Math 17B  
Kouba  
Differentiating an Inverse Trig Function

<table>
<thead>
<tr>
<th>Trig Function</th>
<th>Domain Restriction</th>
<th>Inverse Function</th>
<th>Derivative of Inverse</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = \tan x$</td>
<td>$-\frac{\pi}{2} &lt; x &lt; \frac{\pi}{2}$</td>
<td>$y = \arctan x$</td>
<td>$y' = \frac{1}{1 + x^2}$</td>
</tr>
</tbody>
</table>

Why is $D \arctan x = \frac{1}{1 + x^2}$ ?

PROOF : $y = \arctan x \implies x = \tan y$  (Definition of inverse tangent)

$\implies 1 = \sec^2 y \cdot y'$  (Implicit differentiation)

$\implies y' = \frac{1}{\sec^2 y}$  (Solve for $y$.)

$\implies y' = \frac{1}{\cos^2 y}$  (Definition of secant)

$\implies y' = \cos^2 y$

$\implies y' = (\cos y)^2$

$\implies y' = \left(\frac{1}{\sqrt{1 + x^2}}\right)^2$  (Definition of cosine and Pythagorean Theorem from right triangle)

$\implies y' = \frac{1}{1 + x^2}$