Math 17B
Kouba
Discussion Sheet 5

1.) Solve the following differential equations.

a.) $\frac{dy}{dx} = xe^x$  

b.) $\frac{dy}{dx} = xy\sqrt{y - 4}$  

c.) $\frac{dy}{dx} = \sin^3 x \cos^2 y$

d.) $\frac{dy}{dx} = y^2(y - 1)$  

e.) $\frac{dy}{dx} = \sin x \cos y$  

f.) $\frac{dy}{dx} = xy - y + 3x - 3$

g.) $\frac{dy}{dx} = \frac{x + xy^3 + 1 + y^3}{xy^2 - 2y^2}$  

h.) $\frac{dy}{dx} = e^{2x+3y}$ and $y(1) = 0$

2.) Let $M$ be the total mass (in grams) of a black bullhead (a sport fish common throughout Minnesota’s lakes with sandy, muddy bottoms) at time $t$ (in years). Assume that its growth rate is given by $\frac{dM}{dt} = (1/100)M(400 - M)$. If $M(0) = 2$ grams, solve the D.E. and solve explicitly for mass $M$. What is the bullhead’s mass in 1 year? in 2 years? Determine an upper limit (asymptotic mass) for the mass of this fish.

3.) Solve Problem B on page 2 of this discussion sheet.

4.) Consider the function $f(x) = \cos 2x$ on the interval $[0, 1/2]$. What should $n$ be in order that the Taylor Polynomial of degree $n$ centered at $x = 0$ have a Taylor Error of at most 0.0001?

5.) Consider the function $f(x) = \frac{x}{x + 1}$ on the interval $[0, 3/4]$. What should $n$ be in order that the Taylor Polynomial of degree $n$ centered at $x = 0$ have a Taylor Error of at most 0.0001?

************THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.************

6.) You have 8 black socks, 12 blue socks, 10 gray socks, and 5 white socks randomly scattered in your bureau drawer. If you reach into the drawer without looking, how many socks must you take out to be sure of having a matching pair of socks? a matching pair of white socks?
1.) The graph of a derivative $f'$ is given below. Set up a sign chart for the second derivative $f''$ and sketch a graph of the function $f$, indicating extrema and inflection points.
The following data are plotted on the semi-log graph. Determine $N$ and the growth rate $\frac{dN}{dt}$ (as a function of $N$)

<table>
<thead>
<tr>
<th>$t$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>199</td>
</tr>
<tr>
<td>4</td>
<td>79.2</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>19.9</td>
</tr>
<tr>
<td>9</td>
<td>7.9</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>
Problem B (Solution)

Assume \( \log N = \log C + nt \) (a line),
then \( C = 500 \) so \( \log N = \log 500 + nt \),
and \( t = 10, N = 5 \) →

\[
\begin{align*}
\log 5 &= \log 500 + 10n \\
\log 5 - \log 500 &= 10n \\
\log \frac{5}{500} &= 10n \\
\log \frac{1}{100} &= 10n \\
\log 10^{-2} &= 10n \\
-2 &= 10n \\
n &= -\frac{1}{5} \quad \text{; then}
\end{align*}
\]

\[
\log N = \log 500 - \frac{1}{5} t \quad \rightarrow
\]

\[
10 \log N = 10 \left( \log 500 - \frac{1}{5} t \right) \quad \rightarrow
\]

\[
N = 10 \log 500 \frac{1}{10} \frac{1}{5} t \quad \rightarrow
\]

\[
N = 500 \cdot 10^{-\frac{1}{5} t} \quad ; \quad \text{then}
\]

\[
\frac{dN}{dt} = 500 \cdot 10^{-\frac{1}{5} t} \cdot \ln 10 \cdot \frac{-1}{5} \quad \rightarrow
\]

\[
\frac{dN}{dt} = -\ln 10 \frac{N}{5}
\]