PREREQUISITE REVIEW 10.5

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–6, find a power series representation for the function.

1. $f(x) = e^{3x}$
2. $f(x) = e^{-3x}$
3. $f(x) = \dfrac{4}{x}$
4. $f(x) = \ln 5x$
5. $f(x) = (1 + x)^{1/4}$
6. $f(x) = \sqrt{1 + x}$

In Exercises 7–10, evaluate the definite integral.

7. $\displaystyle\int_0^1 (1 - x + x^2 - x^3 + x^4)\,dx$
8. $\displaystyle\int_0^{1/2} \left(1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{27}\right) dx$
9. $\displaystyle\int_1^2 \left[(x - 1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}\right] dx$
10. $\displaystyle\int_1^{3/2} [1 - (x - 1) + (x - 1)^2 - (x - 1)^3]\,dx$

EXERCISES 10.5

In Exercises 1–6, find the Taylor polynomials (centered at zero) of degrees (a) 1, (b) 2, (c) 3, and (d) 4.

1. $f(x) = e^x$
2. $f(x) = \ln(x + 1)$
3. $f(x) = \sqrt{x + 1}$
4. $f(x) = \dfrac{1}{(x + 1)^2}$
5. $f(x) = \dfrac{x}{x + 1}$
6. $f(x) = \dfrac{4}{x + 1}$

In Exercises 7 and 8, complete the table using the indicated Taylor polynomials as approximations of the function f.

7.

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
$f(x) = e^{x/2}$					
$1 + \frac{x}{2}$					
$1 + \frac{x}{2} + \frac{x^2}{8}$					
$1 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{48}$					
$1 + \frac{x}{2} + \frac{x^2}{8} + \frac{x^3}{48} + \frac{x^4}{384}$					

8.

x	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
$f(x) = \ln(x^2 + 1)$				
x^2				
$x^2 - \frac{x^4}{2}$				
$x^2 - \frac{x^4}{2} + \frac{x^6}{3}$				
$x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \frac{x^8}{4}$				

In Exercises 9 and 10, use a symbolic differentiation utility to find the Taylor polynomials (centered at zero) of degrees (a) 2, (b) 4, (c) 6, and (d) 8.

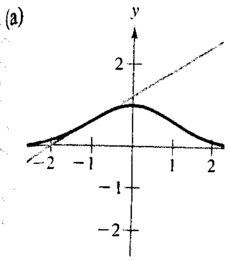
9. $f(x) = \dfrac{1}{1 + x^2}$
10. $f(x) = e^{-x^2}$

In Exercises 11 and 12, use a symbolic differentiation utility to find the fourth-degree Taylor polynomial (centered at zero).

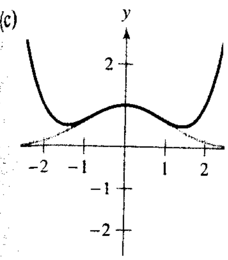
11. $f(x) = \dfrac{1}{x^2 + 1}$
12. $f(x) = xe^x$

In Exercises 13–16, matc of the function $f(x) =$ labeled (a)–(d).] Use a gr

(a)



(c)



13. $y = -\frac{1}{2}x^2 + 1$
14. $y = \frac{1}{8}x^4 - \frac{1}{2}x^2 + 1$
15. $y = e^{-1/2}[(x + 1) +$
16. $y = e^{-1/2}[\frac{1}{3}(x - 1)^3$

In Exercises 17–20, use a si at c for the function f to ol

Function

17. $f(x) = e^{-x}, \quad c = 0$
18. $f(x) = x^2e^{-x}, \quad c = ($
19. $f(x) = \ln x, \quad c = 2$
20. $f(x) = \sqrt{x}, \quad c = 4$

