Worksheet 3

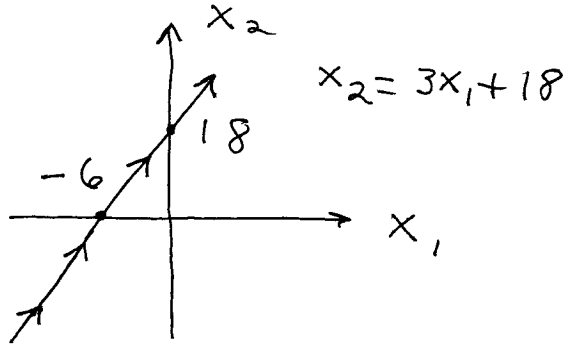
1.) a.) $\begin{cases} x_1 = t - 2 \\ x_2 = 3t + 12 \end{cases}$ for $-\infty < t < \infty$:

$x_1 = t - 2 \rightarrow t = x_1 + 2 \rightarrow$ (SUB)

$x_2 = 3(x_1 + 2) + 12 = 3x_1 + 18 \rightarrow$

$\boxed{x_2 = 3x_1 + 18}$

(line)



b.) $\begin{cases} x_1 = 1 + 2\cos t \\ x_2 = 2\sin t \end{cases}$ for $0 \le t \le 2\pi$:

$\begin{cases} x_1 - 1 = 2\cos t \\ x_2 = 2\sin t \end{cases} \rightarrow$

$(x_1 - 1)^2 + x_2^2 = (2\cos t)^2 + (2\sin t)^2$

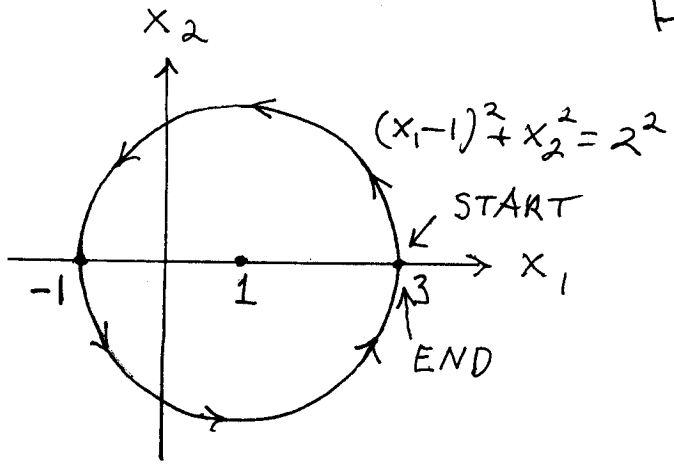
$= 4\cos^2 t + 4\sin^2 t$

$= 4(\cos^2 t + \sin^2 t)$

$= 4(1)$

$= 4 \rightarrow$ $\boxed{(x_1 - 1)^2 + x_2^2 = 2^2}$

(circle)



radius = 2

center: (1, 0)

$t = 0$: $x_1 = 3$, $x_2 = 0$

$t = \frac{\pi}{2}$: $x_1 = 1$, $x_2 = 2$

$t = 2\pi$: $x_1 = 3$, $x_2 = 0$

c.) $\begin{cases} x_1 = t-1 \\ x_2 = t^2 - 5t + 4 \end{cases}$ for $t \geq 0$:

$x_1 = t-1 \rightarrow t = x_1 + 1 \rightarrow$ (SUB)

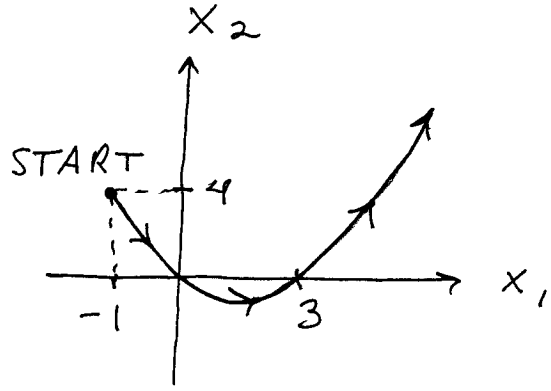
$x_2 = (x_1+1)^2 - 5(x_1+1) + 4$

$= x_1^2 + 2x_1 + 1 - 5x_1 - 5 + 4$

$= x_1^2 - 3x_1 \rightarrow$

$\boxed{x_2 = x_1(x_1 - 3)}$

(parabola)



