35.) \( y = x^2 \) \( a. \) Slope
\[ m = \frac{f(2) - f(0)}{2 - 0} = \frac{4 - 0}{2} = 2 \]

\( b. \) \( f(x) = x^2 \) is continuous on \([0, 2]\) since it is a polynomial; \( f'(x) = 2x \) so \( f \) is differentiable on \((0, 2)\); by MVT there is a \( c \), \( 0 < c < 2 \), satisfying
\[ \frac{f(2) - f(0)}{2 - 0} = f'(c) \rightarrow 2 = 2c \rightarrow c = 1 \]

36.) \( y = \frac{1}{x} \)

\( \begin{array}{c}
\text{a. Slope} \\
\text{b.} \ f(x) = \frac{1}{x} \text{ is continuous on } [1, 2] \text{ (quotient of continuous functions and denominator } \neq 0 \text{.)}; \\
\text{on } (1, 2) \text{;} \text{ by MVT there is a } c, \\
\text{ } 1 < c < 2, \text{ satisfying}
\end{array} \]
\[ \frac{f(2) - f(1)}{2 - 1} = f'(c) \rightarrow -\frac{1}{2} = \frac{-1}{c^2} \rightarrow c^2 = 2 \rightarrow c = \pm \sqrt{2} \rightarrow c = +\sqrt{2} \]

38.) Let \( f(x) = x^2 - x - 2 \) on \([-1, 2]\); \( f \) is continuous on \([-1, 2]\) since it is
a polynomial \( f'(x) = 2x - 1 \) so \( f \) is differentiable on \((-1, 2)\). By MVT there is a \( c \), \(-1 < c < 2\), satisfying
\[
\frac{f(2) - f(-1)}{2 - (-1)} = f'(c) \rightarrow \frac{0 - 0}{3} = f'(c) \rightarrow f'(c) = 0 \] 
(This means \( f \) has a horizontal tangent line at \( x = c \).)
\[
2c - 1 = 0 \rightarrow 2c = 1 \rightarrow c = \frac{1}{2}
\]

Let \( f(x) = \frac{1}{1 + x^2} \) and consider the interval \([-1, 1]\). Then \( f \) is continuous on \([-1, 1]\) (quotient of continuous functions and denominator \( \neq 0 \)); \( f(x) = (1 + x^2)^{-1} \) \( \Rightarrow \)
\[
f'(x) = - (1 + x^2)^{-2} \cdot 2x = \frac{-2x}{(1 + x^2)^2} \] 
\( f \) is differentiable on \((-1, 1)\); by MVT there is a \( c \), \(-1 < c < 1\), satisfying
\[
\frac{f(1) - f(-1)}{1 - (-1)} = f'(c) \rightarrow \frac{\frac{1}{2} - \frac{1}{2}}{2} = f'(c) \rightarrow 0 = f'(c) \rightarrow
\]
\[
-2c = 0 \rightarrow -2c = 0 \rightarrow c = 0
\]
41.) Let \( f(x) = -x^2 + 2 \) on \([-1, 2]\); \( f \) is continuous on \([-1, 2]\) since it is a polynomial; \( f'(x) = -2x \) so \( f \) is differentiable on \((-1, 2)\); by MVT there is a \( c \neq c, -1 < c < 2 \) so that

\[
\frac{f(2) - f(-1)}{2 - (-1)} = f'(c) \rightarrow \frac{-2 - 1}{3} = f'(c) \rightarrow f'(c) = -1
\]

\( f'(c) = -1 \) \( \rightarrow \) \(-2c = -1 \rightarrow c = \frac{1}{2}\)

44.) By MVT there is at least one \( c \neq 0, 0 < c < 1 \) satisfying

\[
\frac{f(1) - f(0)}{1 - 0} = f'(c) \leftarrow \text{SLOPE of tangent line}
\]

\( \uparrow \) \text{SLOPE of second line}

48.) \( s(t) = \frac{1}{10} t^2 \) for \( 0 \leq t \leq 10 \);

a.) ARC on \([0, 10]\) is (average velocity)

\[
\frac{s(10) - s(0)}{10 - 0} = \frac{10 - 0}{10} = 1 \text{ m/sec}
\]

b.) Instantaneous velocity is \( s'(t) = \frac{1}{5} t \) m/sec.
55. Assume that \( |f(x) - f(y)| \leq |x - y|^2 \). Show that \( f(x) = C \) for some constant \( C \):

Let \( x, y \) be real numbers:

\[
|f(x) - f(y)| \leq |x - y|^2 \quad \Rightarrow \quad \frac{|f(x) - f(y)|}{|x - y|} \leq |x - y| \\
\text{(for } x \neq y) \quad \Rightarrow \quad \left| \frac{f(x) - f(y)}{x - y} \right| \leq |x - y| \quad \Rightarrow \\
\lim_{y \to x} \left| \frac{f(y) - f(x)}{y - x} \right| = \lim_{y \to x} |x - y| \quad \Rightarrow \\
|f'(x)| = 0 \quad \Rightarrow \quad f'(x) = 0 \text{ for all } x \text{-values} \\
\Rightarrow \quad f(x) = C \text{ for some constant } C.