In Exercises 21–24, use a six at zero to approximate the

Function

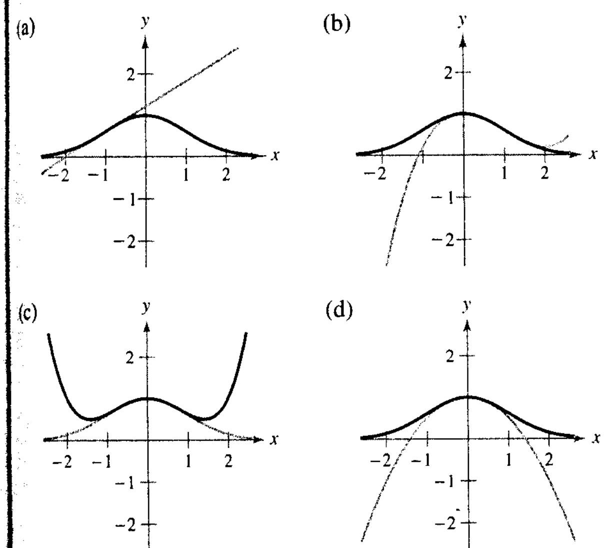
21. $f(x) = e^{-x^2}$
22. $f(x) = \ln(x^2 + 1)$
23. $f(x) = \dfrac{1}{\sqrt{1 + x^2}}$
24. $f(x) = \dfrac{1}{\sqrt[3]{1 + x^2}}$

er sections. You will

ferentiation utility to find
) of degrees (a) 2, (b) 4, (c)

: differentiation utility to
al (centered at zero).

In Exercises 13–16, match the Taylor polynomial approximation of the function $f(x) = e^{-x^2/2}$ with its graph. [The graphs are labeled (a)–(d).] Use a graphing utility to verify your results.



13. $y = -\frac{1}{2}x^2 + 1$

14. $y = \frac{1}{8}x^4 - \frac{1}{2}x^2 + 1$

15. $y = e^{-1/2}[(x + 1) + 1]$

16. $y = e^{-1/2}\left[\frac{1}{3}(x - 1)^3 - (x - 1) + 1\right]$

In Exercises 17–20, use a sixth-degree Taylor polynomial centered at c for the function f to obtain the required approximation.

Function	*Approximation*
17. $f(x) = e^{-x}, \quad c = 0$	$f\left(\frac{1}{2}\right)$
18. $f(x) = x^2e^{-x}, \quad c = 0$	$f\left(\frac{1}{4}\right)$
19. $f(x) = \ln x, \quad c = 2$	$f\left(\frac{3}{2}\right)$
20. $f(x) = \sqrt{x}, \quad c = 4$	$f(5)$

In Exercises 21–24, use a sixth-degree Taylor polynomial centered at zero to approximate the integral.

Function	*Approximation*
21. $f(x) = e^{-x^2}$	$\int_0^1 e^{-x^2}\,dx$
22. $f(x) = \ln(x^2 + 1)$	$\int_{-1/4}^{1/4} \ln(x^2 + 1)\,dx$
23. $f(x) = \dfrac{1}{\sqrt{1 + x^2}}$	$\int_0^{1/2} \dfrac{1}{\sqrt{1 + x^2}}\,dx$
24. $f(x) = \dfrac{1}{\sqrt[3]{1 + x^2}}$	$\int_0^{1/2} \dfrac{1}{\sqrt[3]{1 + x^2}}\,dx$

In Exercises 25 and 26, determine the degree of the Taylor polynomial centered at c required to approximate f in the given interval to an accuracy of ± 0.001.

Function	*Interval*
25. $f(x) = e^x, \quad c = 1$	$[0, 2]$
26. $f(x) = \dfrac{1}{x}, \quad c = 1$	$\left[1, \frac{3}{2}\right]$

In Exercises 27 and 28, determine the maximum error guaranteed by Taylor's Theorem with Remainder when the fifth-degree Taylor polynomial is used to approximate f in the given interval.

27. $f(x) = e^{-x}, \quad [0, 1], \quad$ centered at 0

28. $f(x) = \dfrac{1}{x}, \quad \left[1, \frac{3}{2}\right], \quad$ centered at 1

29. ***Profit*** Let n be a random variable representing the number of units of a certain commodity sold per day in a certain store. The probability distribution of n is shown in the table.

n	0	1	2	3	4, . . .
$P(n)$	$\frac{1}{2}$	$\left(\frac{1}{2}\right)^2$	$\left(\frac{1}{2}\right)^3$	$\left(\frac{1}{2}\right)^4$	$\left(\frac{1}{2}\right)^5, \ldots$

(a) Show that

$$\sum_{n=0}^{\infty} P(n) = 1.$$

(b) Find the expected value of the random variable n.

(c) If there is a \$10 profit on each unit sold, what is the expected daily profit on this commodity?