$t=0$: $x_1 = -1$, $x_2 = 4$

d.) $\begin{cases} x_1 = t^2 \\ x_2 = t^6 - 8 \end{cases}$ for $-1 \leq t \leq 2$:

$x_1 = t^2$ and $x_2 = t^6 - 8 = (t^2)^3 - 8$

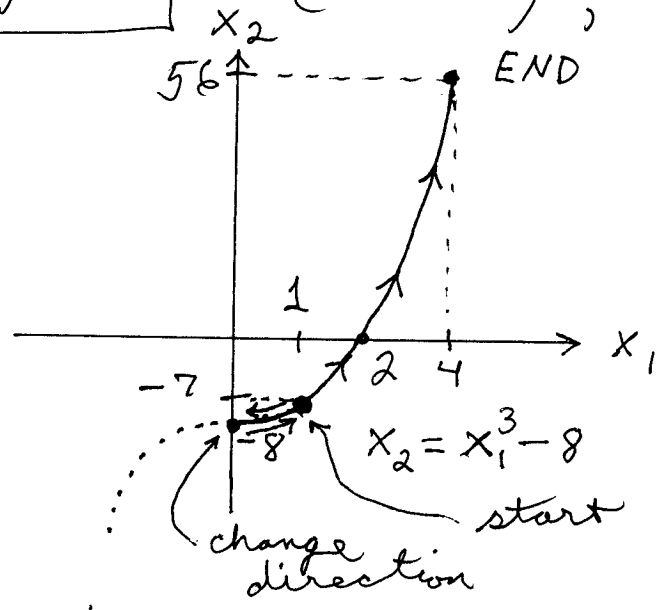
$\rightarrow$ (SUB) $\boxed{x_2 = (x_1)^3 - 8}$ (cubic);

$t=-1$: $x_1 = 1$, $x_2 = -7$

$t=2$: $x_1 = 4$, $x_2 = 56$

$t=0$: $x_1 = 0$, $x_2 = -8$

$t=1$: $x_1 = 1$, $x_2 = -7$



e.) $\begin{cases} x_1 = \sqrt{t+1} \\ x_2 = \sqrt{t} \end{cases}$ for $t \geq 0$:

note: $x_1 \geq 1$ and $x_2 \geq 0$, then

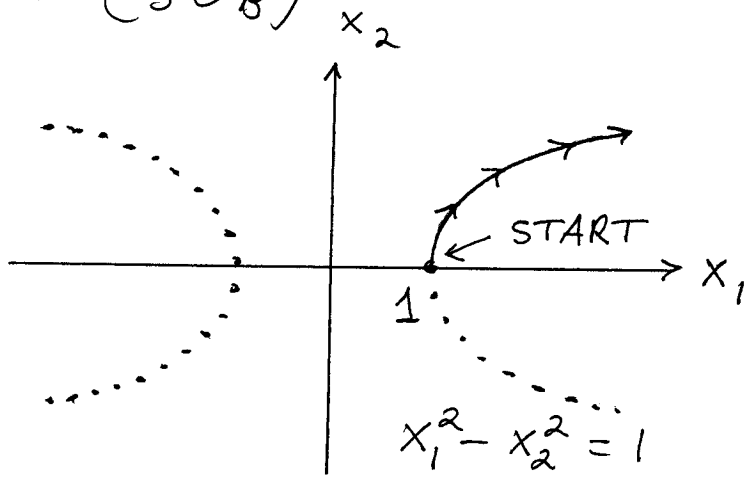
$$\begin{cases} x_1^2 = t+1 \\ x_2^2 = t \end{cases} \longrightarrow \text{(SUB)}$$

$$x_1^2 = x_2^2 + 1$$

$$\longrightarrow \boxed{x_1^2 - x_2^2 = 1}$$

(hyperbola)

$t=0$: $x_1 = 1$, $x_2 = 0$



f.) $$\begin{cases} x_1 = t^2 - 2t \\ x_2 = (t-1)^4 + (t-1)^2 \end{cases} \text{ for } t \geq 0:$$

