Section 12.1

1.) $\underset{\text{light choices}}{\underline{\ 2\ }} \cdot \underset{\text{fert. choices}}{\underline{\ 5\ }} \cdot \underset{\text{\# of rep.}}{\underline{\ 4\ }} = 40$ rep.

4.) choices: $\underline{3} \cdot \underline{3} \cdot \underline{3} \cdots \underline{3} = 3^{50}$ ways
patient #: $1 \quad 2 \quad 3 \quad \cdots \quad 50$

5.) $\underset{\text{bev.}}{\underline{\ 3\ }} \cdot \underset{\text{cereal}}{\underline{\ 7\ }} \cdot \underset{\text{fruit}}{\underline{\ 4\ }} = 84$ choices

6.) $\underset{\text{food}}{\underline{\ 3\ }} \cdot \underset{\text{sex}}{\underline{\ 2\ }} \cdot \underset{\text{\# of rep.}}{\underline{\ 12\ }} = 72$ rats

9.) $P(5,5) = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ routes

10.) $P(5,5) = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ ways

13.) $P(26,4) = \dfrac{26!}{(26-4)!} = \dfrac{26 \cdot 25 \cdot 24 \cdots 3 \cdot 2 \cdot 1}{22 \cdot 21 \cdot 20 \cdots 3 \cdot 2 \cdot 1}$
$= 26 \cdot 25 \cdot 24 \cdot 23 = 358{,}800$ words

14.) $P(10,3) = \dfrac{10!}{7!} = 10 \cdot 9 \cdot 8 = 720$ comm.

16.) $P(10,4) = \dfrac{10!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5040$ ways

19.) $C(10,3) = \dfrac{10!}{3!\,7!} = \dfrac{10\cdot \cancel{9}^{3} \cdot \cancel{8}^{4}}{\cancel{3}\cdot\cancel{2}\cdot 1} = 120$ choices

22.) $C(52,5) = \dfrac{52!}{5!\,47!} = \dfrac{52\cdot 51\cdot \cancel{50}^{10}\cdot 49\cdot \cancel{48}^{2}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1}$

$= 2,598,960$ hands

24.) $C(12,5) = \dfrac{12!}{5!\,7!} = \dfrac{\cancel{12}\cdot 11\cdot \cancel{10}\cdot 9\cdot 8}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 792$ ways

25.) $\dfrac{9!}{4!\,5!} = \dfrac{\cancel{9}^{3}\cdot\cancel{8}\cdot 7\cdot 6}{\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 126$ words

29.) a.) i.) 2 red : $C(5,2) = \dfrac{5!}{2!\,3!} = \dfrac{5\cdot 4}{2} = 10$

ii.) 2 blue : $C(4,2) = \dfrac{4!}{2!\,2!} = \dfrac{4\cdot 3}{2} = 6$

iii.) 1 red, 1 blue :

$\underset{\text{red}}{\underline{C(5,1)}} \cdot \underset{\text{blue}}{\underline{C(4,1)}} = \dfrac{5!}{1!\,4!}\cdot\dfrac{4!}{1!\,3!} = 20$

b.) $C(9,2) = \dfrac{9!}{2!\,7!} = \dfrac{9\cdot 8}{2} = 36$

and $36 = 10 + 6 + 20$

30.) $\underset{\text{pick 5}}{\underline{C(12,5)}} \cdot \underset{\text{pick 4}}{\underline{C(7,4)}} \cdot \underset{\text{pick 3}}{\underline{C(3,3)}}$

$$= \frac{12!}{5!\,7!} \cdot \frac{7!}{4!\,3!} \cdot \frac{3!}{3!\,0!}$$

$$= 792 \cdot 35 \cdot 1 = 27{,}720 \text{ ways}$$

31.) $\dfrac{14!}{5!\,3!\,6!} = 168{,}168$ words

33.) $S = \{a, b, c\} \rightarrow$

Subsets : $\emptyset$, $\{a\}$, $\{b\}$ $\{c\}$,
$\{a,b\}$, $\{a,c\}$, $\{b,c\}$, $\{a,b,c\}$: 8 subsets

$$\underset{\substack{a \text{ is} \\ \text{in or out}}}{\underline{2}} \cdot \underset{\substack{b \text{ is} \\ \text{in or out}}}{\underline{2}} \cdot \underset{\substack{c \text{ is} \\ \text{in or out}}}{\underline{2}} = 8 \text{ subsets}$$

34.) $\underset{\substack{1 \text{ is} \\ \text{in or out}}}{\underline{2}} \cdot \underset{\substack{2 \text{ is} \\ \text{in or out}}}{\underline{2}} \cdot \underset{\substack{3 \text{ is} \\ \text{in or out}}}{\underline{2}} \cdots \underset{\substack{n \text{ is} \\ \text{in or out}}}{\underline{2}} = 2^n$ subsets

35.) $\underline{P}\ \underline{M}\ \underline{?}\ \underline{?}$

If P sits to left of M :