30. ***Profit*** Repeat Exercise 29 for the probability distribution for n that is shown in the table below.

n	0	1	2	3, . . .
$P(n)$	$\frac{1}{2}\left(\frac{2}{3}\right)$	$\frac{1}{2}\left(\frac{2}{3}\right)^2$	$\frac{1}{2}\left(\frac{2}{3}\right)^3$	$\frac{1}{2}\left(\frac{2}{3}\right)^4, \ldots$

PREREQUISITE REVIEW 10.6

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

In Exercises 1–4, evaluate f and f' at the given x-value.

1. $f(x) = x^2 - 2x - 1, \quad x = 2.4$
2. $f(x) = x^3 - 2x^2 + 1, \quad x = -0.6$
3. $f(x) = e^{2x} - 2, \quad x = 0.35$
4. $f(x) = e^{x^2} - 7x + 3, \quad x = 1.4$

In Exercises 5–8, solve for x.

5. $|x - 5| \le 0.1$
6. $|4 - 5x| \le 0.01$
7. $\left|2 - \dfrac{x}{3}\right| \le 0.01$
8. $|2x + 7| \le 0.01$

In Exercises 9 and 10, find the point(s) of intersection of the graphs of the two equations.

9. $y = x^2 - x - 2$
   $y = 2x - 1$
10. $y = x^2$
    $y = x + 1$

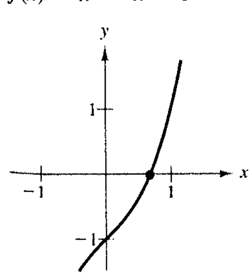
EXERCISES 10.6

In Exercises 1 and 2, complete one iteration of Newton's Method for the function using the given initial estimate.

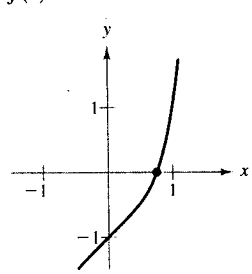
Function	*Initial Estimate*
1. $f(x) = x^2 - 5$	$x_1 = 2.2$
2. $f(x) = 3x^2 - 2$	$x_1 = 1$

In Exercises 3–12, approximate the indicated zero(s) of the function. Use Newton's Method, continuing until two successive approximations differ by less than 0.001.

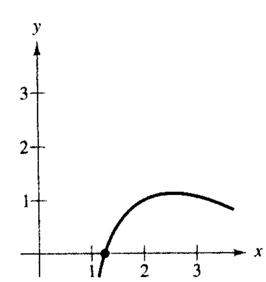
3. $f(x) = x^3 + x - 1$



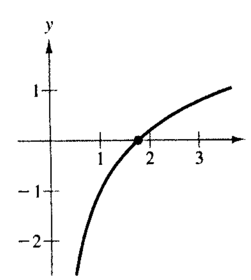
4. $f(x) = x^5 + x - 1$



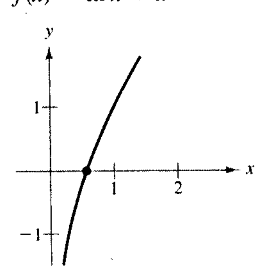
5. $y = 5\sqrt{x - 1} - 2x$



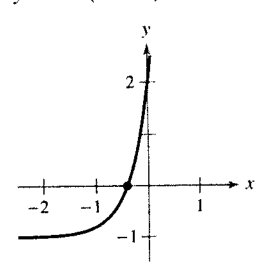
6. $f(x) = \ln x - \dfrac{1}{x}$



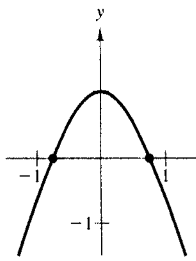
7. $f(x) = \ln x + x$



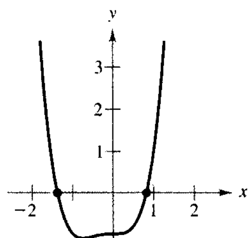
8. $y = e^{3x}(3 - x) - 1$



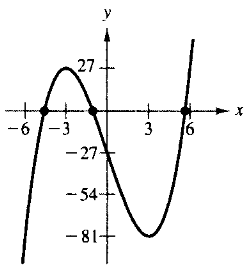
9. $f(x) = e^{-x^2} - x^2$



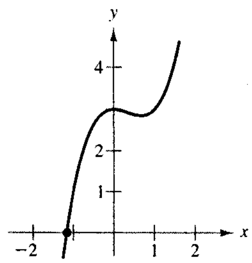
10. $y = x^4 + x^3 - 1$



11. $f(x) = x^3 - 27x - 27$

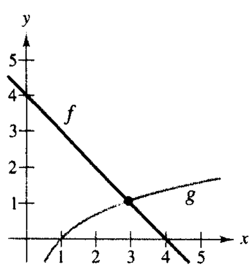


12. $y = x^3 - x^2 + 3$

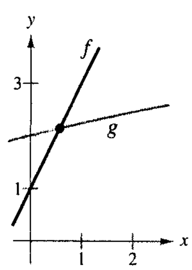


In Exercises 13–16, approximate, to three decimal places, the x-value of the point of intersection of the graphs.

