Section 10.4

1.) $f(x,y) = 2x^3 + y^2 \rightarrow$
$f_x = 6x^2$, $f_y = 2y$ and pt. is
$(1, 2, 6)$ so tangent plane is
$z = 6 + f_x(1,2)(x-1) + f_y(1,2)(y-2) \rightarrow$

$z = 6 + 6(x-1) + 4(y-2) = 6 + 6x - 6 + 4y - 8 \rightarrow$

$\boxed{z = 6x + 4y - 8}$

4.) $f(x,y) = \sin x + \cos y \rightarrow$
$f_x = \cos x$, $f_y = -\sin y$ and pt. is
$(0,0,1)$ so tangent plane is
$z = 1 + f_x(0,0)(x-0) + f_y(0,0)(y-0) \rightarrow$
$z = 1 + (1)(x) + (0)(y) \rightarrow$ $\boxed{z = 1 + x}$

7.) $f(x,y) = e^{x^2+y^2} \rightarrow$
$f_x = 2xe^{x^2+y^2}$, $f_y = 2ye^{x^2+y^2}$ and
pt. is $(1, 0, e)$ so tangent plane is
$z = e + f_x(1,0)(x-1) + f_y(1,0)(y-0) \rightarrow$

$z = e + (2e)(x-1) + (0)(y) \rightarrow$
$z = e + 2ex - 2e \rightarrow$ $\boxed{z = 2ex - e}$

10.) $f(x,y) = \ln(x^2+y^2) \rightarrow$
$f_x = \dfrac{2x}{x^2+y^2}$, $f_y = \dfrac{2y}{x^2+y^2}$ and pt. is

$(1, 1, \ln 2)$ so tangent plane is

$z = \ln 2 + f_x(1,1)\cdot(x-1) + f_y(1,1)\cdot(y-1) \rightarrow$

$z = \ln 2 + (1)(x-1) + (1)(y-1) \rightarrow$

$z = \ln 2 + x - 1 + y - 1 \rightarrow \boxed{z = x + y + \ln 2 - 2}$

19.) $f(x,y) = \sqrt{x} + 2y \rightarrow$

$f_x = \frac{1}{2\sqrt{x}}$, $f_y = 2$ and pt. is $(1,0)$ so

$z = f(1,0) = \sqrt{1} + 2(0) = 1$, then linearization is

$L(x,y) = 1 + f_x(1,0)(x-1) + f_y(1,0)(y-0) \rightarrow$

$L(x,y) = 1 + \frac{1}{2}(x-1) + 2(y) \rightarrow \boxed{L(x,y) = \frac{1}{2}x + 2y + \frac{1}{2}}$

21.) $f(x,y) = \tan(x+y) \rightarrow$

$f_x = \sec^2(x+y)$, $f_y = \sec^2(x+y)$

and pt. is $(0,0)$ so $z = f(0,0) = \tan 0 = 0$,

then linearization is

$L(x,y) = 0 + f_x(0,0)\cdot(x-0) + f_y(0,0)\cdot(y-0) \rightarrow$

$L(x,y) = (1)(x) + (1)(y) \rightarrow \boxed{L(x,y) = x + y}$

24.) $f(x,y) = x^2 e^y \rightarrow$

$f_x = 2xe^y$, $f_y = x^2 e^y$ and pt. is $(1,0)$

so $z = f(1,0) = 1e^0 = 1$, then

linearization is

$$L(x,y) = 1 + f_x(1,0)\,(x-1) + f_y(1,0)\cdot(y-0) \rightarrow$$
$$L(x,y) = 1 + (2)\,(x-1) + (1)\,(y) \rightarrow$$
$$L(x,y) = 1 + 2x - 2 + y \rightarrow$$
$$\boxed{L(x,y) = 2x + y - 1}\,.$$

26.) $f(x,y) = \sin(x+2y) \rightarrow$

$f_x = \cos(x+2y)$, $f_y = 2\cdot\cos(x+2y)$
and pt. is $(0,0)$ so
$z = f(0,0) = \sin 0 = 0$, then linearization
is $L(x,y) = 0 + f_x(0,0)\cdot(x-0) + f_y(0,0)\cdot(y-0) \rightarrow$
$$L(x,y) = (1)(x) + (2)(y) \rightarrow$$
$$\boxed{L(x,y) = x + 2y}\ ;$$

