EX: This example illustrates how some graphs plotted on a rectangular coordinate system become "linearized" when plotted on semi-log graph paper.

1.) Plot the graph of \( N = 3 \cdot 10^t \):

2.) Consider the following \( t \) and \( N \) values:

<table>
<thead>
<tr>
<th>( t )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>0.5</td>
<td>9.5</td>
</tr>
<tr>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>1.5</td>
<td>94.9</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
</tr>
<tr>
<td>2.5</td>
<td>948.7</td>
</tr>
</tbody>
</table>
3.) Plot the values in 2.) on semi-log graph paper:

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Note: The values shown on the vertical axis are values of $N$, but the values plotted on the graph are actually values of $\log N$. For example, notice the values $N=1, N=10, N=100$, and $N=1000$ are equally-spaced. This is because their positions on the vertical axis are actually represented by:

$$
\begin{align*}
\log 1 &= 0, \\
\log 10 &= 1, \\
\log 100 &= 2, \\
\log 1000 &= 3.
\end{align*}
$$

And $\log 1000 = 3$ numbers which are equally-spaced.

4.) Verify that the semi-log graph is a line:
Begin with \( N = 3 \cdot 10^t \)

\[ \rightarrow \log N = \log (3 \cdot 10^t) \]

\[ \rightarrow \log N = \log 3 + \log 10^t \]

\[ \rightarrow \log N = \log 3 + t \cdot \log 10 \]

\[ \rightarrow \log N = (\log 10) \cdot t + (\log 3) \]

and \( Y = m \cdot t + b \)

is the equation of line, where \( Y = \log N \), slope \( m = \log 10 \), and \( Y \)-intercept \( b = \log 3 \).

\[ \text{Def: If } N = N(t), \text{ then its growth rate is } N' = \frac{dN}{dt} \]

\[ \text{Def: If } N' = \frac{dN}{dt} = f(N), \text{ a function of } N \text{ only, then we say the growth rate is autonomous form.} \]

\[ \text{Ex: } \frac{dN}{dt} = 3N, \quad \frac{dN}{dt} = N^2 - N \text{ are in autonomous form.} \]
5.) Find the growth rate for \( N = 3 \cdot 10^{t} \) in autonomous form:

\[
N = 3 \cdot 10^{t} \quad \Rightarrow \quad \frac{dN}{dt} = 3 \cdot 10^{t} \cdot \ln 10
\]

\[
\Rightarrow \quad \frac{dN}{dt} = \ln 10 \cdot (3 \cdot 10^{t})
\]

\[
\Rightarrow \quad \frac{dN}{dt} = \ln 10 \cdot N
\]