Math 17B
Kouba
Discussion Sheet 2

1.) Sketch the graph of $y = 3x^2 + 2$ on the interval $[0, 1]$. Consider the area of the region below the graph and above $[0, 1]$. Use the limit definition of a definite integral to find the exact area of the region.

2.) Use the limit definition of a definite integral to evaluate $\displaystyle\int_{-1}^{2} (x^2 - 2x + 1)\, dx$.

3.) Determine the following indefinite integrals. Do not use u-substitution.

a.) $\displaystyle\int x^2(x+1)\, dx$ b.) $\displaystyle\int (e^x + 2^x)\, dx$ c.) $\displaystyle\int 2x\cos(x^2)\, dx$

d.) $\displaystyle\int \frac{x^2+1}{x^3}\, dx$ e.) $\displaystyle\int \frac{x^2+1}{x+3}\, dx$ f.) $\displaystyle\int \frac{x^2}{x^3+1}\, dx$

4.) Evaluate the following definite integrals. Do not use u-substitution.

a.) $\displaystyle\int_4^9 \frac{1}{x^2}\, dx$ b.) $\displaystyle\int_0^1 3^{x+1}\, dx$ c.) $\displaystyle\int_1^2 \frac{(x+1)^2}{x}\, dx$

d.) $\displaystyle\int_0^5 \sqrt{x+4}\, dx$ e.) $\displaystyle\int_{\pi/6}^{\pi/4} \cos(3x)\, dx$ f.) $\displaystyle\int_{-1}^{0} \frac{x^2}{x-1}\, dx$

g.) $\displaystyle\int_0^{\sqrt{\ln 3}} x e^{x^2}\, dx$ h.) $\displaystyle\int_0^{\ln 2} \frac{e^x}{e^x+1}\, dx$ i.) $\displaystyle\int_0^1 \frac{1}{e^x}\, dx$

j.) $\displaystyle\int_0^{\frac{\pi}{2}} \cos x\, e^{\sin x}\, dx$ k.) $\displaystyle\int_{-1}^{1} 3x^2 \cdot 5^{x^3}\, dx$ l.) $\displaystyle\int_0^{\pi/12} 5\sec^2 3x\, dx$

5.) Differentiate each : a.) $\displaystyle F(x) = \int_{-1}^{3x} \sqrt{1+t^2}\, dt$ b.) $\displaystyle F(x) = \int_{\tan x}^{\sec x} 5^{t^2}\, dt$

6.) Find an equation of the line perpendicular to the graph of

a.) $\displaystyle F(x) = 3 + \int_0^x 2e^{t^2}\, dt$ at $x = 0$.

b.) $\displaystyle F(x) = \int_{2x}^{x^2} \sqrt{t^2+5}\, dt$ at $x = 2$.

7.) Find the average value of each of the following functions over the given interval. Draw a sketch showing the connection between your answer and the definite integral.

a.) $f(x) = x^3 + 1$ on $[-1, 1]$ b.) $f(x) = 5 + \sqrt{x}$ on $[0, 4]$

8.) If $\int_{-2}^{1} f(x)\,dx = 3$ and $\int_{-2}^{3} f(x)\,dx = -2$. What is the value of $\int_{3}^{1} f(x)\,dx$?

9.) A long and thin corn stalk is 100 inches long. Its density x inches from its base is given by $f(x) = 2 - (1/100)x$ ounces per inch. Set up a definite integral and compute the exact weight of the corn stalk.

10.) Consider the region R enclosed by the graphs of the given functions. Describe each region R using

i.) vertical cross-sections.
ii.) horizontal cross-sections.

a.) $y = 2x$, $x = 4$, and $y = 0$
b.) $y = e^x$, $x = 0$, and $y = e^2$
c.) $y = 2/x$, $y = 2x$, and $x = 4$
d.) $y = 2x$, $y = (1/2)x$, and $y = 6 - x$
e.) $y = x^2$ and $y = 4x + 5$

11.) Find the area of the region bounded by the graphs of the given equations.

a.) $y = x, y = 2x$, and $x = 2$ b.) $y = e^x, x = 0$, and $y = 2$
c.) $x = y^2$ and $x = 9$ d.) $y = x$, $y = 0$, $y = 2$, and $y = (1/2)x - 2$

12.) Assume that f is an odd function and $\int_{-2}^{1} f(x)\,dx = 3$. What is the value of $\int_{-1}^{-2} f(x)\,dx$?

13.) The speed s (in miles per hour) of a jogger at time t (in hours) is given by $s(t) = t + \sqrt{t}$.
a.) Find the jogger's average speed between $t = 0$ hrs. and $t = 4$ hrs.
b.) Find the total distance traveled by the jogger between $t = 0$ hrs. and $t = 4$ hrs.

14.) A heavy snow begins to fall at Squaw Valley Ski Resort. If snow falls at time t hours at the rate of $(1/2)t + 1$ in./hr. for $t \geq 0$, then what is the total accumulated snowfall for $t = 0$ to $t = 8$ hours ?

15.) Find the volume of the solid formed by revolving each region bounded by the given graphs about the given axis.

a.) $y = x^2 - 1$ and the x−axis about the x−axis

b.) $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the x−axis

c.) $y = \sqrt{x}$, $y = 0$, and $x = 4$ about the y−axis

d.) $y = 3x$, $y = 6$, and $x = 0$ about the x−axis

e.) $y = 2x$, $y = 5 - (1/2)x$, and $y = 0$ about the y−axis f.) $y = x^2$ and $y = x + 2$ about the line $y = 4$

g.) $y = x^2$ and $y = x^3$ about the line $y = 2$

h.) $y = x^2$ and $y = x^3$ about the line $y = -1$

i.) $y = x^2$ and $y = x^3$ about the line $x = 3$

j.) $y = x^2$ and $y = x^3$ about the line $x = -2$

16.) Find the length of each graph on the given interval.

a.) $y = x^{3/2}$ on the interval $[0, 4]$ b.) $y = (2/3)(x^2 + 1)^{3/2}$ on the interval $[0, 2]$

c. $y = \dfrac{x^4}{4} + \dfrac{1}{8x^2}$ on the interval $[2, 4]$

d.) $y = (1/2)(e^x + e^{-x})$ on the interval $[0, \ln 2]$

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THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

15.) Count the total number of squares (including overlapping squares) in the following diagram.