I.) Properties of the Definite Integral

a.) \( \int_a^a f(x) \, dx = 0 \)

b.) \( \int_a^b f(x) \, dx = -\int_b^a f(x) \, dx \)

c.) \( \int_a^b cf(x) \, dx = c \int_a^b f(x) \, dx \)

d.) \( \int_a^b (f(x) \pm g(x)) \, dx = \int_a^b f(x) \, dx \pm \int_a^b g(x) \, dx \)

e.) If \( f(x) \geq 0 \) then \( \int_a^b f(x) \, dx \geq 0 \) (if \( a < b \))

f.) If \( f(x) \geq g(x) \) then \( \int_a^b f(x) \, dx \geq \int_a^b g(x) \, dx \) (if \( a < b \))

g.) \( \int_a^b f(x) \, dx = \int_a^c f(x) \, dx + \int_c^b f(x) \, dx \)

h.) If \( m \leq f(x) \leq M \) then \( m(b - a) \leq \int_a^b f(x) \, dx \leq M(b - a) \)

II.) Applications of the Definite Integral

a.) Area of region: If \( f(x) \) is the height of region \( S \) at \( x \), then total area of \( S \) from \( a \) to \( b \) is

\[
\text{AREA} = \int_a^b f(x) \, dx
\]

b.) Mass of string: If \( f(x) \) is the density (mass/length units) of string at \( x \), then total mass of string from \( a \) to \( b \) is

\[
\text{MASS} = \int_a^b f(x) \, dx
\]

c.) Distance traveled: If \( f(t) \) is the speed of an object at time \( t \), then total distance traveled from time \( a \) to time \( b \) is

\[
\text{DISTANCE} = \int_a^b f(t) \, dt
\]

d.) Volume of solid: If \( A(x) \) is the cross-sectional area of a solid \( S \) at \( x \), then total volume of \( S \) from \( a \) to \( b \) is

\[
\text{VOLUME} = \int_a^b A(x) \, dx
\]