Math 17C
Kouba
Describing an Equilibrium at (0,0)

Recall: If $X' = AX$ and λ_1 and λ_2 are eigenvalues for A, then solution is
$$X = c_1 \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} e^{\lambda_1 t} + c_2 \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} e^{\lambda_2 t} .$$

Case 1: If $\lambda_1 < 0$ and $\lambda_2 < 0$ are real and distinct, then we say $(0,0)$ is a stable equilibrium and refer to $(0,0)$ as a sink.

Case 2: If $\lambda_1 > 0$ and $\lambda_2 > 0$ are real and distinct, then we say $(0,0)$ is an unstable equilibrium and refer to $(0,0)$ as a source.

Case 3: If $\lambda_1 < 0$ and $\lambda_2 > 0$ are real, then we say $(0,0)$ is an unstable equilibrium and refer to $(0,0)$ as a saddle.

Fact: (Euler) $e^{a+bi} = e^a e^{bi} = e^a(\cos b + i \sin b)$.

If eigenvalues are complex conjugates $\lambda_1 = a+bi$ and $\lambda_2 = a-bi$, then
$$X = c \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} e^{(a+bi)t} = c \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} e^{at}(\cos bt + i \sin bt) .$$

Case 4: If $a<0$, then we say $(0,0)$ is a stable equilibrium and refer to $(0,0)$ as a stable spiral.

Case 5: If $a>0$, then we say $(0,0)$ is an unstable equilibrium and refer to $(0,0)$ as an unstable spiral.

Case 6: If $a=0$, then we say $(0,0)$ is an unstable equilibrium and refer to $(0,0)$ as a neutral spiral

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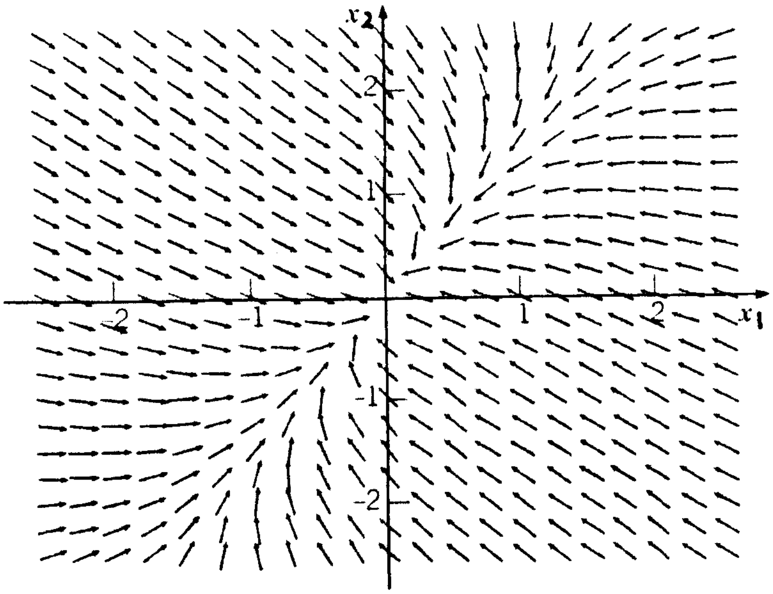
Kouba

