

Math 17A  
Kouba  
Discussion Sheet 5

1.) Use the Intermediate Value Theorem to prove that each of the following equations is solvable.

a.)  $x^3 - x = 7$  on the interval  $[2, 3]$

b.)  $1 - x = \sqrt{x}$

c.)  $\frac{x^2}{x^2 + 1} = 2 - x$

2.) Use the Bisection Method to estimate the value of the solution to  $x^3 - x - 1 = 0$  to two decimal places.

3.) Use the Bisection Method to estimate the value of the solution to  $x^3 + 1 = x^2 - x$  to two decimal places.

4.) Use the limit definition of derivative to compute  $f'(x)$  for each of the following functions.

a.)  $f(x) = 3$       b.)  $f(x) = 3x + 5$       c.)  $f(x) = 2x^2 - 3x + 7$

d.)  $f(x) = 3 + \sqrt{x}$       e.)  $f(x) = \frac{x+1}{2-x}$       f.)  $f(x) = \sqrt{x^2 + 4}$

5.) Find an equation of the line tangent to the graph of the function in problem 4.)c.) at the point  $x = 2$ .

6.) Find an equation of the line normal (perpendicular) to the graph of the function in problem 4.)d.) at the point  $x = 1/4$ .

7.) Let  $f(x) = 2x^3 + 3x^2 - 12x$ . Solve  $f'(x) = 0$  for  $x$  and set up a sign chart for  $f'$ .

8.) Let  $f(x) = x - 2\sqrt{x}$ . Solve  $f'(x) = 0$  for  $x$  and set up a sign chart for  $f'$ .

9.) Let  $f(x) = \begin{cases} x^2 + 1, & \text{if } x > 0 \\ 1 - x^2, & \text{if } x \leq 0. \end{cases}$  Use the limit definition to show that  $f$  is differentiable at  $x = 0$ .

10.) Let  $f(x) = x^{1/3}$ . Use the limit definition to show that  $f$  is NOT differentiable at  $x = 0$ .

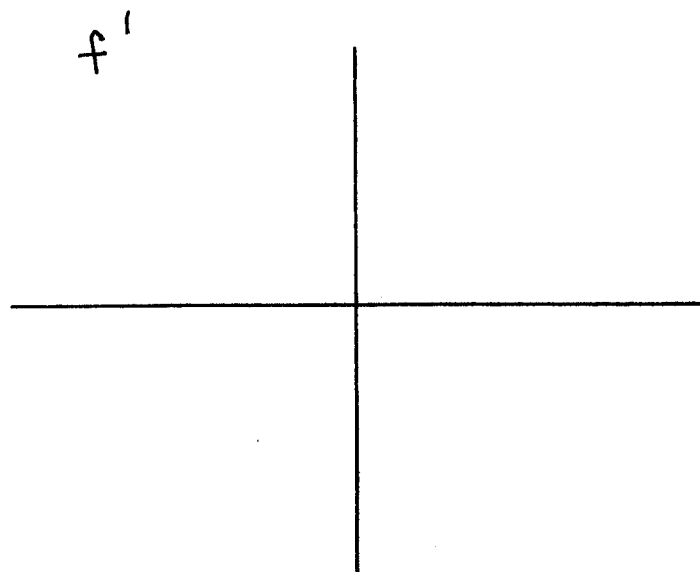
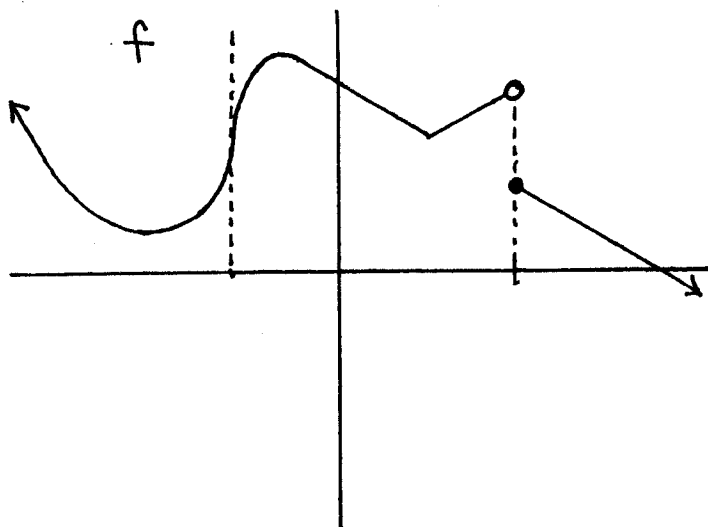
11.) Derive an equation of the line tangent to the graph of  $y = \frac{x+1}{2-x}$  at  $x = -1$ .