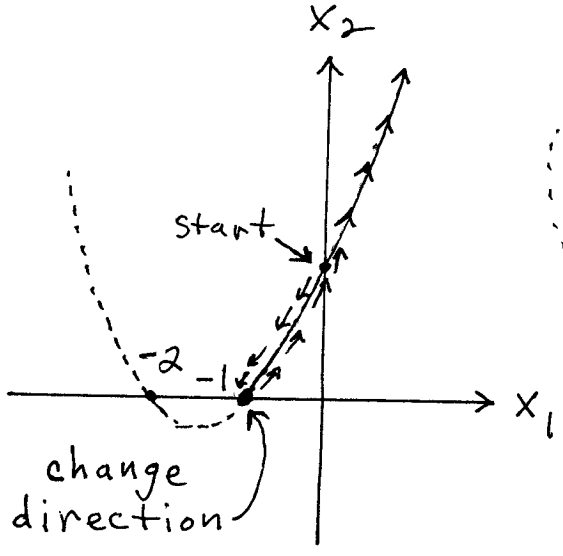
$x_1 = t^2 - 2t = (t^2 - 2t + 1) - 1 = (t-1)^2 - 1 \longrightarrow$

$x_1 = (t-1)^2 - 1 \longrightarrow x_1 + 1 = (t-1)^2 \longrightarrow$ (SUB)

$x_2 = (t-1)^4 + (t-1)^2 = ((t-1)^2)^2 + (t-1)^2 \longrightarrow$

$x_2 = (x_1+1)^2 + (x_1+1) = (x_1+1)((x_1+1)+1) \longrightarrow$

$\boxed{x_2 = (x_1+1)(x_1+2)}$ (parabola)



note: $t=0$: $x_1 = 0$, $x_2 = 2$

$t=1$: $x_1 = -1$, $x_2 = 0$

$t=2$: $x_1 = 0$, $x_2 = 2$

$t=3$: $x_1 = 3$, $x_2 = 20$

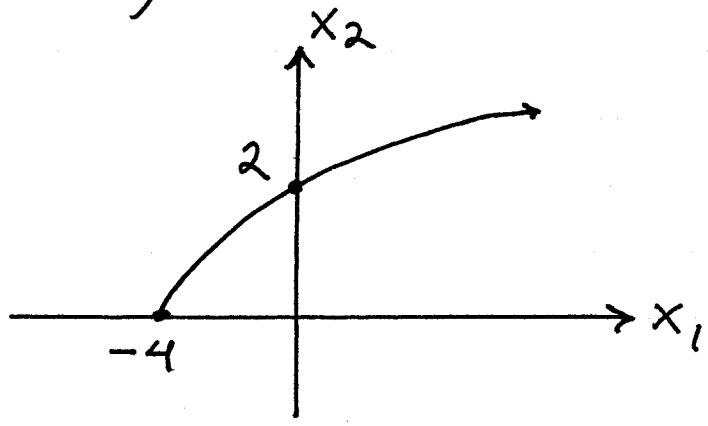
so particle changes direction !! here !!

2.) a.) $\begin{cases} x_1 = t - 3 \\ x_2 = \sqrt{t+1} \end{cases}$ for $t \geq 0$

$\rightarrow$ $t = x_1 + 3$ so (sub) $\rightarrow$

$x_2 = \sqrt{(x_1+3)+1}$ $\rightarrow$

$\boxed{x_2 = \sqrt{x_1+4}}$



b.) $X(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t-3 \\ \sqrt{t+1} \end{bmatrix}$ for $t \geq 0$

c.) $X'(t) = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2}(t+1)^{-1/2} \end{bmatrix}$

d.) $t = 0$: $X(0) = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$, $X'(0) = \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}$, so

