Section 11.1

1.) $\begin{cases} \dfrac{dx_1}{dt} = 2x_1 + 3x_2 \\ \dfrac{dx_2}{dt} = -4x_1 + x_2 \end{cases}$

$$\rightarrow \begin{pmatrix} \dfrac{dx_1}{dt} \\ \dfrac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rightarrow X' = AX$$

2.) $\begin{cases} \dfrac{dx_1}{dt} = x_1 + x_2 \\ \dfrac{dx_2}{dt} = -2x_2 \end{cases}$

$$\rightarrow \begin{pmatrix} \dfrac{dx_1}{dt} \\ \dfrac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \rightarrow X' = AX$$

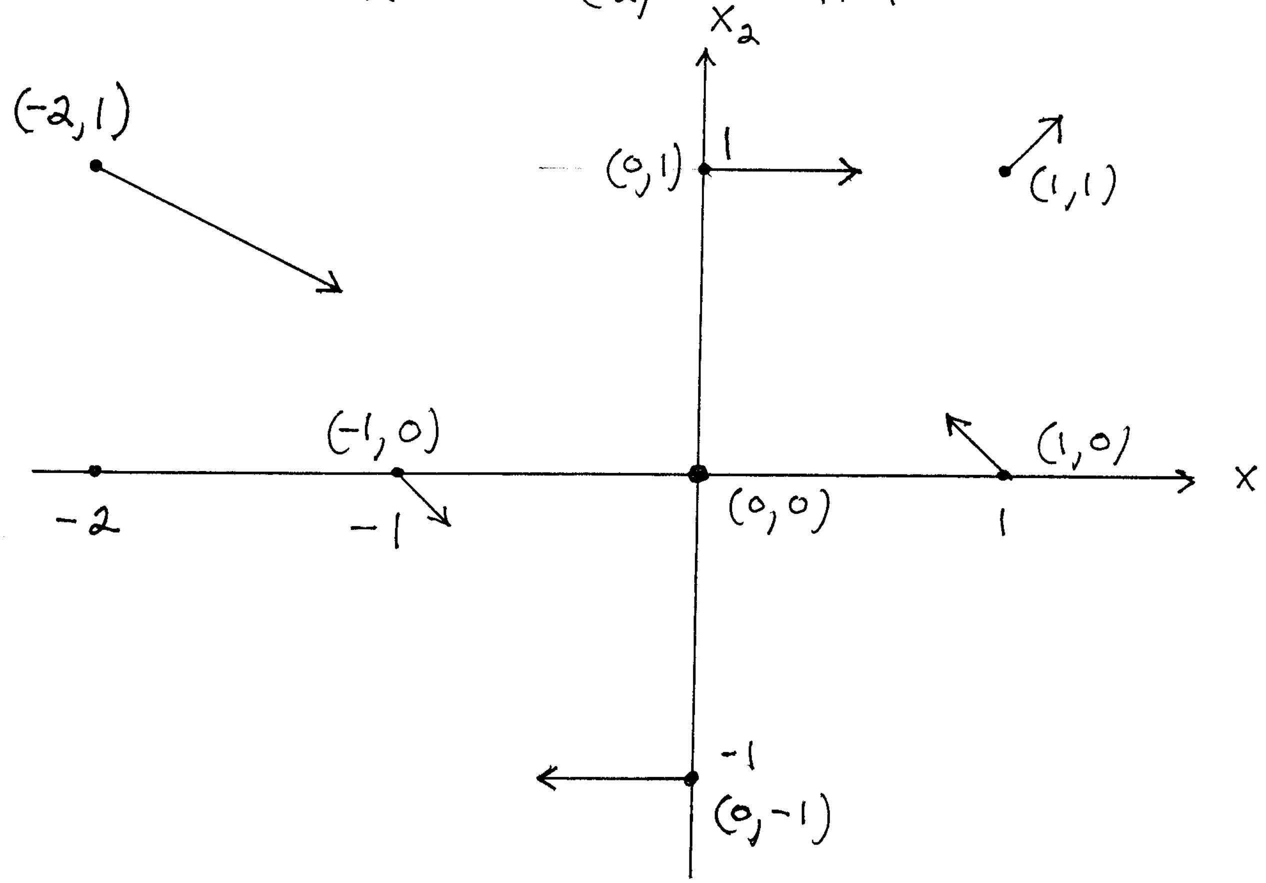
5.) $\begin{cases} \dfrac{dx_1}{dt} = -x_1 + 2x_2 \\ \dfrac{dx_2}{dt} = x_1 \end{cases}$

SLOPE : $\dfrac{dx_2}{dx_1} = \dfrac{\frac{dx_2}{dt}}{\frac{dx_1}{dt}} = \dfrac{x_1}{-x_1 + 2x_2}$ and

Direction Vector : $X' = \begin{pmatrix} \dfrac{dx_1}{dt} \\ \dfrac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} -x_1 + 2x_2 \\ x_1 \end{pmatrix}$,

Speed : $|X'| = \sqrt{\left(\dfrac{dx_1}{dt}\right)^2 + \left(\dfrac{dx_2}{dt}\right)^2}$

x_1	x_2	Slope	Dir. Vec.	Speed
1	0	-1	$\binom{-1}{1}$	$\sqrt{1+1} = \sqrt{2}$
0	1	0	$\binom{2}{0}$	$\sqrt{4+0} = 2$
-1	0	-1	$\binom{1}{-1}$	$\sqrt{1+1} = \sqrt{2}$
0	-1	0	$\binom{-2}{0}$	$\sqrt{4+0} = 2$
1	1	1	$\binom{1}{1}$	$\sqrt{1+1} = \sqrt{2}$
0	0	DNE	$\binom{0}{0}$	$\sqrt{0+0} = 0$
-2	1	$-1/2$	$\binom{4}{-2}$	$\sqrt{16+4} = \sqrt{20}$



