Section 12.3

1.) 1st card is club, leaving 51 cards:
P(2nd card is spade) $= \frac{13}{51}$

4.) 1st 2 cards are clubs, leaving 50 cards:
P(3rd card is club) $= \frac{11}{50}$

5.) 5 blue, 6 green balls : 1st ball is blue, leaving 4 blue, 6 green balls; then P(2nd ball green) $= \frac{6}{10} = \frac{3}{5}$

6.) 5 green, 6 blue, 4 red balls : 1st 2 balls are red, leaving 5 green, 6 blue, 2 red balls; then
P(3rd ball green) $= \frac{5}{13}$

7.) Having 2 children are independent events : P(older child is girl) $= \frac{1}{2}$

8.) Sample Space $\Omega = \{bB, bG, gB, gG\}$;
younger older

let A: one of the children is a girl so
$A = \{bG, gB, gG\}$
let B: both children are girls so
$B = \{gG\}$; then the probability of having 2 girls knowing there is at least 1 girl is

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(9G)}{P(A)}$$

$$= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{3}$$

9.) Sample Space has 36 outcomes:
Let $A = \{41, 42, 43, 44, 45, 46\}$
let $B = \{61, 52, 43, 34, 25, 16\}$, then

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(43)}{P(B)}$$

$$= \frac{\frac{1}{36}}{\frac{6}{36}} = \frac{1}{36} \cdot \frac{36}{6} = \frac{1}{6}$$

10.) Sample Space has 36 outcomes:
Let $A = \{51, 52, 53, 54, 55, 56\}$
let $B = \{33, 34, 35, 36, 43, 44, 45, 46, 53, 54, 55, 56, 63, 64, 65, 66\}$, then

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\{53, 54, 55, 56\})}{P(B)}$$

$$= \frac{\frac{4}{36}}{\frac{16}{36}} = \frac{4}{36} \cdot \frac{36}{16} = \frac{1}{4}$$

11.) Sample Space has $2 \cdot 2 \cdot 2 = 8$ outcomes:
Let $A = \{HHH, HHT, HTH, HTT\}$,
let $B = \{HTT, THT, TTH, HHT, HTH, THH, HHH\}$, then

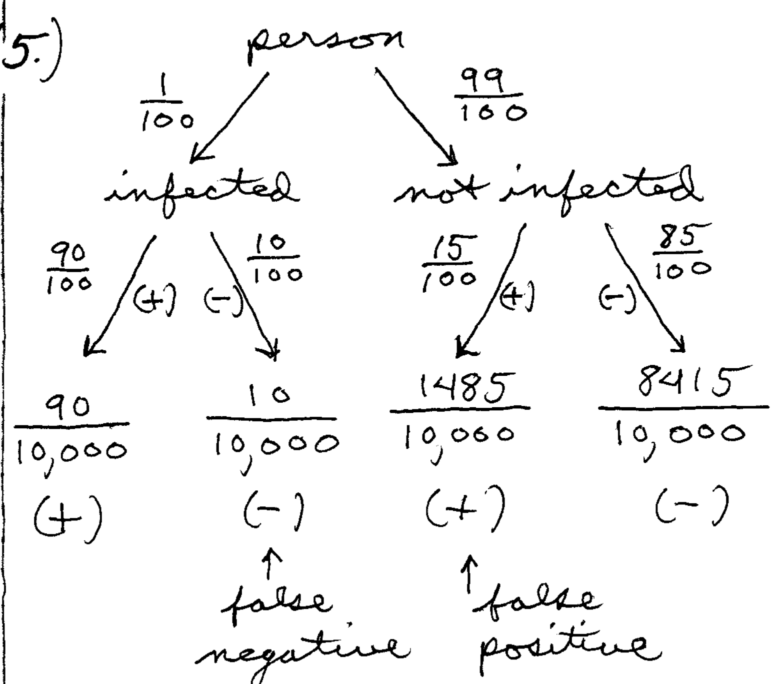
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{\frac{4}{8}}{\frac{7}{8}} = \frac{4}{7}$$

12.) Sample Space has $2 \cdot 2 \cdot 2 = 8$ outcomes:
Let $A = \{HHT, HTH, THH, HHH\}$,
let $B = \{HHH, HHT, THH, THT\}$, then

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\{HHT, THH, HHH\})}{P(B)}$$

$$= \frac{\frac{3}{8}}{\frac{4}{8}} = \frac{3}{4}$$

15.)



$$P(-) = \frac{8415}{10,000} + \frac{10}{10,000}$$

$$= 0.8425$$

Test

18.) Let <u>prevalence</u> be $\boxed{k\% = \frac{k}{100}}$;

person

$\frac{k}{100}$ infected $\qquad$ $\frac{100-k}{100}$ not infected

infected: $\frac{95}{100}$ (+) $\qquad$ $\frac{5}{100}$ (−)

not infected: $\frac{20}{100}$ (+) $\qquad$ $\frac{80}{100}$ (−)

$$\frac{95k}{10{,}000}\ (+) \qquad \frac{5k}{10{,}000}\ (-) \qquad \frac{2000-20k}{10{,}000}\ (+) \qquad \frac{8000-80k}{10{,}000}\ (-)$$

false negative $\qquad$ false positive

Given $\frac{95k}{10{,}000} + \frac{2000-20k}{10{,}000} = 21.5\% \rightarrow$

$$\frac{2000+75k}{10{,}000} = \frac{21.5}{100} \rightarrow 100\,(2000+75k) = 21.5\,(10{,}000)$$