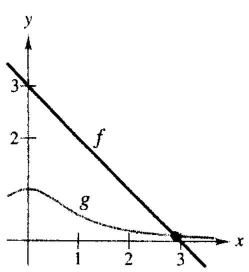
13. $f(x) = 4 - x$
$g(x) = \ln x$



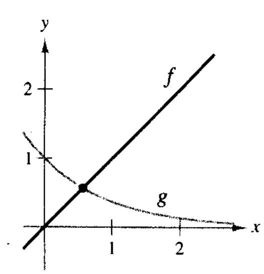
14. $f(x) = 2x + 1$
$g(x) = \sqrt{x + 4}$



15. $f(x) = 3 - x$
$g(x) = \dfrac{1}{x^2 + 1}$



16. $f(x) = x$
$g(x) = e^{-x}$



In Exercises 17–28, use a graphing utility to approximate all the real zeros of the function by Newton's Method. Graph the function to make the initial estimate of a zero.

17. $f(x) = \frac{1}{4}x^3 - 3x^2 + \frac{3}{4}x - 2$

18. $f(x) = \frac{1}{2}x^3 - \frac{11}{3}x^2 + \frac{10}{3}x + \frac{32}{3}$

19. $f(x) = -x^4 + 5x^2 - 5$

20. $f(x) = \frac{4}{11}x^4 + \frac{3}{7}x^2 - 2$

21. $f(x) = x^3 - 3.9x^2 + 4.79x - 1.881$

22. $f(x) = 0.16e^{x^2} - 4.1x^3 + 6.8$

23. $f(x) = 3\sqrt{x - 1} - x$

24. $f(x) = x^4 - 10x^2 - 11$

25. $f(x) = x^2 - \ln x - \frac{3}{2}$

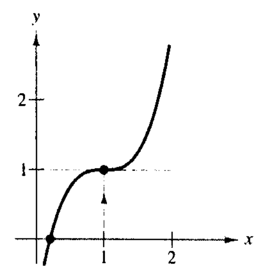
26. $f(x) = x + \sin(x + 1)$

27. $f(x) = x^3 - \cos x$

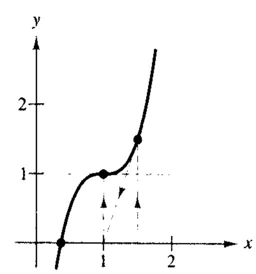
28. $f(x) = \sin \pi x + x - 1$

In Exercises 29–32, apply Newton's Method using the indicated initial estimate. Then explain why the method fails.

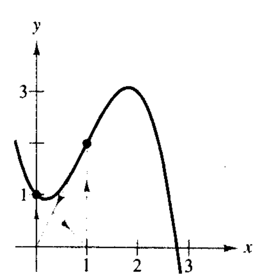
29. $y = 2x^3 - 6x^2 + 6x - 1, \quad x_1 = 1$



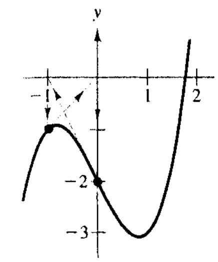
30. $y = 4x^3 - 12x^2 + 12x - 3, \quad x_1 = \frac{3}{2}$



31. $y = -x^3 + 3x^2 - x + 1, \quad x_1 = 1$



32. $y = x^3 - 2x - 2,$



In Exercises 33 and 34, us
rule for approximating th

33. $\sqrt{a}$ [*Hint:* Conside

34. $\sqrt[n]{a}$ [*Hint:* Conside

In Exercises 35–38, use
approximate the indicate

35. $\sqrt{7}$

36. $\sqrt{5}$

37. $\sqrt[4]{6}$

38. $\sqrt[3]{15}$

39. Use Newton's Meth

$x_{n+1} = x_n(2 - a$

can be used to appr
the reciprocal of a. N
reciprocals uses onl
subtraction. [*Hint:* C

40. Use the result of E
decimal places.

41. ***Distance*** A man
point on the coast (
which is 3 miles dow
row at 3 miles per
toward what point o
reach point Q in the

42. ***Average Cost*** A c
lars of producing x u

$C = 0.0001x^3 +$

Find the production
per unit.

