Math 17C
Kouba
Double Integrals

Recall:

\[ y = f(x) \]

\[ a \leq x \leq b \]

Divide interval \([a, b]\) into \(n\) equal parts each of length \(\frac{b-a}{n}\). Then the area of the shaded region is given by

\[
\text{Area} = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} f(x_i) \cdot \frac{b-a}{n} = \int_{a}^{b} f(x) \, dx
\]

Fact: If \(A(x)\) is the area of a "slice" of a solid taken perpendicular to the \(x\)-axis at \(x\), then the volume of the solid is given by

\[
\text{Volume} = \int_{a}^{b} A(x) \, dx
\]

Double integral: Consider region \(R\) bounded by \(a \leq x \leq b\) and \(g(x) \leq y \leq h(x)\) and let \(z = f(x, y)\) be a surface defined on \(R\). We seek to compute the volume of the
solid region lying below the surface and above region $R$:

\[ z = f(x, y) \]

\[ Y = h(x) \]
\[ Y = g(x) \]

\[ a \quad x \quad b \]

Pick an $x$-value and make a slice perpendicular to the $x$-axis. Let $A(x)$ be the area of this slice. Then

\[ A(x) = \int_{g(x)}^{h(x)} f(x, y) \, dy \]

and the volume of the solid is

\[ \text{Volume} = \int_{a}^{b} A(x) \, dx \], or

\[ \text{Volume} = \int_{a}^{b} \int_{g(x)}^{h(x)} f(x, y) \, dy \, dx \].

\[ \int_{a}^{b} \int_{g(x)}^{h(x)} \]