$$\rightarrow 2000 + 75k = 21.5\,(100) = 2150$$

$$\rightarrow 75k = 150 \rightarrow k = 2 \rightarrow$$

prevalence is 2%

19.) pick 1 bag

$\frac{1}{3}$ bag 1 $\qquad$ $\frac{1}{3}$ bag 2 $\qquad$ $\frac{1}{3}$ bag 3

pick 1 ball

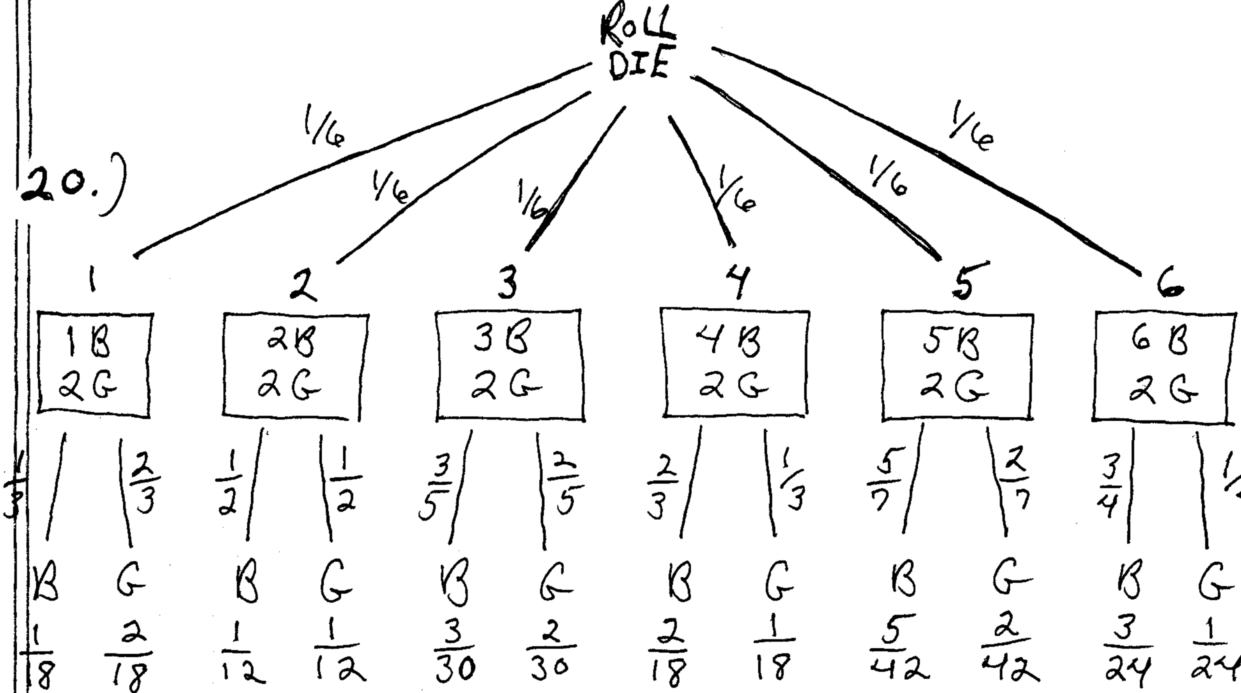
bag 1: $\frac{3}{3}$ blue, $\frac{3}{9}$

bag 2: $\frac{4}{4}$ green, $\frac{4}{12}$

bag 3: $\frac{1}{3}$ green, $\frac{1}{9}$; $\quad$ $\frac{2}{3}$ blue, $\frac{2}{9}$

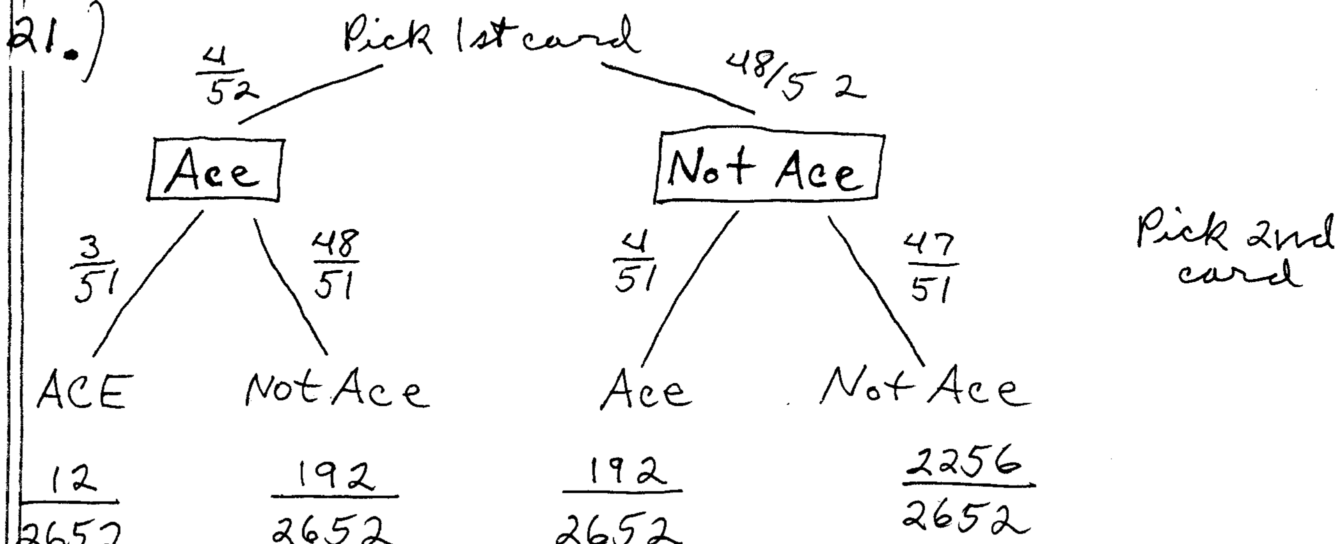
$$P(\text{blue}) = \frac{3}{9} + \frac{2}{9} = \frac{5}{9}$$