$f(-0.1, 0.2) = \sin(-0.1 + 0.4) = \sin(0.3) \approx 0.296$;
$L(-0.1, 0.2) = -0.1 + 0.4 = 0.300$

27.) $f(x,y) = \ln(x^2 - 3y) \rightarrow$

$f_x = \dfrac{2x}{x^2-3y}$, $f_y = \dfrac{-3}{x^2-3y}$ and pt. is
$(1,0)$ so $z = f(1,0) = \ln 1 = 0$, then
linearization is
$$L(x,y) = 0 + f_x(1,0)\,(x-1) + f_y(1,0)\cdot(y-0) \rightarrow$$
$$L(x,y) = (2)(x-1) + (-3)(y) \rightarrow$$

$L(x,y) = 2x - 2 - 3y \longrightarrow \boxed{L(x,y) = 2x - 3y - 2}$;

$f(1.1, 0.1) = \ln\left(1.1^2 - 3(0.1)\right) \approx -0.0943$;

$L(1.1, 0.1) = 2(1.1) - 3(0.1) - 2 = -0.1000$.

29.) $f(x,y) = \begin{bmatrix} x+y \\ x^2-y^2 \end{bmatrix} \longrightarrow$

$$Df(x,y) = \begin{bmatrix} 1 & 1 \\ 2x & -2y \end{bmatrix}$$

31.) $f(x,y) = \begin{bmatrix} e^{x-y} \\ e^{x+y} \end{bmatrix} \longrightarrow$

$$Df(x,y) = \begin{bmatrix} e^{x-y} & -e^{x-y} \\ e^{x+y} & e^{x+y} \end{bmatrix}$$

34.) $f(x,y) = \begin{bmatrix} \ln(x+y) \\ e^{x+y} \end{bmatrix} \longrightarrow$

$$Df(x,y) = \begin{bmatrix} \frac{1}{x+y} & \frac{1}{x+y} \\ e^{x+y} & e^{x+y} \end{bmatrix}$$

35.) $f(x,y) = \begin{bmatrix} 2x^2y - 3y + x \\ e^x \sin y \end{bmatrix} \longrightarrow$

$$Df(x,y) = \begin{bmatrix} 4xy + 1 & 2x^2 - 3 \\ e^x \sin y & e^x \cos y \end{bmatrix}$$

37.) $f(x,y) = \begin{bmatrix} 2x^2y \\ \frac{1}{xy} \end{bmatrix}$ and pt. $(1,1) \longrightarrow$

$Df(x,y) = \begin{bmatrix} 4xy & 2x^2 \\ \frac{-1}{x^2y} & \frac{-1}{xy^2} \end{bmatrix}$ so

$Df(1,1) = \begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix}$; then linearization of f at $(x,y)=(1,1)$ is

$$L(x,y) = f(1,1) + Df(1,1)\cdot\begin{bmatrix} x-1 \\ y-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x-1 \\ y-1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 4(x-1)+2(y-1) \\ -(x-1)-(y-1) \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 4x-4+2y-2 \\ -x+1-y+1 \end{bmatrix} \longrightarrow$$

$$L(x,y) = \begin{bmatrix} 4x+2y-4 \\ -x-y+3 \end{bmatrix}.$$

41.) $f(x,y) = \begin{bmatrix} x/y \\ y/x \end{bmatrix}$ and pt. $(1,1) \rightarrow$

$$Df(x,y) = \begin{bmatrix} \frac{1}{y} & \frac{-x}{y^2} \\ \frac{-y}{x^2} & \frac{1}{x} \end{bmatrix} \text{ so}$$