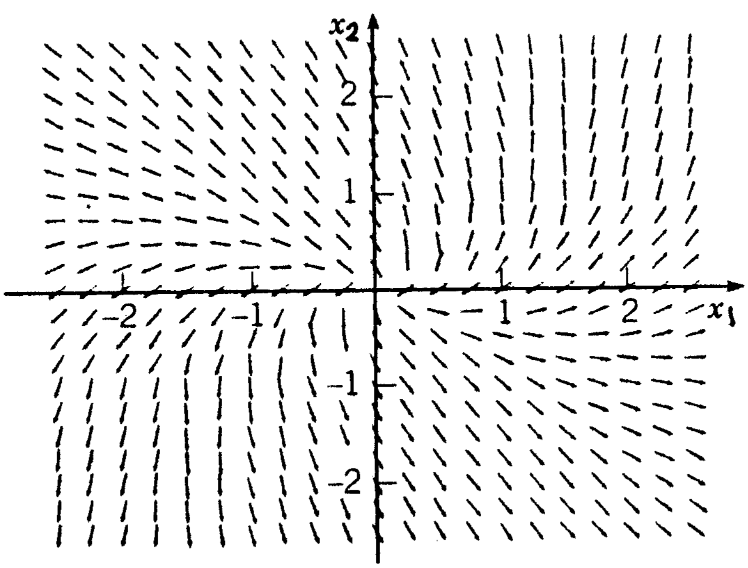
$X' = AX$

λ_1, λ_2 eigenvalues for A

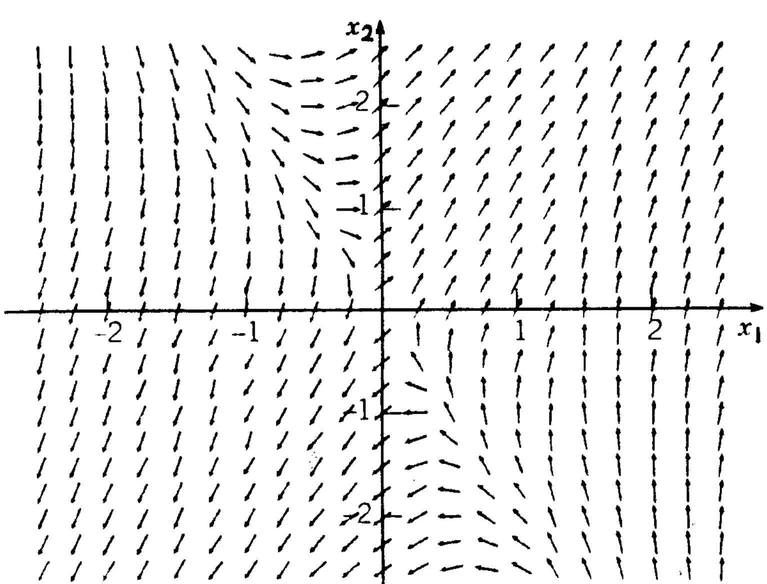
Describing an Equilibrium at $(0,0)$



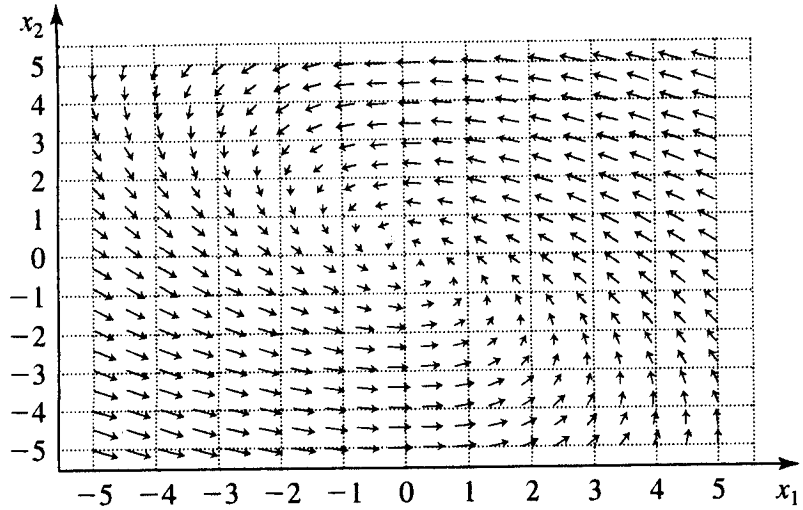
stable
(sink)
λ_1, λ_2 real
$\lambda_1 < 0, \ \lambda_2 < 0$



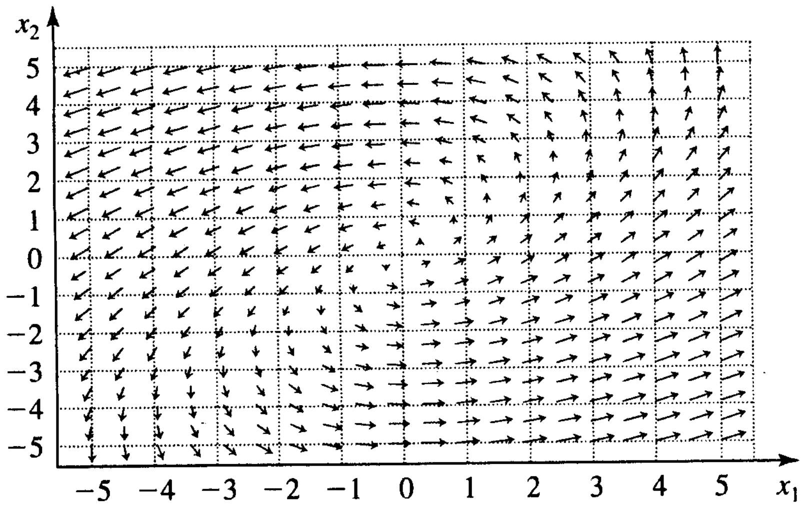
unstable
(source)
λ_1, λ_2 real
$\lambda_1 > 0, \ \lambda_2 > 0$



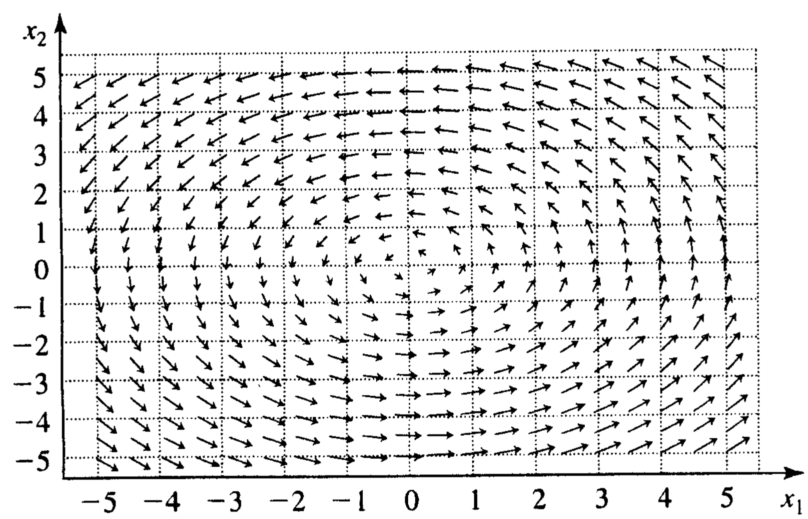
unstable
(saddle)
λ_1, λ_2 real
$\lambda_1 > 0, \ \lambda_2 < 0$



stable
(stable spiral)
$\lambda = a \pm bi$ complex
$a < 0$



unstable
(unstable spiral)
$\lambda = a \pm bi$ complex
$a > 0$



unstable
(neutral spiral)
$\lambda = a \pm bi$ complex
$a = 0$