20.)



$$P(\text{Blue}) = \frac{1}{18} + \frac{1}{12} + \frac{3}{30} + \frac{2}{18} + \frac{5}{42} + \frac{3}{24} \approx 0.59$$

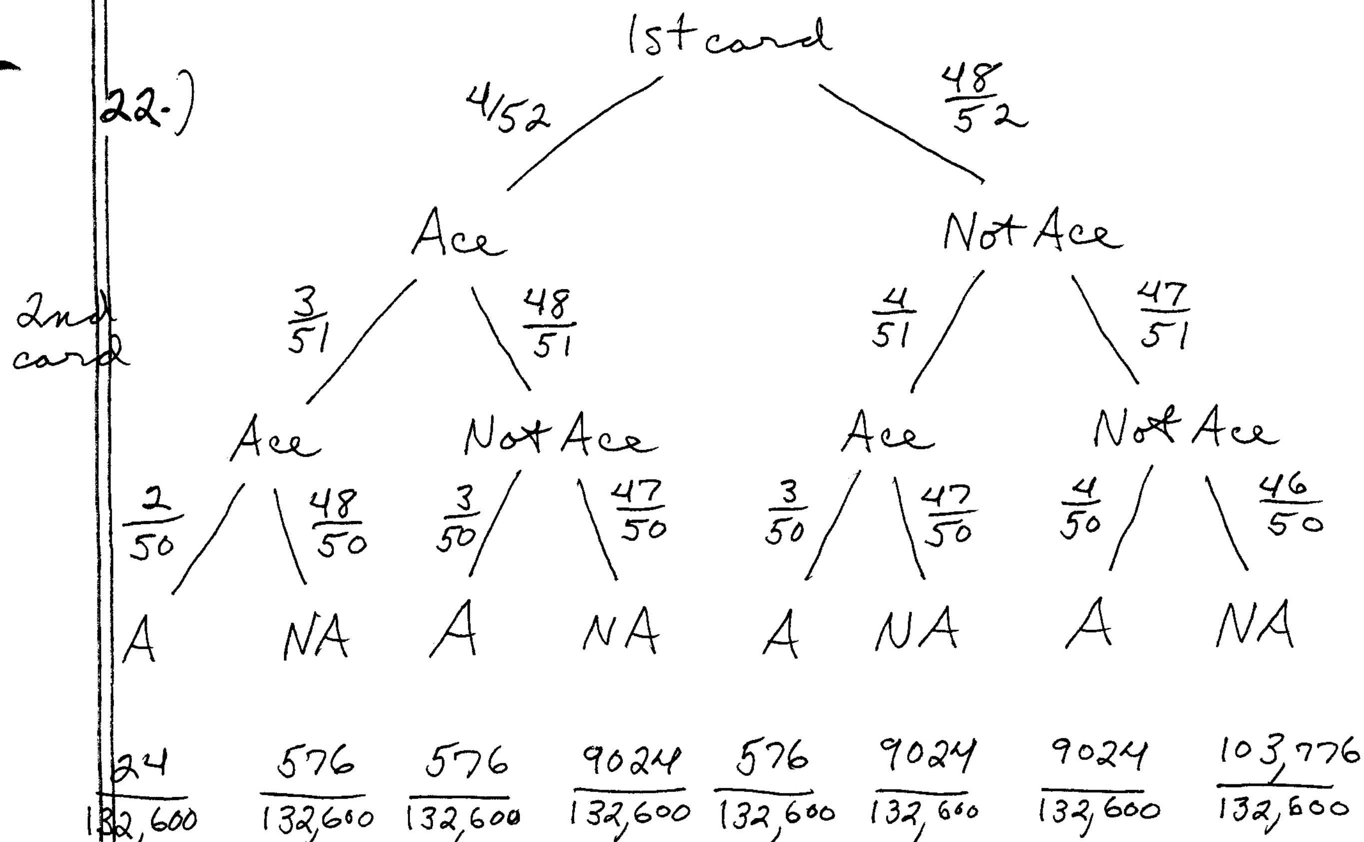
21.)



$$P(\text{Ace on 2nd pick}) = \frac{12}{2652} + \frac{192}{2652} = \frac{204}{2652} = \frac{1}{13},$$

$$P(\text{Ace on 1st pick}) = \frac{4}{52} = \frac{1}{13}$$

22.)

