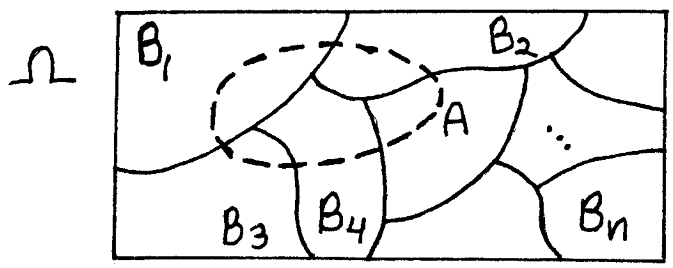
Math 17C
Kouba
Bayes Formula

Let Ω be a sample space and let $B_1, B_2, B_3,$ and B_n be a partition for Ω. Let A be an event in Ω. Recall that $P(A|B_i) = \dfrac{P(A \cap B_i)}{P(B_i)}$ so that $P(A \cap B_i) = P(A|B_i) \cdot P(B_i)$ for $i = 1, 2, 3, \cdots, n$. By the Law of Total Probability we have



$$P(A) = P(A \cap B_1) + P(A \cap B_2) + P(A \cap B_3) + \cdots + P(A \cap B_n)$$
$$= P(A|B_1) \cdot P(B1) + P(A|B_2) \cdot P(B_2) + P(A|B_3) \cdot P(B_3) + \cdots + P(A|B_n) \cdot P(B_n) .$$

Note also that

$$P(B_i|A) = \frac{P(B_i \cap A)}{P(A)} = \frac{P(A \cap B_i)}{P(A)}$$

$$= \frac{P(A|B_i) \cdot P(B_i)}{P(A|B_1) \cdot P(B_1) + P(A|B_2) \cdot P(B_2) + P(A|B_3) \cdot P(B_3) + \cdots + P(A|B_n) \cdot P(B_n)} ,$$

i.e.,

$$P(B_i|A) = \frac{P(A|B_i) \cdot P(B_i)}{\Sigma_{i=1}^{n} P(A|B_i) \cdot P(B_i)} \quad \text{for } i = 1, 2, 3, \cdots, n \quad \text{(Bayes Formula)}$$

EXAMPLE: A blood test for the HIV virus shows a positive (+) result in 99% of all cases when the virus is actually present in an individual and in 5% of all cases when the virus is NOT present in an individual (false positive). Assume that 1 out of every 200 people are carriers of the virus.

(I.) (Law of Total Probability) If a person is selected at random and this test is administered, what is the probability that the test result is positive (+) ? Assuming that these tests are independent events, what is the probability that the same person tests positive twice ?

(II.) (Bayes Formula) If a person tests positive for HIV, what is the probability that this person is a carrier of the HIV virus ? If a person tests positive for HIV, what is the probability that this person is NOT a carrier of the HIV virus ? If a person tests negative for HIV, what is the probability that this person is NOT a carrier of the HIV virus ?