^2}}$

43. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt[3]{n}}$

44. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{5}{6}\right)^n$

45. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{3^n}$

46. $\displaystyle\sum_{n=2}^{\infty} \ln n$

47. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^2} - \frac{1}{n^3}\right)$

48. $\displaystyle\sum_{n=1}^{\infty} \frac{n3^n}{n!}$

49. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n$

50. $\displaystyle\sum_{n=1}^{\infty} n(0.4)^n$

51. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{3^{n-1}}$

52. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{0.95}}$

53. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{2^{n+1} + 1}$

54. $\displaystyle\sum_{n=1}^{\infty} 2e^{-n}$

55. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{5^{n-1}}$

56. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n^2 + 1}$

In Exercises 57 and 58, use a computer to confirm the sum of the convergent series.

57. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

58. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n)^2} = \frac{\pi^2}{24}$

59. ***Expenses*** The table lists the research and development expenses at universities for life sciences (in billions of dollars) for the years 1996 through 2001. *(Source: U.S. National Science Foundation)*

Year	1996	1997	1998	1999	2000	2001
Amount	12.70	13.61	14.55	15.59	17.46	19.19

(a) Find a quadratic model for the data with $n = 1$ corresponding to 1996. Use this quadratic model to find an infinite series to model the data.

(b) Can you determine whether the series is converging or diverging by using the Ratio Test? Explain.

60. ***Research Project*** Use your school's library, the Internet, or some other reference source to find data about a college or university that can be modeled by a series. Write a summary of your findings, including a description of the convergence or divergence of the series and how it was determined.

10.

Power Series

In the preceding tw
stants. In this secti
Specifically, you wil
Informally, you can

Definition of Po

An infinite serie

$\sum_{n=0}^{\infty} a_n x^n = \ldots$

is called a **powe**

$\sum_{n=0}^{\infty} a_n(x - \ldots$

is called a **powe**

EXAMPLE 1

(a) The power serie

$\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1$

is centered at ze

(b) The power serie

$\sum_{n=1}^{\infty} \frac{(x-1)}{n}$

is centered at 1.

(c) The power serie

$\sum_{n=1}^{\infty} \frac{(x+1)}{n}$

is centered at −

PREREQUISITE REVIEW 10.4

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, find $f(g(x))$ and $g(f(x))$.

1. $f(x) = x^2, \quad g(x) = x - 1$
2. $f(x) = 3x, \quad g(x) = 2x + 1$
3. $f(x) = \sqrt{x + 4}, \quad g(x) = x^2$
4. $f(x) = e^x, \quad g(x) = x^2$

In Exercises 5–8, find $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$.

5. $f(x) = 5e^x$
6. $f(x) = \ln x$
7. $f(x) = 3e^{2x}$
8. $f(x) = \ln 2x$

In Exercises 9 and 10, simplify the expression.

9. $\dfrac{3^n}{n!} \div \dfrac{3^{n+1}}{(n + 1)!}$
10. $\dfrac{n!}{(n + 2)!} \div \dfrac{(n + 1)!}{(n + 3)!}$

EXERCISES 10.4

In Exercises 1–4, write the first five terms of the power series.

1. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{x}{4}\right)^n$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(x - 2)^n}{3^n}$
3. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x + 1)^n}{n!}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{(n - 1)!}$

In Exercises 5–24, find the radius of convergence for the series.

5. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$
6. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{x}{k}\right)^n$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{3n}$
8. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} n x^n$
9. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$
10. $\displaystyle\sum_{n=0}^{\infty} \frac{(3x)^n}{n!}$
11. $\displaystyle\sum_{n=0}^{\infty} n!\left(\frac{x}{2}\right)^n$
12. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n + 1)(n + 2)}$
13. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{4^n}$
14. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n!(x - 4)^n}{3^n}$
15. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x - 5)^n}{n5^n}$
16. $\displaystyle\sum_{n=0}^{\infty} \frac{(x - 2)^{n+1}}{(n + 1)3^{n+1}}$
17. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x - 1)^{n+1}}{n + 1}$
18. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x - c)^n}{nc^n}$
19. $\displaystyle\sum_{n=1}^{\infty} \frac{(x - c)^{n-1}}{c^{n-1}}, \quad 0 < c$
20. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{2n - 1}$
21. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n + 1)!}(-2x)^{n-1}$
22. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$
23. $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n + 1)!}$
24. $\displaystyle\sum_{n=0}^{\infty} \frac{n! x^n}{(n + 1)!}$

In Exercises 25–32, apply Taylor's Theorem to find the power series (centered at c) for the function, and find the radius of convergence.