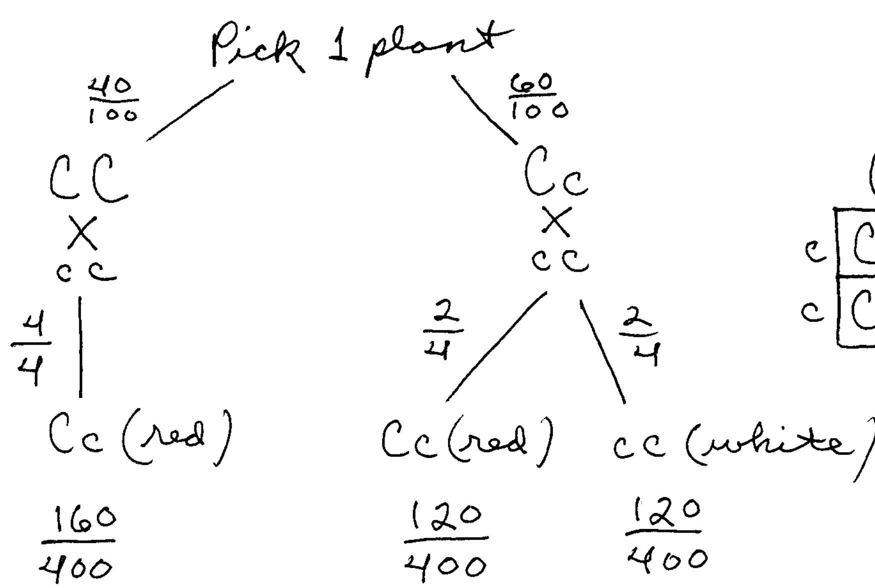


$$P(\text{Ace on 3rd pick}) = \frac{24+576+576+9024}{132,600} = \frac{1}{13},$$

$$P(\text{Ace on 1st pick}) = \frac{4}{52} = \frac{1}{13}$$

23.)

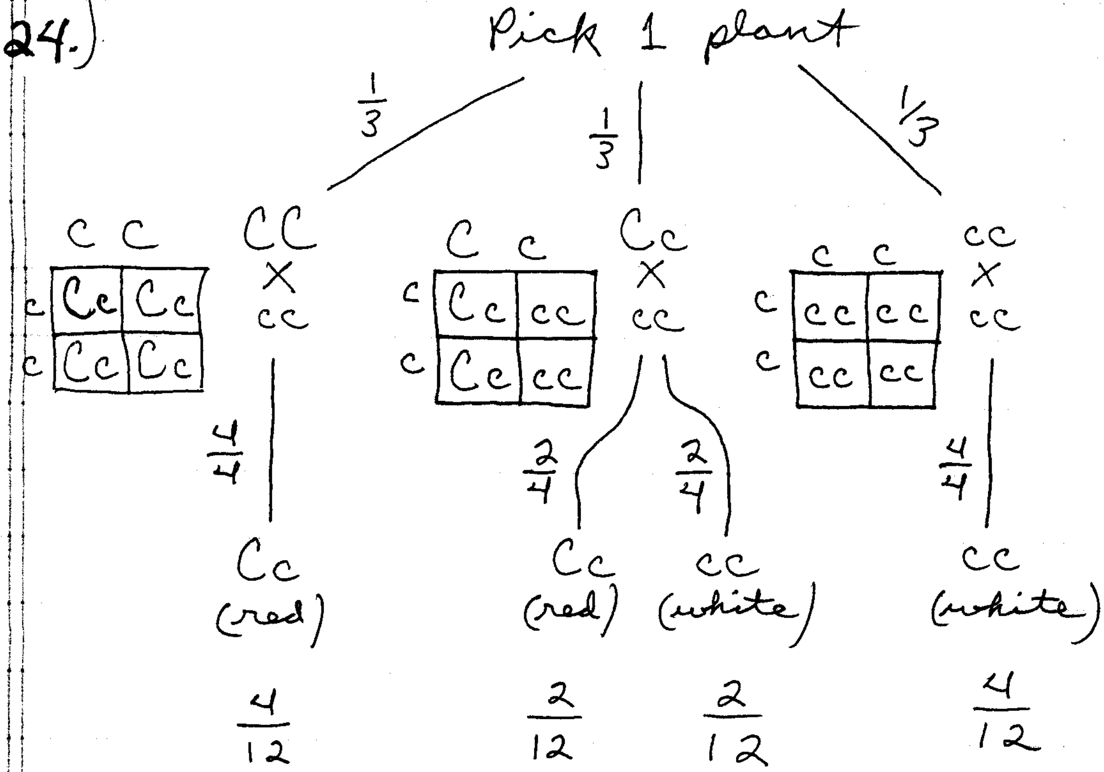
	C	C
c	Cc	Cc
c	Cc	Cc



	C	c
c	Cc	cc
c	Cc	cc

$$P(\text{white}) = \frac{120}{400} = \frac{3}{10}$$

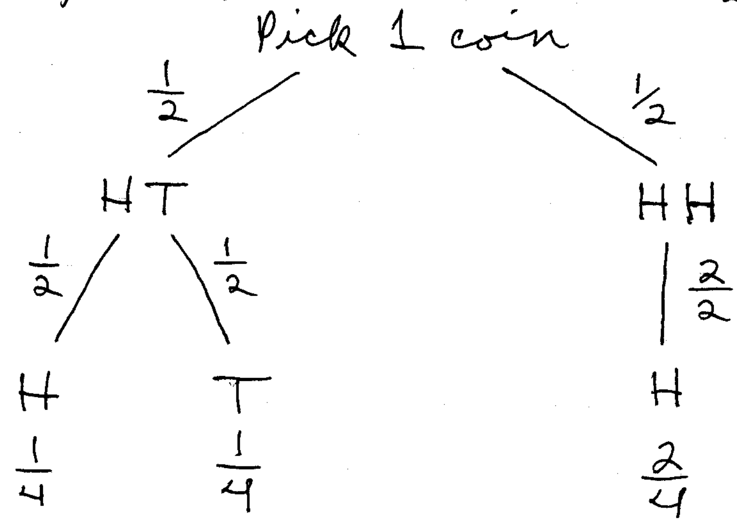
24.)



