Math 17A (Fall 2006)
Kouba
Exam 2

Please PRINT your name here : KEY

Your Exam ID Number ____________

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM ANOTHER STUDENT'S EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO HAVE ANOTHER STUDENT TAKE YOUR EXAM FOR YOU. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

5. Make sure that you have 7 pages, including the cover page.

6. You may NOT use L'Hopital's Rule on this exam.

7. Put units on answers where units are appropriate.

8. Using only a calculator to determine limits will receive little or no credit.

9. You will be graded on proper use of limit and derivative notation.

10. You have until 8:50 a.m. sharp to finish the exam.

1.) (7 pts. each) Differentiate each of the following functions. DO NOT SIMPLIFY ANSWERS.

a.) $y = x^{3/4} + \sqrt{123} - 2x^{-7}$

$\xrightarrow{D}$ $Y' = \frac{3}{4}X^{-1/4} + 0 - 2 \cdot -7X^{-8}$

b.) $f(x) = \dfrac{6 + 7x}{3 - x}$

$\xrightarrow{D}$ $f'(x) = \dfrac{(3-x)(7) - (6+7x)(-1)}{(3-x)^2}$

c.) $f(x) = \sqrt{4 + \sqrt{x^2 + 9}}$

$\xrightarrow{D}$ $f'(x) = \frac{1}{2}(4 + \sqrt{x^2+9})^{-1/2} \cdot \frac{1}{2}(x^2+9)^{-1/2} \cdot 2x$

2.) (5 pts. each) Let $f(x) = x(5 - x)^4$.

a.) Solve $f'(x) = 0$ for x .

$\xrightarrow{D}$ $f'(x) = x \cdot 4(5-x)^3 \cdot (-1) + (1)(5-x)^4$

$= (5-x)^3 \cdot [-4x + (5-x)]$

$= (5-X)^3 \cdot [5 - 5X] = 0 \rightarrow X = 5, X = 1$

b.) Solve $f''(x) = 0$ for x .

$\xrightarrow{D}$ $f''(x) = (5-x)^3[-5] + 3(5-x)^2(-1)[5-5X]$

$= -(5-x)^2 \cdot [5(5-x) + 3(5-5x)]$

$= -(5-x)^2 [25 - 5X + 15 - 15X]$

$= -(5-x)^2 [40 - 20X] = 0 \rightarrow X = 5, X = 2$

3.) (9 pts.) Use the Intermediate Value Theorem to prove that the equation $x^3 = 4 - x$ is solvable. This is a writing exercise. You will be scored on proper style and mathematical correctness.

$x^3 = 4 - x \rightarrow x^3 + x - 4 = 0$ so let $f(x) = x^3 + x - 4$ and $m = 0$. Function f is continuous for all x-values since f is a polynomial. and $f(1) = -2$, $f(2) = 6$, and $m = 0$ is between $f(1)$ and $f(2)$. Choose interval $[1, 2]$. Thus, by IMVT there is a # c, $1 \le c \le 2$, so that $f(c) = m$, i.e., $c^3 + c - 4 = 0$ and the equation is solvable.

4.) (8 pts.) The following table of values uses the Bisection Method to estimate the solution to the equation $f(x) = 0$. Fill in the missing 10 numbers in the table.

a	b	(1/2)(a+b)	f(a)	f(b)	f((1/2)(a+b))
0	1	0.5	+0.234	−0.569	−0.432
0	0.5	0.25	+0.234	−0.432	+0.145
0.25	0.5	0.375	+0.145	−0.432	−0.317
0.25	0.375	0.313	+0.145	−0.317	+0.098

5.) (9 pts.) Assume that y is a function of x and $xy^2 + y = x + 3$. Determine an equation of the line perpendicular to this graph at $x = 0$.

