**Beverton–Holt Growth Curve**

Recall: For discrete exponential growth/decay we have

$$N_t = N_0 \cdot R^t \text{ for } t = 0, 1, 2, 3, \ldots$$

and for some constant $R$. Note also that

$$\begin{cases} 
N_t \text{ increases if } R > 1 \\
N_t \text{ decreases if } 0 < R < 1 \\
N_t \text{ is constant if } R = 1
\end{cases}$$

In addition, we have

$$N_{t+1} = N_0 \cdot R^{t+1} = N_0 \cdot R \cdot R^t = R(N_0 R^t) = R \cdot N_t$$

so that

$$N_{t+1} = RN_t$$

and

$$\frac{N_t}{N_{t+1}} = \frac{1}{R}$$

for $t = 0, 1, 2, 3, \ldots$.

We call $\frac{N_t}{N_{t+1}}$ the parent–offspring ratio. Since $\frac{1}{R}$ is a constant, we
say the growth is density independent, i.e., the growth rate does not depend on the size of $N_t$ at time $t$. In many cases, this is unrealistic due to limitations imposed by space, habitat, food, etc. It is more realistic to assume that the growth rate decreases as $N_t$ increases.

It is useful to plot $\frac{N_t}{N_{t+1}}$ vs. $N_t$.

For discrete exponential growth/decay we have

\[
\frac{N_t}{N_{t+1}} = \frac{1}{R}
\]

Beaverton-Holt Growth Curve

This discrete model assumes that the graph of $\frac{N_t}{N_{t+1}}$ vs. $N_t$
is an increasing linear function, which implies:
1.) $R > 1$ but decreases over time.
2.) The growth rate of $N_t$ decreases over time.
3.) The growth of $N_t$ is density dependent.

Note: If $\frac{N_t}{N_{t+1}} = 1$, then $N_{t+1} = N_t$,
which means there are no new members and $N_t$ has reached its carrying capacity, $K$, i.e.,
$\lim_{t \to \infty} N_t = K$. The following graph is consistent with this information:
The equation of the line is:
\[ Y = mX + b \rightarrow \]
\[ \frac{N_t}{N_{t+1}} = \left(1 - \frac{1}{R}\right) \cdot \frac{N_t}{K} + \frac{1}{R} \rightarrow \]
\[ \frac{R \cdot N_t}{N_{t+1}} = \frac{R-1}{K} \cdot N_t + 1 \rightarrow \]
\[ N_{t+1} = \frac{R \cdot N_t}{1 + \frac{R-1}{K} \cdot N_t} \quad \text{for } t=0,1,2,\ldots \]

This is the Beverton-Holt Growth Recursion.