I.) 

volume $64\pi = \pi r^2 h \rightarrow h = \frac{64}{r^2}$,

minimize surface area

$S = \pi r^2 + 2\pi r h = \pi r^2 + 2\pi r\left(\frac{64}{r^2}\right) \rightarrow$

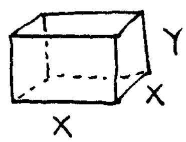
$S = \pi r^2 + \frac{128\pi}{r} \rightarrow S' = 2\pi r - \frac{128\pi}{r^2} = \frac{2\pi r^3 - 128\pi}{r^2}$

$= \frac{2\pi(r^3 - 64)}{r^2} = 0$

$-$ 0 $+$ S'

$r=0$ $r = 4$ in. and $h = 4$ in. and

abs. min. $S = 48\pi$ in.^{2}

II.) 

volume $X^2 Y = 80 \rightarrow Y = \frac{80}{X^2}$,

minimize cost

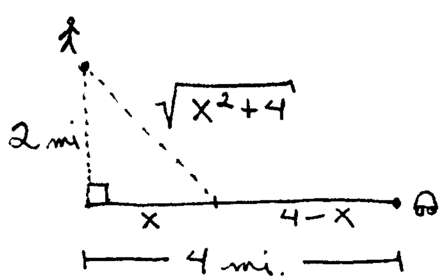
$C = 5(X^2) + 2(4XY) = 5X^2 + 8X\left(\frac{80}{X^2}\right) = 5X^2 + \frac{640}{X} \rightarrow$

$C' = 10X - \frac{640}{X^2} = \frac{10X^3 - 640}{X^2} = \frac{10(X^3 - 64)}{X^2} = 0$

$-$ 0 $+$ C'

$X = 4$ ft. , $Y = 5$ ft. and

abs. min. $C = \$240$

III.) 

woods: 3 mph

road: 5 mph

$T = \frac{D}{R}$

minimize time

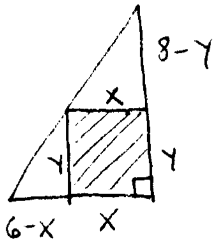
$T = \frac{\sqrt{X^2+4}}{3} + \frac{4-X}{5} \rightarrow T' = \frac{1}{3}\cdot\frac{1}{2}(X^2+4)^{-\frac{1}{2}}\cdot 2X - \frac{1}{5}$

$= \frac{X}{3\sqrt{X^2+4}} - \frac{1}{5} = 0 \rightarrow 5X = 3\sqrt{X^2+4} \longrightarrow$

$25X^2 = 9(X^2+4) \rightarrow 16X^2 = 36 \rightarrow X = 3/2$

$-$ 0 $+$ T'

$X = 3/2$ mi. and abs. min. $T = \frac{4}{3}$ hr.

IV.) A.) 

By similar Δ's

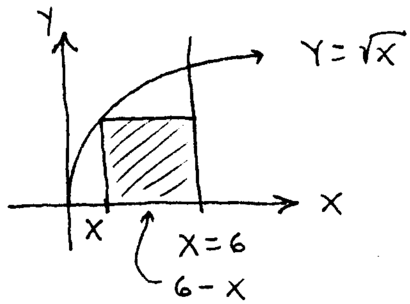
$$\frac{8}{6} = \frac{Y}{6-X} \rightarrow 48 - 8X = 6Y$$
$$\rightarrow Y = 8 - \tfrac{4}{3}X$$

maximize area $A = XY = X\left(8 - \frac{4}{3}X\right) = 8X - \frac{4}{3}X^2 \rightarrow$

$$A' = 8 - \frac{8}{3}X = 0 \rightarrow X = 3$$

$$\underset{X=3}{\overset{+ \quad 0 \quad -}{\longrightarrow}} \; A'$$

$Y = 4$ and abs. max. $A = 12$.

B.) 

maximize area

$$A = (6-X)\,Y$$
$$= (6-X)\sqrt{X} = 6\sqrt{X} - X^{3/2} \rightarrow$$
$$A' = 6\frac{1}{2\sqrt{X}} - \frac{3}{2}\sqrt{X} = \frac{3}{\sqrt{X}} - \frac{3\sqrt{X}}{2}$$

$$\rightarrow A' = \frac{6-3X}{2\sqrt{X}} = 0 \rightarrow X = 2$$

$$\underset{X=2}{\overset{+ \quad 0 \quad -}{\longrightarrow}} \; A'$$

so dimensions are 4 by $\sqrt{2}$ and abs. max. $A = 4\sqrt{2}$.

V.) Let X be the number of \$5 increases, then maximize revenue

$$R = (\#\text{ of rooms})(\text{charge per room})$$
$$= (100 - 4X)(50 + 5X)$$
$$= 5000 + 300X - 20X^2 \xrightarrow{D}$$
$$R' = 300 - 40X = 0 \rightarrow X = \frac{30\cancel{0}}{4\cancel{0}} = 7\tfrac{1}{2}$$

$$\underset{X = 7\frac{1}{2}}{\overset{+ \quad 0 \quad -}{\longrightarrow}} \; R'$$

$X = 7\frac{1}{2}$ \$5 increases

max revenue $R = \$6125$,

rooms $= 70$, charge per room $= \$87.50$

VI. x: demand   p: price

$X = \frac{c}{p^2}$ and $p = \$20$, $X = 125$ boxes so

$125 = \frac{c}{400} \rightarrow c = 50{,}000$ so

$X = \frac{50{,}000}{p^2}$ or price $p = \sqrt{\frac{50{,}000}{X}}$;

cost $C = 750 + 5X$ so profit

$P_r = (\text{revenue}) - (\text{cost})$

$= pX - (750 + 5X)$

$= \sqrt{\frac{50{,}000}{X}} \cdot X - 750 - 5X = \sqrt{50{,}000} \cdot \sqrt{X} - 750 - 5X$;

$P_r' = \sqrt{50{,}000} \cdot \frac{1}{2\sqrt{X}} - 5 = 0 \rightarrow \cdots \rightarrow X = 500$ boxes

$+ \quad 0 \quad -$ P'
$X = 500$ boxes
$p = \$10$

and max. profit is

$P_r = \$1750$