12.) Derive equations of all lines which are tangent to the graph of  $y = -7 - x^2$  and passing through the point  $(3, 0)$ .

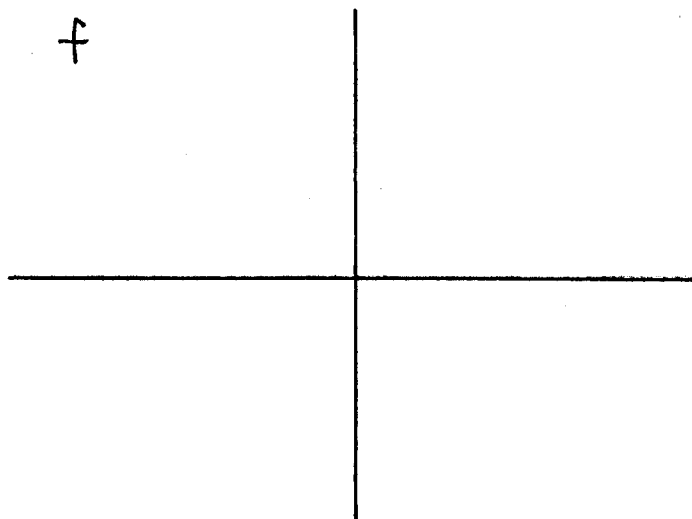
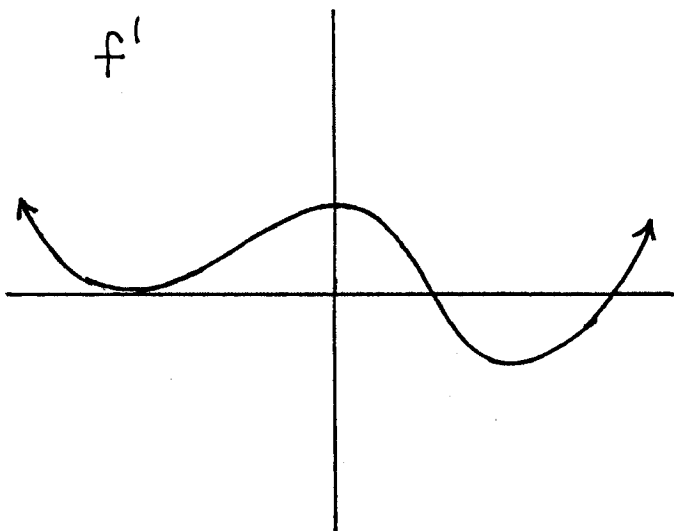
13.) After a long day of collecting nectar a tired honeybee is headed to its home beehive (sitting somewhere on the  $x$ -axis). It is flying (left to right) along the curve  $y = 3 \cos x$ . At the point  $x = \frac{\pi}{3}$  it sees home and travels straight towards the hive along a line tangent

to the graph of  $y = 3 \cos x$ . At what point  $x$  is the home beehive? (You may assume that  $D \cos x = -\sin x$ .)

14. Sketch the graph of  $f'$  from the graph of  $f$ .



15. Sketch the graph of  $f$  from the graph of  $f'$ . Assume that  $f(0) = 1$ .



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The following problem is for recreational purposes only.

16.) Assume that you have three boxes labeled and filled with fruit. One box contains APPLES only. One box contains ORANGES only. One box contains APPLES and ORANGES. Unfortunately, ALL of the boxes are labeled incorrectly. Explain how to correctly relabel all of the boxes by (without peeking into any box) selecting exactly one fruit from exactly one box.