

Math 17C

Kouba

Creating
Direction Field for $X' = \begin{bmatrix} 3 & -4 \\ 4 & -7 \end{bmatrix} X$

$$\text{or } \begin{cases} x_1' = 3x_1 - 4x_2 \\ x_2' = 4x_1 - 7x_2 \end{cases} ;$$

$$\text{Slope : } \frac{dx_2}{dx_1} = \frac{x_2'}{x_1'} = \frac{4x_1 - 7x_2}{3x_1 - 4x_2} ,$$

$$\text{Direction Vector : } X' = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 3x_1 - 4x_2 \\ 4x_1 - 7x_2 \end{bmatrix} ,$$

$$\text{Speed : } \frac{ds}{dt} = \sqrt{(x_1')^2 + (x_2')^2} :$$

<u>x_1</u>	<u>x_2</u>	<u>Slope</u>	<u>Dir. Vec.</u>	<u>Speed</u>
3	2	$-\frac{2}{1} = -2$	$\begin{bmatrix} 1 \\ -2 \end{bmatrix}$	$\sqrt{1+4} = \sqrt{5}$
0	2	$-\frac{14}{8} = \frac{7}{4}$	$\begin{bmatrix} -8 \\ -14 \end{bmatrix}$	$\sqrt{64+196} = \sqrt{260}$
-3	-4	$\frac{16}{7}$	$\begin{bmatrix} 7 \\ 16 \end{bmatrix}$	$\sqrt{49+256} = \sqrt{305}$
2	-2	$\frac{22}{14} = \frac{11}{7}$	$\begin{bmatrix} 14 \\ 22 \end{bmatrix}$	$\sqrt{196+484} = \sqrt{680}$
2	1	$\frac{1}{2}$	$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$	$\sqrt{4+1} = \sqrt{5}$
-2	-1	$-\frac{1}{2} = \frac{1}{2}$	$\begin{bmatrix} -2 \\ -1 \end{bmatrix}$	$\sqrt{4+1} = \sqrt{5}$
2	4	$-\frac{20}{-10} = 2$	$\begin{bmatrix} -10 \\ -20 \end{bmatrix}$	$\sqrt{100+400} = \sqrt{500}$
-1	-2	$\frac{10}{5} = 2$	$\begin{bmatrix} 5 \\ 10 \end{bmatrix}$	$\sqrt{25+100} = \sqrt{125}$
5	-4	$\frac{48}{31}$	$\begin{bmatrix} 31 \\ 48 \end{bmatrix}$	$\sqrt{961+2304} = \sqrt{3265}$
-6	6	$-\frac{66}{-42} = \frac{11}{7}$	$\begin{bmatrix} -42 \\ -66 \end{bmatrix}$	$\sqrt{1764+4356} = \sqrt{6120}$
30	30	$-\frac{90}{-30} = 3$	$\begin{bmatrix} -30 \\ -90 \end{bmatrix}$	$\sqrt{900+8100} = \sqrt{9000}$
0	0	DNE	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	$\sqrt{0+0} = 0$

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Showing Some Solutions with
Different Initial Conditions

