Math 17B
Kouba
Discussion Sheet 3

1.) Use any method to determine the following indefinite integrals (antiderivatives).

a.) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$ b.) $\displaystyle\int \frac{e^{2x}}{1+e^{2x}}\,dx$ c.) $\displaystyle\int \cos 5x\,dx$ d.) $\displaystyle\int \frac{1+\sin x}{\cos^2 x}\,dx$

e.) $\displaystyle\int (\sec x + \sec^2 x)\,dx$ f.) $\displaystyle\int \tan^2 x\ \sec^2 x\,dx$ g.) $\displaystyle\int \frac{(x+2)(x+3)}{x+1}\,dx$

h.) $\displaystyle\int (x^2+1)(x^3+3x)^{10}\,dx$ i.) $\displaystyle\int \frac{x+6}{(x+5)^2}\,dx$ j.) $\displaystyle\int \frac{(\ln x)^4}{x}\,dx$

k.) $\displaystyle\int \sec^2(3x)\ 2^{\tan(3x)}\,dx$ l.) $\displaystyle\int (x+3)\sqrt{x-2}\,dx$ m.) $\displaystyle\int (\sec(2x)-\tan(2x))\,dx$

n.) $\displaystyle\int \cot x\ (\cot x + 3)\,dx$ o.) $\displaystyle\int \sec^3 x\ \tan x\,dx$ p.) $\displaystyle\int \sec x\ \tan^3 x\,dx$

q.) $\displaystyle\int (\csc x + 1)(\cot x - 1)\,dx$ r.) $\displaystyle\int \tan^3 x\,dx$ s.) $\displaystyle\int \sec^3 x\,dx$ (challenging)

t.) $\displaystyle\int \sin(3x)\ \cos^2(3x)\,dx$ u.) $\displaystyle\int \cos^2 x\,dx$ v.) $\displaystyle\int (\sin x + \cos x)^2\,dx$

w.) $\displaystyle\int \frac{1}{9+x^2}\,dx$ x.) $\displaystyle\int \frac{1}{1+16x^2}\,dx$ y.) $\displaystyle\int \frac{7}{9+25x^2}\,dx$

2.) Use partial fractions to determine the following indefinite integrals (antiderivatives).

a.) $\displaystyle\int \frac{1}{x^2-x}\,dx$ b.) $\displaystyle\int \frac{x+3}{x^2-x-2}\,dx$ c.) $\displaystyle\int \frac{x^2+x-1}{x^2(x+1)}\,dx$ d.) $\displaystyle\int \frac{x-3}{x(x^2+1)}\,dx$

e.) $\displaystyle\int \frac{2x-1}{x^2(x^2+1)}\,dx$ f.) $\displaystyle\int \frac{x^2+4}{x^2(x-2)^2}\,dx$ g.) $\displaystyle\int \frac{(x-1)^2}{(x^2-x)^2}\,dx$ h.) $\displaystyle\int \frac{x^2+2x}{x^2-1}\,dx$

i.) $\displaystyle\int \frac{1}{x^4+1}\,dx$ (Challenging– Just set up the partial fractions decomposition, but do not solve for the unknown constants.)

3.) Find the following antiderivative two ways– using u-substitution with a back substitute and using integration by parts : $\displaystyle\int x^3(1+x^2)^4\,dx$

4.) Find the following antiderivatives by using integration by parts twice, then using algebraic manipulation (integration by parts with a twist).

a.) $\int e^{2x} \sin x \, dx$ b.) $\int \sin x \cos 3x \, dx$

5.) Find the average value of $f(x) = x \ln x$ on the interval $[1, e]$.

6.) A three-dimensional solid object lies above the x-axis from $x = 0$ to $x = 4$ centimeters. The cross-sectional area of the solid taken perpendicular to the x-axis at x is $A(x) = 6x^3$ square centimeters. Compute the volume of the solid.

7.) Assume that snow is falling at the rate of $t + \sqrt{t}$ in./hr. at time t hours. Determine a definite integral and compute the total amount of snowfall between $t = 0$ and $t = 4$ hours.

8.) Use a power u-substitution to integrate each of the following.

a.) $\int \frac{1}{1+\sqrt{x}} \, dx$ b.) $\int \sqrt{4+\sqrt{x}} \, dx$ c.) $\int \frac{\sqrt{x}}{4+x^{1/3}} \, dx$

9.) The base of a solid lies in the region bounded by the graphs of $y = 1/x$, $y = x^3$, and $x = 2$. Find the volume of the solid if cross-sections taken perpendicular to the x-axis at x are

i.) squares. ii.) rectangles of height 4. iii.) semi-circles.

10.) The base of a solid lies in the region bounded by the graphs of $y = e^x$, $y = 1$, and $x = 3$. Find the volume of the solid if cross-sections taken perpendicular to the x-axis at x are

i.) triangles of height 5. ii.) equilateral triangles.

11.) Determine a function having the following properties :

$f''(x) = 1 + e^{x/2}, f'(0) = -1,$ and $f(0) = 3$

12.) Wildebeests (Gnus) are migratory animals and are an important part of the African ecosystem, since their dung fertilizes the soil and their grazing and trampling encourage new growth along migratory paths. Assume that a herd of wildebeests migrates along a path given by $y = (1/6)x^3 + \frac{1}{2x}$ from $x = 1$ to $x = 50$ miles. Determine the total length of this path.

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THE FOLLOWING PROBLEMS ARE FOR RECREATIONAL PURPOSES ONLY.

13.) Connect 6 toothpicks end-to-end to form 4 triangles.

14.) Determine the exact value of the following expression :

$$\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}}}$$

THESE ARE OPTIONAL PRACTICE PROBLEMS

1.) Use any method to determine the following indefinite integrals (antiderivatives).

a.) $\displaystyle\int \frac{(\arctan x)^2}{x^2+1}\,dx$ b.) $\displaystyle\int \frac{1}{x^2+1}\,dx$ c.) $\displaystyle\int \ln x\,dx$ d.) $\displaystyle\int x(\ln x)^2\,dx$

e.) $\displaystyle\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx$ f.) $\displaystyle\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$ g.) $\displaystyle\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx$ h.) $\displaystyle\int \frac{1}{x\sqrt{x-1}}\,dx$

i.) $\displaystyle\int \frac{2}{x^2+4x+13}\,dx$ j.) $\displaystyle\int \frac{x^2}{x^2+1}\,dx$ k.) $\displaystyle\int \frac{x^5}{1+x^6}\,dx$ l.) $\displaystyle\int \frac{x^2}{1+x^6}\,dx$

m.) $\displaystyle\int \arctan x\,dx$ n.) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$ o.) $\displaystyle\int \frac{\cos x}{1+\sin^2 x}\,dx$ p.) $\displaystyle\int \frac{3}{x^2+4}\,dx$

q.) $\displaystyle\int \frac{1}{x^2-2x+5}\,dx$ r.) $\displaystyle\int \frac{1}{x^2-6x+5}\,dx$ s.) $\displaystyle\int \frac{e^{-x}+1}{xe^{-x}+1}\,dx$ (Challenging)

t.) $\displaystyle\int e^x\cos(e^x)\,dx$ u.) $\displaystyle\int e^{\sqrt{x}}\,dx$ v.) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

w.) $\displaystyle\int \sin 2x\cos 2x\,dx$ x.) $\displaystyle\int \frac{\sin 2x}{\cos 2x}\,dx$ y.) $\displaystyle\int \sin\sqrt{x}\,dx$