Function	Center
25. $f(x) = e^x$	$c = 0$
26. $f(x) = e^{-x}$	$c = 0$
27. $f(x) = e^{2x}$	$c = 0$
28. $f(x) = e^{-2x}$	$c = 0$
29. $f(x) = \dfrac{1}{x+1}$	$c = 0$
30. $f(x) = \dfrac{1}{2-x}$	$c = 0$
31. $f(x) = \sqrt{x}$	$c = 1$
32. $f(x) = \sqrt{x}$	$c = 4$

In Exercises 33–36, apply Taylor's Theorem to find the binomial series (centered at $c = 0$) for the function, and find the radius of convergence.

33. $f(x) = \dfrac{1}{(1+x)^3}$

34. $f(x) = \sqrt{1+x}$

35. $f(x) = \dfrac{1}{\sqrt{1+x}}$

36. $f(x) = \sqrt[3]{1+x}$

In Exercises 37–40, find the radius of convergence of (a) $f(x)$, (b) $f'(x)$, (c) $f''(x)$, and (d) $\int f(x)\,dx$.

37. $f(x) = \displaystyle\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$

38. $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n5^n}$

39. $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^{n+1}}{n+1}$

40. $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}$

In Exercises 41–52, find the power series for the function using the suggested method. Use the basic list of power series for elementary functions on page 687.

41. Use the power series for e^x.

$f(x) = e^{x^3}$

42. Use the series found in Exercise 41.

$f(x) = e^{-x^3}$

43. Differentiate the series found in Exercise 41.

$f(x) = 3x^2e^{x^3}$

44. Use the power series for e^x and e^{-x}.

$$f(x) = \frac{e^x + e^{-x}}{2}$$

45. Use the power series for $\dfrac{1}{x+1}$.

$$f(x) = \frac{1}{x^4+1}$$

46. Use the power series for $\dfrac{1}{x+1}$.

$$f(x) = \frac{2x}{x+1}$$

47. Use the power series for $\dfrac{1}{x+1}$.

$$f(x) = \ln(x^2+1)$$

48. Integrate the series for $\dfrac{1}{x+1}$.

$$f(x) = \ln(x+1)$$

49. Integrate the series for $\dfrac{1}{x}$.

$$f(x) = \ln x$$

50. Differentiate the series found in Exercise 44.

$$f(x) = \frac{e^x - e^{-x}}{2}$$

51. Differentiate the series for $-\dfrac{1}{x}$.

$$f(x) = \frac{1}{x^2}$$

52. Differentiate the power series for e^x term by term and use the resulting series to show that

$$\frac{d}{dx}[e^x] = e^x.$$

In Exercises 53 and 54, use the Taylor series for the exponential function to approximate the expression to four decimal places.

53. $e^{1/2}$

54. e^{-1}

In Exercises 55–58, use a symbolic algebra utility and 50 terms of the series to approximate the function

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}, \quad 0 < x \le 2.$$

55. $f(0.5)$ (Actual sum is ln 0.5.)

56. $f(1.5)$ (Actual sum is ln 1.5.)

57. $f(0.1)$ (Actual sum is ln 0.1.)

58. $f(1.95)$ (Actual sum is ln 1.95.)

10.5

Taylor Polynom

In Section 10.4, you s
series representation c

$f(x) = e^{-x}$

can be represented ex

$e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

$= 1 - x + \frac{x}{2}$

The problem with usin
depends on the summ
feasible, and you mus
function rather than re
partial sums below.

$S_0(x) = 1$

$S_1(x) = 1 - x$

$S_2(x) = 1 - x +$

$S_3(x) = 1 - x +$

$S_4(x) = 1 - x +$

$S_5(x) = 1 - x +$

⋮

$S_n(x) = 1 - x +$

Each of these polynom
As n approaches infini
and closer approximati

$f(x) = e^{-x}$.