Math 17B
Kouba
Discussion Sheet 4

1.) Compute the Midpoint Estimate, M_6, for $\displaystyle\int_0^1 \frac{1}{x^2+1}\,dx$. Compare your answer with the exact value of the integral.

2.) Compute the Trapezoidal Estimate, M_5, for $\displaystyle\int_{-1}^1 \sqrt{1-x}\,dx$. Compare your answer with the exact value of the integral.

3.) Determine the value of n so that the Trapezoidal Estimate, T_n, estimates the exact value of $\displaystyle\int_0^{1/2} e^{-2x^2}\,dx$ with absolute error at most 0.00001.

4.) Determine the value of n so that the Midpoint Estimate, T_n, estimates the exact value of $\displaystyle\int_0^3 \frac{x+1}{x+5}\,dx$ with absolute error at most 0.00001.

5.) Compute the following improper integrals.

a.) $\displaystyle\int_0^4 \frac{1}{\sqrt{x}}\,dx$ b.) $\displaystyle\int_1^\infty \frac{3}{x^2}\,dx$ c.) $\displaystyle\int_0^1 \frac{3}{x^2}\,dx$

d.) $\displaystyle\int_{\sqrt{3}}^\infty \frac{1}{1+x^2}\,dx$ e.) $\displaystyle\int_e^\infty \frac{1}{x\ln x}\,dx$ f.) $\displaystyle\int_{-\infty}^0 \frac{e^x}{1+e^{2x}}\,dx$

g.) $\displaystyle\int_1^\infty \frac{1}{x(x+4)}\,dx$ h.) $\displaystyle\int_{-\infty}^0 e^{3x}\,dx$ i.) $\displaystyle\int_{-1}^\infty \frac{1}{\sqrt{x+1}}\,dx$

j.) $\displaystyle\int_{-\infty}^{\sqrt{3}} \frac{1}{x^2+9}\,dx$ k.) $\displaystyle\int_1^{e^2+1} \frac{7}{x-1}\,dx$

6.) Consider the region R (in the first quadrant) bounded by the graphs of $y=\dfrac{1}{x}$, $x=1$, and $y=0$.

a.) Determine if R has finite or infinite area.

b.) Form a solid by revolving R about the x-axis. Determine if the resulting volume is finite or infinite.

7.) Find the following Taylor polynomials of degree n about $a=0$, $P_n(x)$, for the indicated functions.

a.) $f(x)=x^4+x^3-x^2+3x-5$, $n=2$ b.) $f(x)=x^4+x^3-x^2+3x-5$, $n=4$

c.) $f(x)=xe^x$, $n=3$ d.) $f(x)=\sqrt{x+4}$, $n=2$ d.) $f(x)=\ln(x+1)$, $n=3$

8.) Find the following Taylor polynomials about $a=0$ for the function $f(x)=\dfrac{x-2}{x+1}$:

$P_0(x)$, $P_1(x)$, $P_2(x)$, $P_3(x)$. Compare the values of the function and its Taylor polynomials at $x = 0.1$ and $x = 2$. What conclusion do you draw ?

9.) It is well known that the integral $\int_0^1 e^{x^2}\,dx$ has no closed-form anti-derivative. Replace $f(x) = e^{x^2}$ with $P_4(x)$, its fourth-degree Taylor Polynomial centered at $x = 0$, to get an estimate for this definite integral. Compare this value with one obtained by a calculator which computes definite integrals and determine the absolute percentage error in your estimate.

****

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

10.) A nonnegative integer I is a perfect square, triangular (PST) number if I is equal to the square of a nonnegative integer AND is also equal to one-half the product of consecutive nonnegative integers. Find the first four PST numbers.