Math 17A
Kouba
Discussion Sheet 7

1.) Solve for x.

a.) $\ln x = 3$ b.) $\ln x = \ln 3$ c.) $\ln(2x+1) - \ln(x+3) = 0$
d.) $\ln(x-1) + \ln(x-2) = 0$ e.) $\ln(x-2) + \ln(x+2) - \ln x = \ln 3$

2.) Let $f(x) = x^3 \ln x$. Solve $f'(x) = 0$ for x and solve$f''(x) = 0$ for x. Then set up a sign chart for each derivative.

3.) Let $f(x) = x^2 e^{-x}$. Solve $f'(x) = 0$ for x and $f''(x) = 0$ for x. Then set up a sign chart for each derivative.

4.) Find $y' = \dfrac{dy}{dx}$ as simply as possible. Do not simplify your answers.

a.) $y = \ln(5x+7)$ b.) $y = x^{\ln 7}$ c.) $y = \ln(x \tan x)$
d.) $y = \ln\left(\dfrac{x^2}{x^2+3}\right)$ e.) $y = \ln(\ln(\ln(\sin x)))$

5.) Solve for x.

a.) $e^x = 2$ b.) $7 \cdot e^{2x+3} = e^{3-x}$ c.) $(e^x+1)(e^x-5) = 0$
d.) $e^{2x} - 4 \cdot e^x = 0$ e.) $e^{2x} - 5 \cdot e^x + 6 = 0$

6.) Find $y' = \dfrac{dy}{dx}$. Do not simplify your answers.

a.) $y = 7 \cdot e^{5x-4}$ b.) $y = e^{x^2} \cdot \tan(3x)$ c.) $y = \dfrac{2^x + 4^x}{3^x + 5^x}$

7.) Assume that y is a function of x. Compute $y' = \dfrac{dy}{dx}$ and $y'' = \dfrac{d^2y}{dx^2}$ (You need not simplify y''.) for each equation.

a.) $y = e^{x^2}$ b.) $x = e^{y^2}$ c.) $y = \ln(3x-2)$ d.) $x = \ln(3y-2)$
e.) $y = \ln(3x-y)$ f.) $\sin x = y - \cos xy$ g.) $xy^2 = x + \tan y$

8.) Assume that $x^2 + (5-y)^3 = 2x + 125$.

a.) Find $y' = \dfrac{dy}{dx}$ at the point $(0,0)$ using implicit differentiation.

b.) Solve the original equation for y. Then find $y' = \dfrac{dy}{dx}$ at the point $(0,0)$ using ordinary differentiation.

9.) A herd of elk initially has 500 elk. After 5 years there are 750 elk. Assuming exponential growth, how many elk will there be after 8 years ?

10.) Double Trouble Bubble gum is sweet and delicious. You are chewing on a big wad of this bubble gum. The half-life of the sugar in your gum is 2 minutes. After chewing your gum for 10 minutes you measure the amount of sugar to be 3.5 grams. Assuming exponential decay, what was the initial amount of sugar in your gum ?

11.) Assume that a fossilized bone found today contains 35% of it original amount of carbon-14. If the half-life of carbon-14 is 5730 years, estimate the age of the fossil.

12.) Let $f(x) = x^3 + x + 5$.

a.) Use a derivative to verify that f is one-to-one. Thus, $y = f(x)$ has an inverse function, f^{-1}.

b.) Note that $f(1) = 7$ and then find $D\{f^{-1}(7)\}$.

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The following problem is for recreational purposes only.

13.) A horse is tethered by a rope to the corner of a small shed with a square 10 ft. by 10 ft. floor. If the rope is 40 feet long, sketch the shape of the horse's grazing area. How close can you plant flowers to the shed and keep the horse from eating them ?