If $x=0$, then $(0)y^2 + y = 0 + 3 \rightarrow y = 3$;

$xy^2 + y = x + 3 \xrightarrow{D} x \cdot 2yy' + (1)y^2 + y' = 1 + 0 \rightarrow$

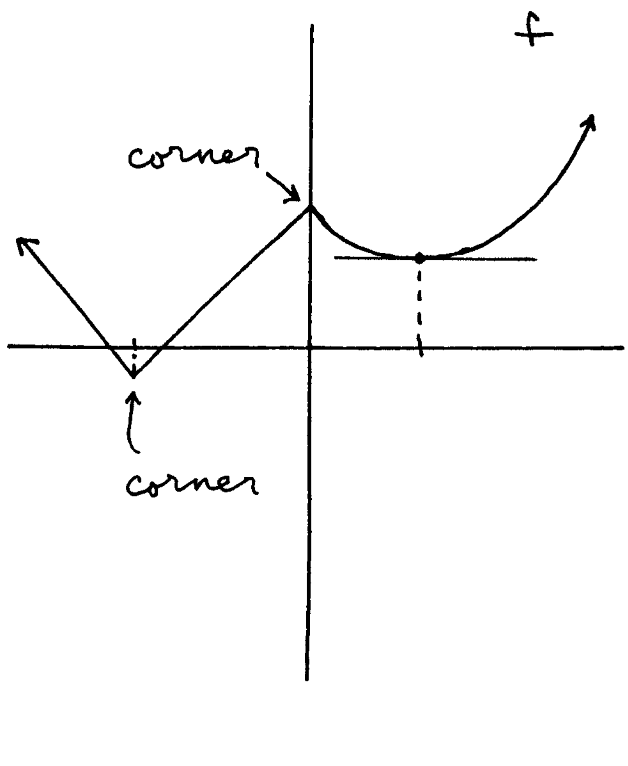
$y'(2xy+1) = 1 - y^2 \rightarrow y' = \dfrac{1-y^2}{2xy+1}$ and

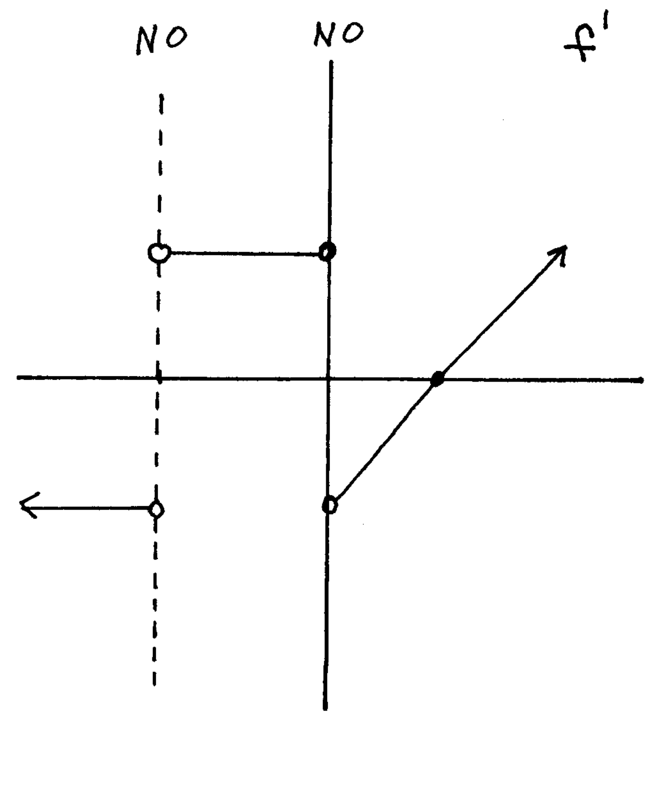
$x=0, y=3 \rightarrow y' = \dfrac{1-9}{2(0)(3)+1} = -8$ so $\perp$ slope