1.881

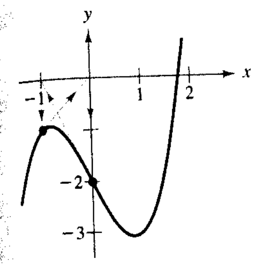
Method using the indicated method fails.

$= 1$

$= \frac{3}{2}$

1

32. $y = x^3 - 2x - 2, \quad x_1 = 0$



In Exercises 33 and 34, use Newton's Method to obtain a general rule for approximating the indicated radical.

33. $\sqrt{a}$ [*Hint:* Consider $f(x) = x^2 - a$.]

34. $\sqrt[n]{a}$ [*Hint:* Consider $f(x) = x^n - a$.]

In Exercises 35–38, use the results of Exercises 33 and 34 to approximate the indicated radical to three decimal places.

35. $\sqrt{7}$

36. $\sqrt{5}$

37. $\sqrt[4]{6}$

38. $\sqrt[3]{15}$

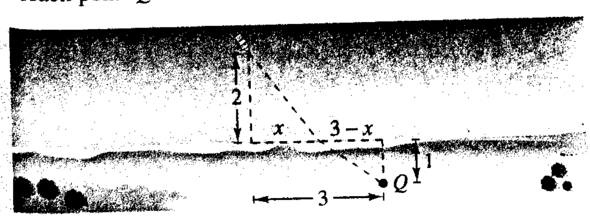
39. Use Newton's Method to show that the equation

$$x_{n+1} = x_n(2 - ax_n)$$

can be used to approximate $1/a$ if x_1 is an initial guess of the reciprocal of a. Note that this method of approximating reciprocals uses only the operations of multiplication and subtraction. [*Hint:* Consider $f(x) = (1/x) - a$.]

40. Use the result of Exercise 39 to approximate $\frac{1}{11}$ to three decimal places.

41. ***Distance*** A man is in a boat 2 miles from the nearest point on the coast (see figure). He is to go to a point Q, which is 3 miles down the coast and 1 mile inland. If he can row at 3 miles per hour and walk at 4 miles per hour, toward what point on the coast should he row in order to reach point Q in the shortest time?



42. ***Average Cost*** A company estimates that the cost in dollars of producing x units of a product is given by the model

$$C = 0.0001x^3 + 0.02x^2 + 0.4x + 800.$$

Find the production level that minimizes the average cost per unit.

43. ***Medicine*** The concentration C of an antibiotic in the bloodstream t hours after injection into muscle tissue is given by

$$C = \frac{3t^2 + t}{50 + t^3}.$$

When is the concentration the greatest?

44. ***Cost*** The ordering and transportation cost C of the components used in manufacturing a product is given by

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x + 30}\right), \quad x \ge 1$$

where C is measured in thousands of dollars and x is the order size in hundreds. Find the order size that minimizes the cost.

45. ***Forestry*** The value of a tract of timber is given by

$$V(t) = 100{,}000e^{0.8\sqrt{t}}$$

where t is the time in years, with $t = 0$ corresponding to 2000. If money earns interest at a rate of $r = 10\%$ compounded continuously, then the present value of the timber at any time t is given by

$$A(t) = V(t)e^{-0.10t}.$$

Assume the cost of maintenance of the timber to be a constant cash flow at the rate of $1000 per year. Then the total present value of this cost for t years is given by

$$C(t) = \int_0^t 1000e^{-0.10u}\,du$$

and the net present value of the tract of timber is given by

$$P(t) = A(t) - C(t).$$

Find the year when the timber should be harvested to maximize the present value function P.

46. ***Forestry*** Repeat Exercise 45 for

$$V(t) = 100{,}000e^{0.9\sqrt{t}}.$$

True or False? In Exercises 47–50, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

47. The zeros of

$$f(x) = \frac{p(x)}{q(x)}$$

coincide with the zeros of $p(x)$.

48. The roots of $\sqrt{f(x)} = 0$ coincide with the roots of $f(x) = 0$.

49. Every cubic polynomial has at least one real zero.

50. When using Newton's Method to find the zero of a polynomial, any initial approximation will work.