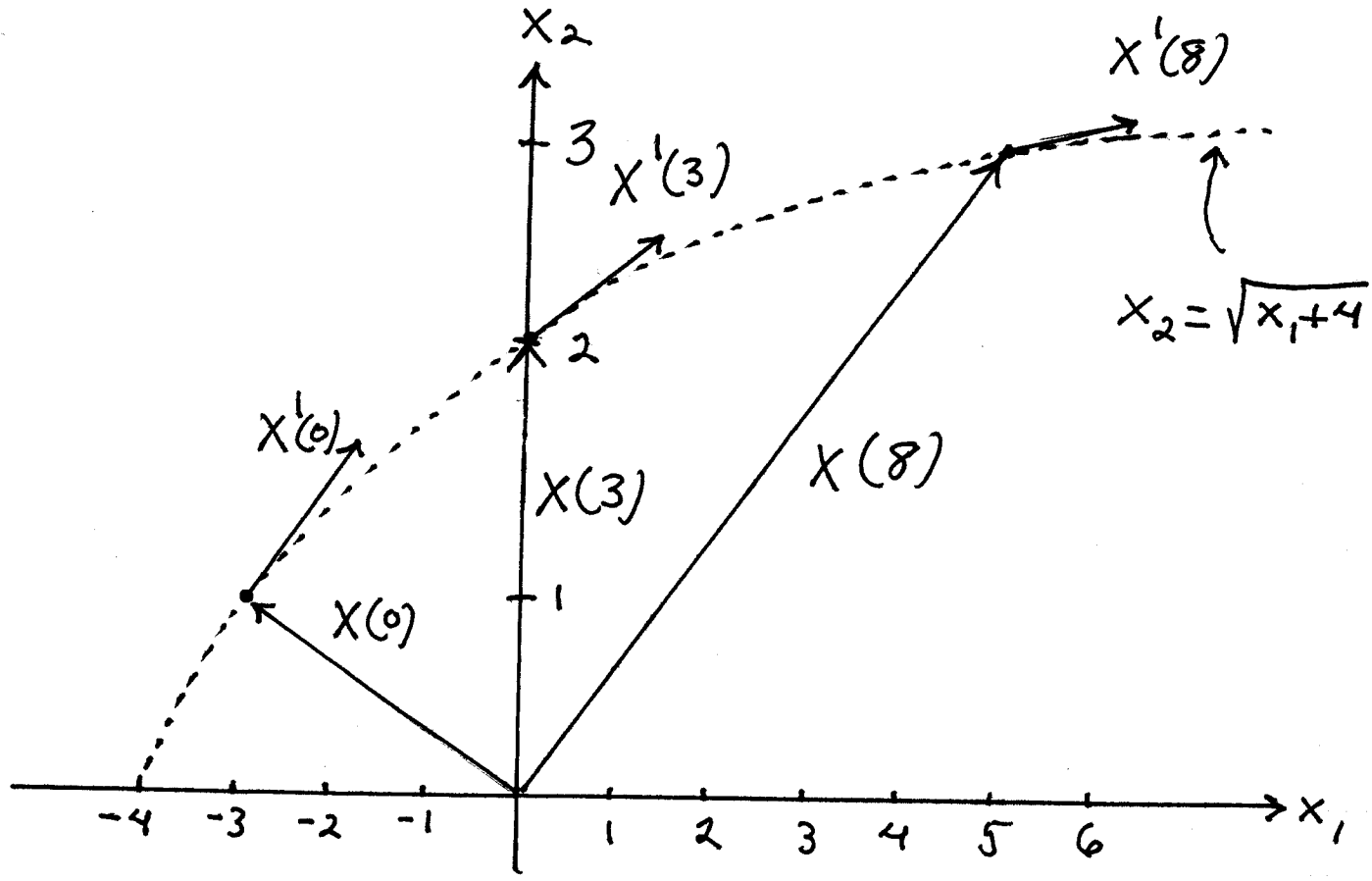
speed $\frac{ds}{dt} = \sqrt{(1)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{5/4} = \frac{\sqrt{5}}{2} \approx 1.118$;

$t = 3$: $X(3) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $X'(3) = \begin{bmatrix} 1 \\ 1/4 \end{bmatrix}$, so

speed $\frac{ds}{dt} = \sqrt{(1)^2 + \left(\frac{1}{4}\right)^2} = \sqrt{\frac{17}{16}} = \frac{\sqrt{17}}{4} \approx 1.031$;

$t = 8$: $X(8) = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, $X'(3) = \begin{bmatrix} 1 \\ 1/6 \end{bmatrix}$, so

speed $\frac{ds}{dt} = \sqrt{(1)^2 + \left(\frac{1}{6}\right)^2} = \sqrt{\frac{37}{36}} = \frac{\sqrt{37}}{6} \approx 1.014$


x_2
3
2
1
$x'(8)$
$x'(3)$
$x'(0)$
$x(3)$
$x(8)$
$x(0)$
$x_2 = \sqrt{x_1+4}$
-4
-3
-2
-1
1
2
3
4
5
6
x_1