is $m = \dfrac{1}{8}$ and line is

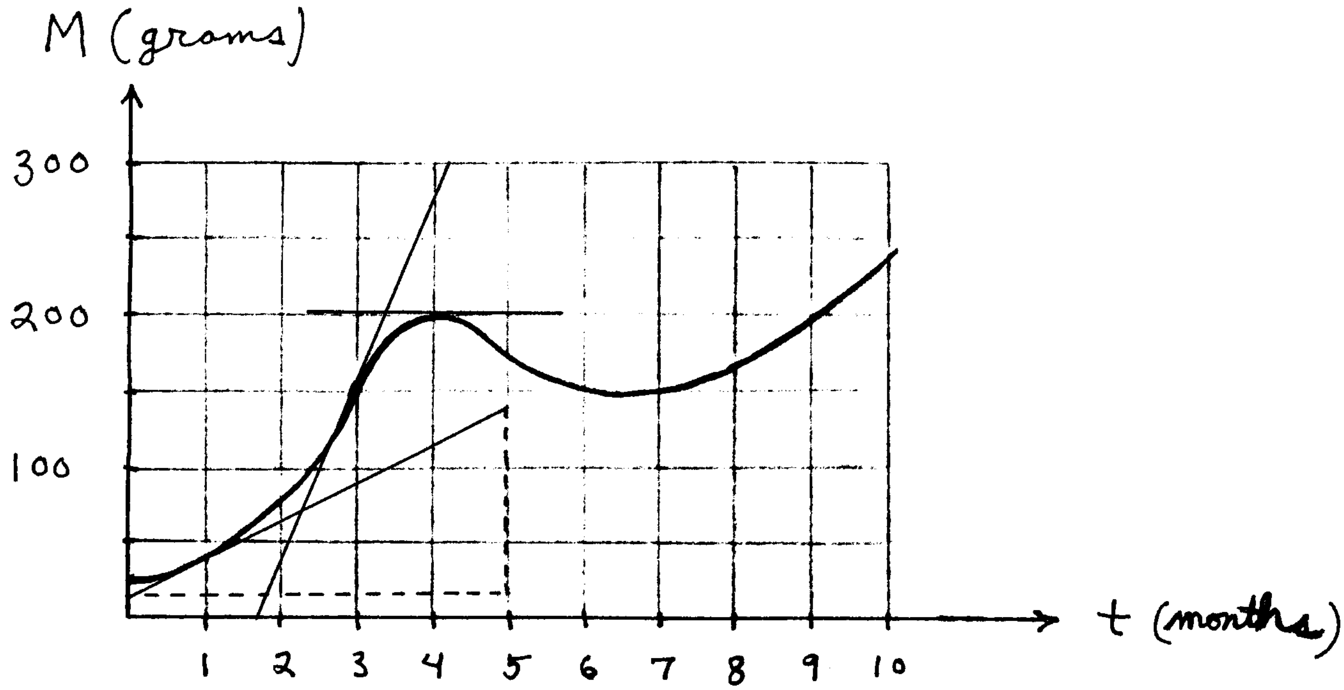
$$y - 3 = \frac{1}{8}(x - 0) \text{ or } y = \frac{1}{8}x + 3$$

6.) (9 pts.) Sketch a graph of the derivative f' using the given graph of f.





7.) The given graph represents the mass M (grams) of a benign tumor from $t = 0$ months to $t = 10$ months.



a.) (3 pts. each) Estimate the tumor's average growth rate (grams/month) for

i.) $t = 0$ to $t = 3$ months.

$$ARC = \frac{M(3) - M(0)}{3 - 0} \approx \frac{150 - 25}{3} = \frac{125}{3} \approx 41.7 \ \frac{\text{grams}}{\text{mo.}}$$

ii.) $t = 4$ to $t = 7$ months.

$$ARC = \frac{M(7) - M(4)}{7 - 4} \approx \frac{150 - 200}{3} = -\frac{50}{3} \approx -16.7 \ \frac{\text{grams}}{\text{mo.}}$$

b.) (3 pts. each) Estimate the instantaneous growth rate of the tumor when

i.) $t = 1$ month.

$$IRC = M'(1) \approx \frac{140 - 15}{5 - 0} = 25 \ \frac{\text{grams}}{\text{mo.}}$$

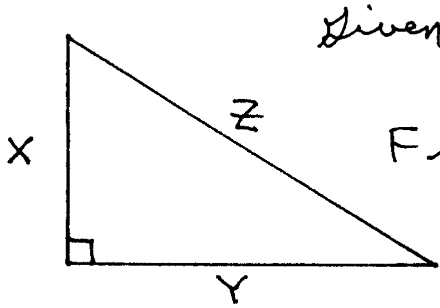
ii.) $t = 4$ months.

$$IRC = M'(4) \approx 0 \ \frac{\text{grams}}{\text{mo.}}$$

c.) (4 pts.) Estimate the specific time t at which the tumor is growing most rapidly, and estimate the value of this rate.