$P(\text{white}) = \frac{2}{12} + \frac{4}{12} = \frac{6}{12} = \frac{1}{2}$ and

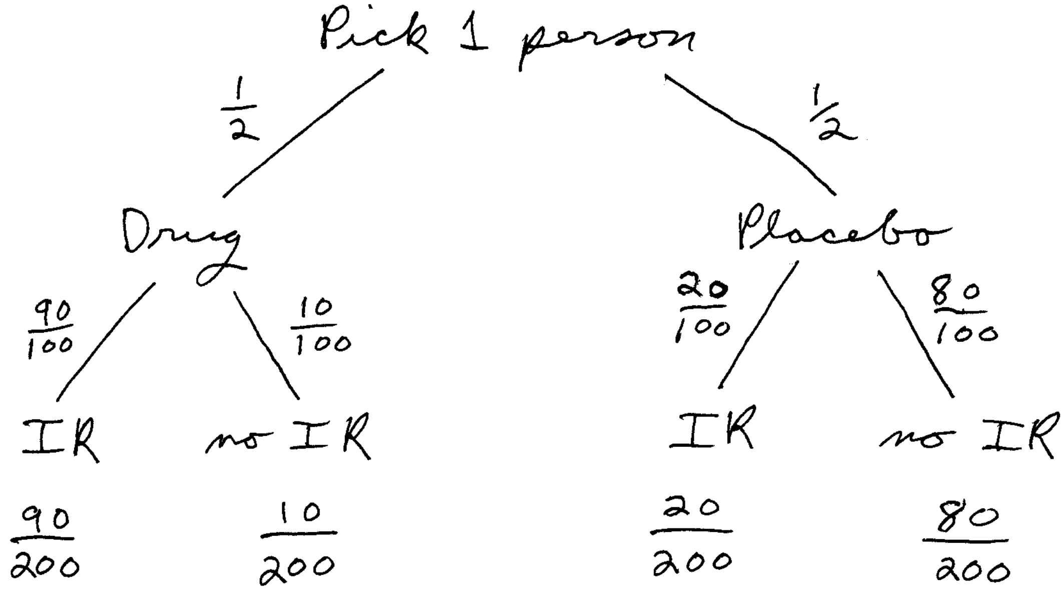
$P(\text{red}) = \frac{4}{12} + \frac{2}{12} = \frac{6}{12} = \frac{1}{2}$

25.)



$P(H) = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

26.)



$$P(IR) = \frac{90}{200} + \frac{20}{200} = \frac{110}{200} = \frac{11}{20}$$

27.) Let A: spade, B: Ace, ~~then~~

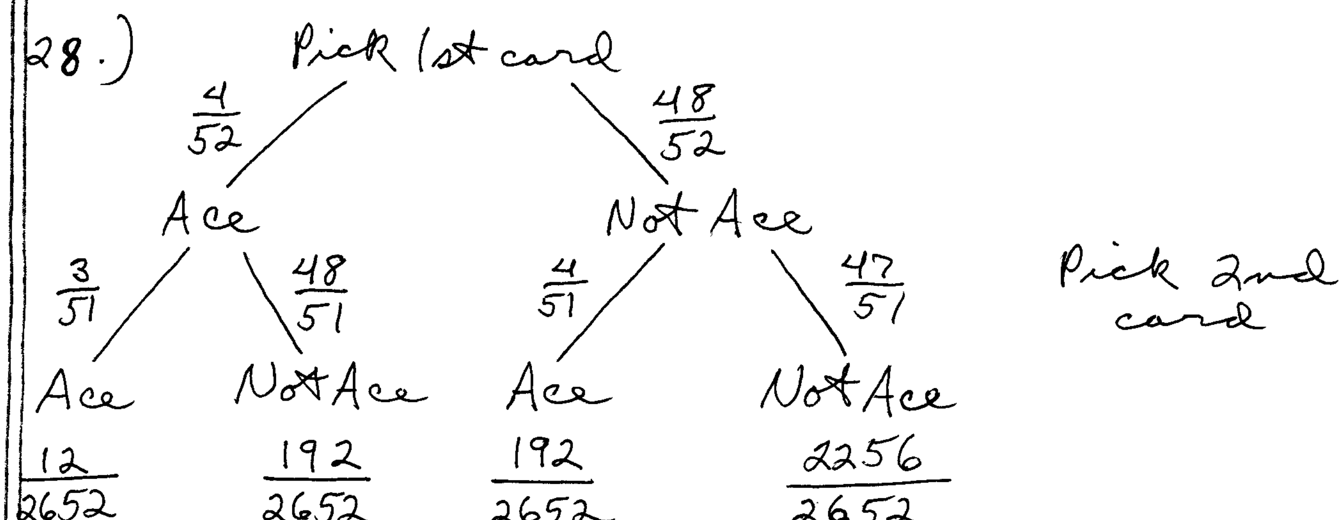
$P(A) = {}^{13}/_{52} = {}^{1}/_{4}$, $P(B) = {}^{4}/_{52} = {}^{1}/_{13}$,

$P(A \cap B) = P(\text{Ace of } \spadesuit) = {}^{1}/_{52}$ and

$P(A) \cdot P(B) = \frac{1}{4} \cdot \frac{1}{13} = \frac{1}{52} = P(A \cap B)$,

so A and B are independent.

28.)