$Df(1,1) = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$; then linearization of f at $(x,y)=(1,1)$ is

$$L(x,y) = f(1,1) + Df(1,1)\begin{bmatrix} x-1 \\ y-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x-1 \\ y-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} (x-1) - (y-1) \\ -(x-1) + (y-1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} x - 1 - y + 1 \\ -x + 1 + y - 1 \end{bmatrix} \longrightarrow$$

$$L(x,y) = \begin{bmatrix} x - y + 1 \\ -x + y + 1 \end{bmatrix}$$

42.) $f(x,y) = \begin{bmatrix} (x+y)^2 \\ xy \end{bmatrix}$ and pt. $(-1,1) \longrightarrow$

$$Df(x,y) = \begin{bmatrix} 2(x+y) & 2(x+y) \\ y & x \end{bmatrix}$$ so

$$Df(-1,1) = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix};$$ then linearization of f at $(x,y) = (-1,1)$ is

$$L(x,y) = f(-1,1) + Df(-1,1) \begin{bmatrix} x+1 \\ y-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x+1 \\ y-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 + 0 \\ (x+1) - (y-1) \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 \\ x - y + 2 \end{bmatrix} \longrightarrow$$

$$L(x,y) = \begin{bmatrix} 0 \\ x - y + 1 \end{bmatrix}.$$

43.) $f(x,Y) = \begin{bmatrix} x^2 - xY \\ 3Y^2 - 1 \end{bmatrix}$ and pt. $(1,2) \rightarrow$

$Df(x,Y) = \begin{bmatrix} 2X - Y & -X \\ 0 & 6Y \end{bmatrix}$ so

$Df(1,2) = \begin{bmatrix} 0 & -1 \\ 0 & 12 \end{bmatrix}$; then linearization of f at $(x,Y) = (1,2)$ is

$$L(X,Y) = f(1,2) + Df(1,2) \begin{bmatrix} X-1 \\ Y-2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 11 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & 12 \end{bmatrix} \begin{bmatrix} X-1 \\ Y-2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 11 \end{bmatrix} + \begin{bmatrix} 0 - (Y-2) \\ 0 + 12(Y-2) \end{bmatrix} = \begin{bmatrix} -1 \\ 11 \end{bmatrix} + \begin{bmatrix} -Y+2 \\ 12Y-24 \end{bmatrix} \rightarrow$$

$L(X,Y) = \begin{bmatrix} -Y+1 \\ 12Y-13 \end{bmatrix}$; then

$L(1.1, 1.9) = \begin{bmatrix} -1.9 + 1 \\ 12(1.9) - 13 \end{bmatrix} = \begin{bmatrix} -0.9 \\ 9.8 \end{bmatrix}$ and

$f(1.1, 1.9) = \begin{bmatrix} (1.1)^2 - (1.1)(1.9) \\ 3(1.9)^2 - 1 \end{bmatrix} = \begin{bmatrix} -0.88 \\ 9.83 \end{bmatrix}$

46.) $f(X,Y) = \begin{bmatrix} \sqrt{2X+Y} \\ X - Y^2 \end{bmatrix}$ and pt. $(1,2) \rightarrow$

$$Df(X,Y) = \begin{bmatrix} \frac{1}{\cancel{2}}(2X+Y)^{-1/2}\cancel{(2)} & \frac{1}{2}(2X+Y)^{-1/2} \\ 1 & -2Y \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2X+Y}} & \frac{1}{2\sqrt{2X+Y}} \\ 1 & -2Y \end{bmatrix}$$ so

$$Df(1,2) = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ 1 & -4 \end{bmatrix}$$; then linearization

of f at $(X,Y) = (1,2)$ is

$$L(X,Y) = f(1,2) + Df(1,2)\begin{bmatrix} X-1 \\ Y-2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 1/2 & 1/4 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} X-1 \\ Y-2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}(X-1) + \frac{1}{4}(Y-2) \\ (X-1) - 4(Y-2) \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -3 \end{bmatrix} + \begin{bmatrix} 1/2 X - 1/2 + 1/4 Y - 1/2 \\ X - 1 - 4Y + 8 \end{bmatrix} \rightarrow$$

$$L(X,Y) = \begin{bmatrix} 1/2 X + \frac{1}{4} Y + 1 \\ X - 4Y + 4 \end{bmatrix}$$; then

$$L(1.05, 2.05) = \begin{bmatrix} 2.0375 \\ -3.15 \end{bmatrix}$$ and

$$f(1.05, 2.05) \approx \begin{bmatrix} 2.0372 \\ -3.1525 \end{bmatrix}$$