$t \approx 3$ mo. and $IRC = M'(3) \approx \dfrac{275}{2.3} \approx 119.5 \ \dfrac{\text{grams}}{\text{mo.}}$

8.) (9 pts.) Consider the given right triangle with edges x and y. If length x is *increasing* at the rate of 2 ft./sec. and length y is *decreasing* at the rate of 5 ft./sec., determine the rate at which the *perimeter* of the triangle is changing when $x = 3$ feet and $y = 4$ feet.



Given: $\frac{dX}{dt} = 2$ ft./sec., $\frac{dY}{dt} = -5$ ft./sec.;

Find $\frac{dP}{dt}$ when $X = 3$ ft., $Y = 4$ ft. :

1st $X^2 + Y^2 = Z^2 \longrightarrow Z = \sqrt{X^2+Y^2}$

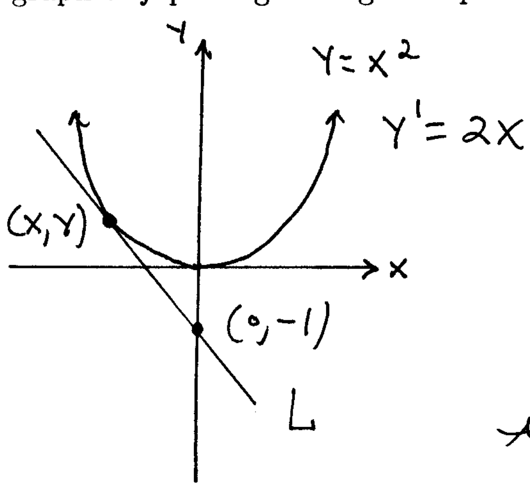
so perimeter $P = X + Y + Z = X + Y + \sqrt{X^2+Y^2} \xrightarrow{D}$

$$\frac{dP}{dt} = \frac{dX}{dt} + \frac{dY}{dt} + \frac{1}{2}(X^2+Y^2)^{-1/2} \cdot \left\{2X \cdot \frac{dX}{dt} + 2Y \cdot \frac{dY}{dt}\right\}$$

$$= 2 + (-5) + \frac{1}{2}(9+16)^{-1/2} \cdot \{2(3)(2) + 2(4)(-5)\}$$

$$= -3 + \frac{1}{10}\{12 - 40\} = -3 - 2.8 = -5.8 \text{ ft./sec.}$$

9.) (9 pts.) Find all points (x, y) on the graph of $f(x) = x^2$ with tangent lines to the graph of f passing through the point $(0, -1)$.



SLOPE of line L is

1. $m = 2X$
2. $m = \dfrac{Y-(-1)}{X-0}$

$= \dfrac{Y+1}{X} = \dfrac{X^2+1}{X}$;

then set equal $\rightarrow$

$2X = \dfrac{X^2+1}{X} \rightarrow 2X^2 = X^2 + 1 \rightarrow X^2 = 1 \rightarrow X = \pm 1 \rightarrow$

$X = 1, Y = 1$ or $X = -1, Y = 1$

The following EXTRA CREDIT PROBLEM is worth 10 points. This problem is OPTIONAL.

1.) Evaluate the following limit : $\lim_{x\to 0} \frac{\sin x^2 \cdot \sin^2(\sin x^2)}{\cos^2 x^2 - 1}$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{\sin x^2 \cdot \sin(\sin x^2) \cdot \sin(\sin x^2)}{-\sin^2 x^2}$$

$$= \lim_{x\to 0} \frac{-\cancel{\sin} x^2}{\cancel{\sin} x^2} \cdot \frac{\sin(\sin x^2)}{\sin x^2} \cdot \sin(\sin x^2)$$

$$= (-1) \cdot (1) \cdot \sin(\sin 0)$$

$$= -1 \sin 0$$

$$= -1 (0) \quad = 0$$