Let A: 1st card Ace, B: 2nd card Ace

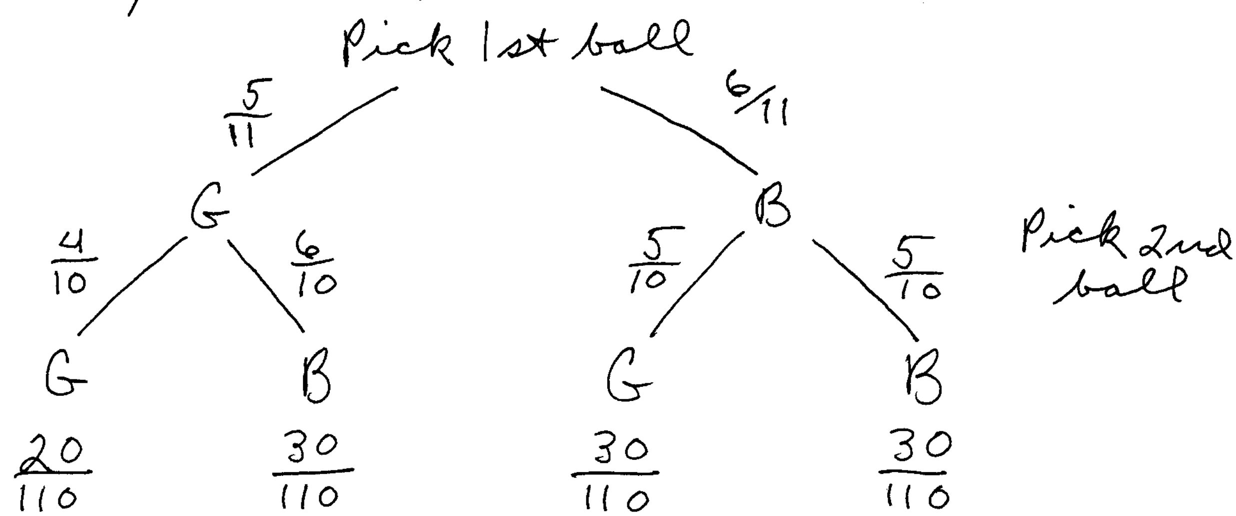
$P(A) = \frac{4}{52} = \frac{1}{13}$, $P(B) = \frac{12}{2652} + \frac{192}{2652} = \frac{1}{13}$,

$P(A \cap B) = \frac{12}{2652} = \frac{1}{221}$, then

$P(A) \cdot P(B) = \frac{1}{13} \cdot \frac{1}{13} = \frac{1}{169} \neq \frac{1}{221} = P(A \cap B)$,

so A and B are <u>not</u> independent.

29.) (5G, 6B balls)



Let A: 1st ball G, B: 2nd ball G, then

$P(A) = 5/11$, $P(B) = \frac{20}{110} + \frac{30}{110} = \frac{50}{110} = 5/11$,

$P(A \cap B) = \frac{20}{110} = \frac{2}{11}$ and

$P(A) \cdot P(B) = \frac{5}{11} \cdot \frac{5}{11} = \frac{25}{121} \neq \frac{2}{11} = P(A \cap B)$

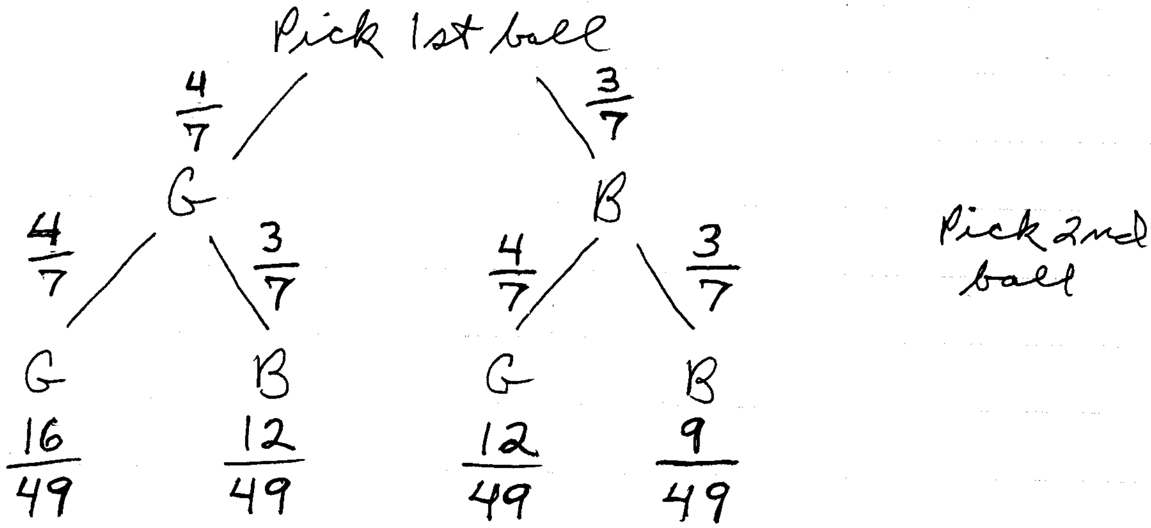
so A and B are <u>not</u> independent

30.) (4G, 3B balls)

Let A: 1st ball G, B: 2nd ball G, then

$P(A) = \frac{4}{7}$, $P(B) = \frac{16}{49} + \frac{12}{49} = \frac{28}{49} = \frac{4}{7}$

(SEE diagram on next page.)



$P(A \cap B) = \frac{4}{7} \cdot \frac{4}{7} = \frac{16}{49}$, ~~then~~

$P(A) \cdot P(B) = \frac{4}{7} \cdot \frac{4}{7} = \frac{16}{49} = P(A \cap B)$, so

A and B are independent.