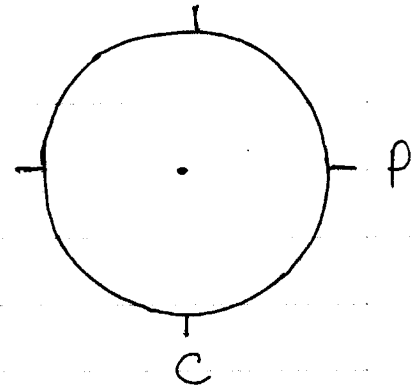
$$\underset{\substack{\text{seat PM} \\ \text{1st}}}{\underline{3}} \cdot \underset{\substack{\text{next} \\ \text{person}}}{\underline{2}} \cdot \underset{\substack{\text{next} \\ \text{person}}}{\underline{1}} = 6 \text{ ways};$$

If P sits to right of M: 6 ways $\rightarrow$
12 ways total

36.) If C sits on P's left :

$$\underset{\text{seat CP}}{\underline{4}} \cdot \underset{\text{next person}}{\underline{2}} \cdot \underset{\text{next person}}{\underline{1}}$$

$= 8$ ways



37.) 7 people : 5 and A and B, where A and B will <u>not</u> be on 3-person committee together :

comm. with A : $C(5,2) = 10$,
comm. with B : $C(5,2) = 10$,
comm. with neither A nor B : $C(5,3) = 10 \longrightarrow$
total # of committees $= 30$

39.) 4 annuals, 3 perennials

exactly 1 per. : $\underset{\text{pick 1 per.}}{\underline{3}} \cdot \underset{\text{pick 2 ann.}}{\underline{C(4,2)}} = 3 \cdot 6 = 18$,

exactly 2 per. : $\underset{\text{pick 2 per.}}{\underline{C(3,2)}} \cdot \underset{\text{pick 1 ann.}}{\underline{C(4,1)}} = 3 \cdot 4 = 12$,

exactly 3 per. : $\underset{\text{pick 3 per.}}{\underline{C(3,3)}} = 1 \longrightarrow$

total is 31 choices

45.) (52 cards: 13 diamonds (red) 13 hearts (red), 13 spades (black), 13 clubs (black))

$\underset{\text{pick 4 red}}{\underline{C(26,4)}} \cdot \underset{\text{pick 5 black}}{\underline{C(26,5)}} = 14{,}950 \cdot 65{,}780 = 983{,}411{,}000$ ways

46.) (4 aces, 4 kings)

$\underset{\text{pick 2 aces}}{\underline{C(4,2)}} \cdot \underset{\text{pick 3 kings}}{\underline{C(4,3)}} = 6 \cdot 4 = 24$ ways

47.) (52 cards: there are 4 each of A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K)

a poker hand is a 5-card combination ∴

$\underset{\text{pick 2 face values}}{\underline{C(13,2)}} \cdot \underset{\text{pick pair 1}}{\underline{C(4,2)}} \cdot \underset{\text{pick pair 2}}{\underline{C(4,2)}} \cdot \underset{\text{pick card 5, which can't be same face value } (52-8=44)}{\underline{44}}$

$= 78 \cdot 6 \cdot 6 \cdot 44 = 123{,}552$ hands

48.) $\underset{\text{pick the suit}}{\underline{4}} \cdot \underset{\text{pick 5 cards}}{\underline{C(13,5)}} = 4 \cdot 1287 = 5148$ hands

49.) $\underset{\text{pick face value}}{\underline{13}} \cdot \underset{\text{pick 4 cards}}{\underline{C(4,4)}} \cdot \underset{\text{pick card 5}}{\underline{48}} = 624$ hands

50.) (a straight can <u>start</u> with
A, 2, 3, 4, 5, 6, 7, 8, 9, 10.)

$$\underset{\substack{\text{pick}\\\text{face}\\\text{value}\\\text{of}\\\text{start}\\\text{card}}}{\underline{10}} \cdot \underset{\substack{\text{pick}\\\text{card}\\1}}{\underline{C(4,1)}} \cdot \underset{\substack{\text{pick}\\\text{card}\\2}}{\underline{C(4,1)}} \cdot \underset{\substack{\text{pick}\\\text{card}\\3}}{\underline{C(4,1)}} \cdot \underset{\substack{\text{pick}\\\text{card}\\4}}{\underline{C(4,1)}} \cdot \underset{\substack{\text{pick}\\\text{card}\\5}}{\underline{C(4,1)}}$$

$= 10 \cdot 4^5 = 10{,}240$, but this is too many! It includes straight flushes (straights with all same suit), so we must throw away the straight flushes:

$$\underset{\substack{\text{pick}\\\text{suit}}}{\underline{4}} \cdot \underset{\substack{\text{pick}\\\text{start}\\\text{card}}}{\underline{10}} \cdot \underset{\substack{\text{pick}\\\text{card}\\2}}{\underline{1}} \cdot \underset{\substack{\text{pick}\\\text{card}\\3}}{\underline{1}} \cdot \underset{\substack{\text{pick}\\\text{card}\\4}}{\underline{1}} \cdot \underset{\substack{\text{pick}\\\text{card}\\5}}{\underline{1}} = 40,$$

so total number of straights is $10{,}240 - 40 = 10{,}200$.