31.) Let A: girl, B: boy, then $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$ and A and B are independent

a.) $P(A_1 \cap A_2 \cap A_3) = P(A_1) \cdot P(A_2) \cdot P(A_3) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

b.) $P(\text{at least } 1 \text{ boy}) = 1 - P(\text{no boys})$

$= 1 - P(A_1 \cap A_2 \cap A_3) = 1 - \frac{1}{8} = \frac{7}{8}$

c.) $P(\text{at least } 2 \text{ girls}) = P(\text{exactly } 2 \text{ girls})$

$+ P(\text{exactly } 3 \text{ girls})$

$= P(A_1 \cap A_2 \cap B_3) + P(A_1 \cap B_2 \cap A_3) + P(B_1 \cap A_2 \cap A_3)$

$+ P(A_1 \cap A_2 \cap A_3) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$

d.) $P(\text{at most 2 boys}) = P(\text{at least 1 girl})$
$= 1 - P(\text{no girl}) = 1 - P(B_1 \cap B_2 \cap B_3)$
$= 1 - \frac{1}{8} = \frac{7}{8}$

32.) A: parasitized ; B: not parasitized
$P(A) = 20\% = \frac{1}{5}$, $P(B) = 80\% = 4/5$ so pick 10:
$P(\text{at most 2 parasitized})$
$= P(0A, 10B) + P(1A, 9B) + P(2A, 8B)$
$= \left(\frac{4}{5}\right)^{10} + (9)\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^9 + \frac{10!}{8!\,2!}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^8$
$= \frac{1}{5^{10}}\left(4^{10} + 9 \cdot 4^9 + 45 \cdot 4^8\right) = \frac{6{,}356{,}992}{9{,}765{,}625} \approx 0.651$

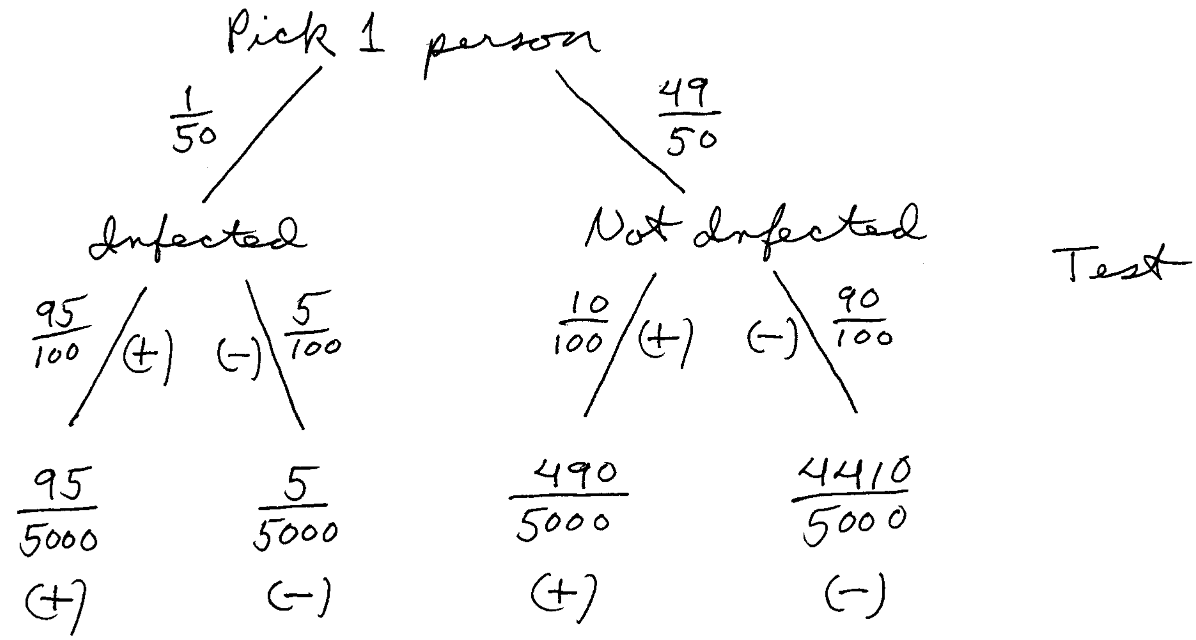
33.) Let A: correct answer, $P(A) = \frac{1}{4}$,
for 10 questions
$P(\text{10 correct answers})$
$= P(\underbrace{A_1 \cap A_2 \cap \cdots \cap A_{10}}_{\text{10 times}}) = P(A_1) \cdot P(A_2) \cdots P(A_{10})$
$= \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdots \frac{1}{4} = \left(\frac{1}{4}\right)^{10} \approx 0.0000009$

34.) Assume $A \cap B = \emptyset$, $P(A) > 0$, and $P(B) > 0$.
Then
$P(A \cap B) = P(\emptyset) = 0 \neq P(A) \cdot P(B) > 0$,
so A and B are <u>not</u> independent

35.) Let A: lives > 5 days, then
$P(A) = 0.1$ so pick 10 insects $\rightarrow$
$P(\text{at least 1 alive} > 5 \text{ days})$
$= 1 - P(\text{no alive} > 5 \text{ days})$

$= 1 - P(10 \text{ not alive} > 5 \text{ days})$
$= 1 - (0.9)^{10} \approx 0.651 = 65.1\%$

37.)



Let A: person tests (+),
B_1: person is infected,
B_2: person is not infected

$$P(B_1|A) = \frac{P(A|B_1)\cdot P(B_1)}{P(A|B_1)P(B_1) + P(A|B_2)\cdot P(B_2)}$$

$$= \frac{(0.95)(1/50)}{(0.95)(\frac{1}{50}) + (0.10)(\frac{49}{50})} \approx 0.1624 = 16.24\%$$

38.) (Use tree from problem 31.)
Let C: person tests (+) twice,
A: person tests (+) once,
B_1: person is infected
B_2: person is not infected;

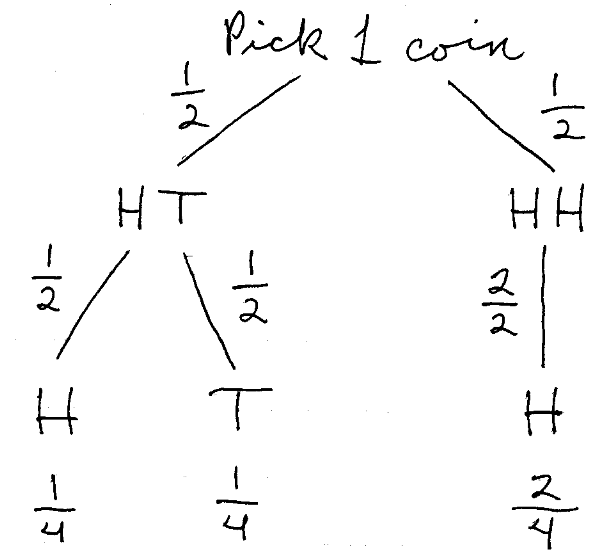
assume tests are independent events, then:

$$P(A|B_1) = 95\% = 0.95 \to$$
$$P(C|B_1) = P(A|B_1)\cdot P(A|B_1) = (0.95)^2 \text{ and}$$
$$P(A|B_2) = 10\% = 0.1 \to$$
$$P(C|B_2) = P(A|B_2)\cdot P(A|B_2) = (0.1)^2, \text{ then}$$

$$P(B_1|C) = \frac{P(C|B_1)\cdot P(B_1)}{P(C|B_1)\cdot P(B_1) + P(C|B_2)\cdot P(B_2)}$$

$$= \frac{(0.95)^2(1/50)}{(0.95)^2(1/50)+(0.1)^2(49/50)} \approx 0.6481 = 64.81\%$$

39.)



Flip coin

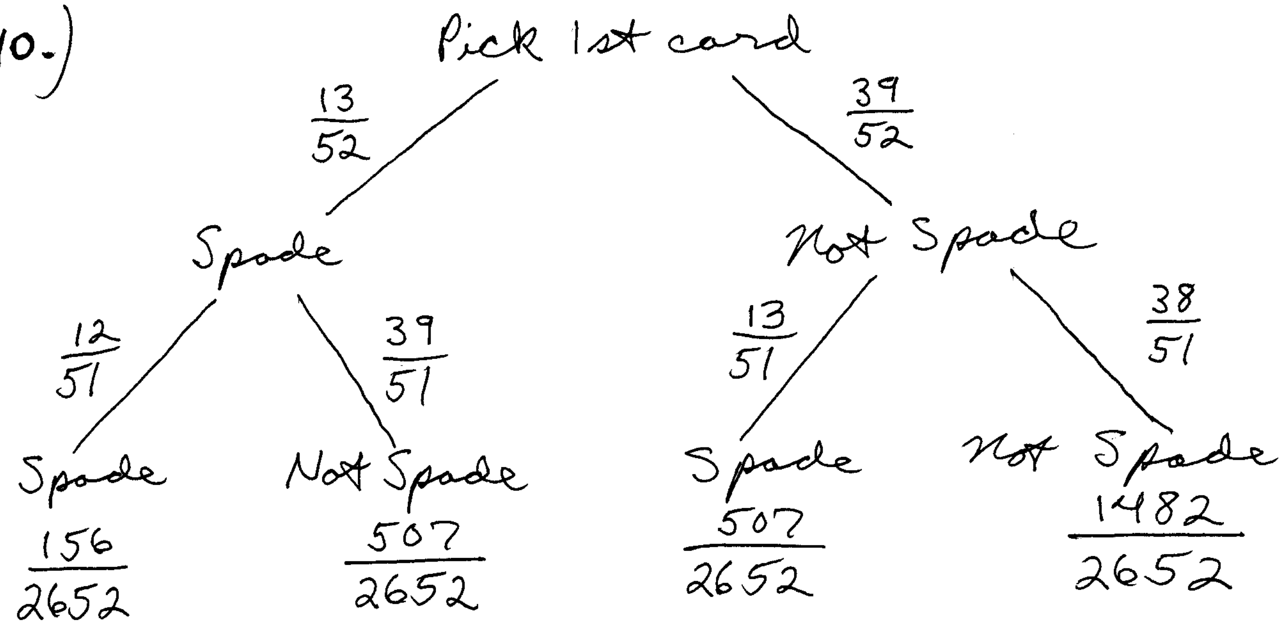
Let A: toss H,
B_1: pick fair coin (HT),
B_2: pick unfair coin (HH),

$$P(B_1 \mid A) = \frac{P(A \mid B_1)\cdot P(B_1)}{P(A \mid B_1)\cdot P(B_1) + P(A \mid B_2)P(B_2)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{2}{2}\right)\left(\frac{1}{2}\right)} = \frac{\frac{1}{4}}{\frac{1}{4}+\frac{2}{4}} = \frac{1}{4}\cdot\frac{4}{3} = \frac{1}{3}$$

40.)

Pick 2nd card



Let A: 2nd card a spade,
B_1: 1st card a spade,
B_2: 1st card not a spade; then

$$P(B_1 \mid A) = \frac{P(A \mid B_1)\cdot P(B_1)}{P(A \mid B_1)P(B_1) + P(A \mid B_2)\cdot P(B_2)}$$

$$= \frac{\left(\frac{12}{51}\right)\left(\frac{13}{52}\right)}{\left(\frac{12}{51}\right)\left(\frac{13}{52}\right) + \left(\frac{13}{51}\right)\cdot\left(\frac{39}{52}\right)} = \frac{\frac{156}{2652}}{\frac{156}{2652}+\frac{507}{2652}}$$

$$= \frac{156}{663} = \frac{4}{17}$$