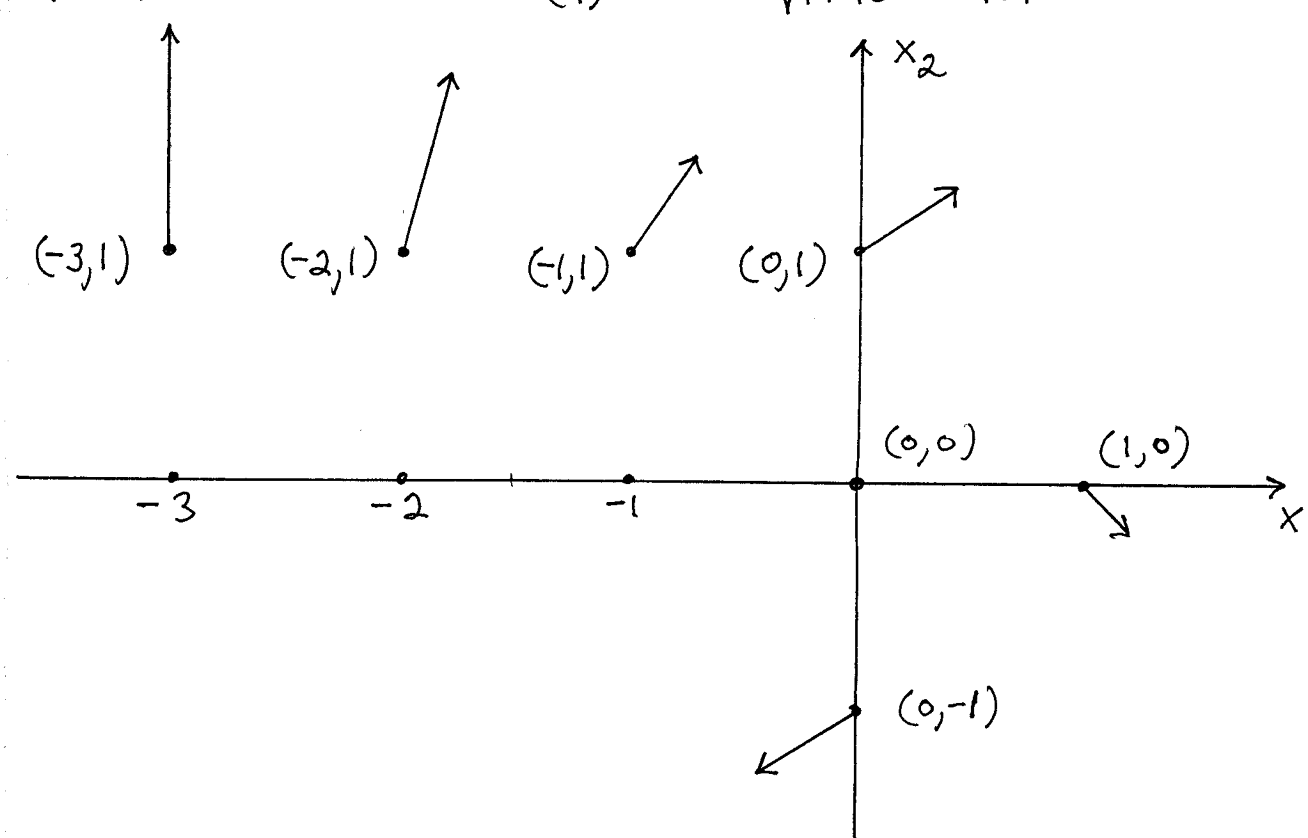
7.) $\begin{cases} \dfrac{dx_1}{dt} = x_1 + 3x_2 \\ \dfrac{dx_2}{dt} = -x_1 + 2x_2 \end{cases}$

SLOPE : $\dfrac{dx_2}{dx_1} = \dfrac{\frac{dx_2}{dt}}{\frac{dx_1}{dt}} = \dfrac{-x_1 + 2x_2}{x_1 + 3x_2}$ and

Direction Vector: $X' = \begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} x_1 + 3x_2 \\ -x_1 + 2x_2 \end{pmatrix}$,

Speed: $|X'| = \sqrt{\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2}$

x_1	x_2	Slope	Dir. Vec.	Speed
1	0	-1	$\binom{1}{-1}$	$\sqrt{1+1} = \sqrt{2}$
0	1	$2/3$	$\binom{3}{2}$	$\sqrt{9+4} = \sqrt{13}$
-1	1	$3/2$	$\binom{2}{3}$	$\sqrt{4+9} = \sqrt{13}$
0	-1	$2/3$	$\binom{-3}{-2}$	$\sqrt{9+4} = \sqrt{13}$
-3	1	DNE	$\binom{0}{5}$	$\sqrt{0+25} = 5$
0	0	DNE	$\binom{0}{0}$	$\sqrt{0+0} = 0$
-2	1	4	$\binom{1}{4}$	$\sqrt{1+16} = \sqrt{17}$



9.) Note direction vector at $x_1 = 1,\ x_2 = 1$ in all 4 figures:

d.) $\begin{cases} \dfrac{dx_1}{dt} = -x_2 \\ \dfrac{dx_2}{dt} = x_1 \end{cases}$ $\rightarrow$ direction vector at $x_1 = 1,\ x_2 = 1$ is

$X' = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ so fig. 11.18 ;

b.) $\begin{cases} \dfrac{dx_1}{dt} = x_1 + 2x_2 \\ \dfrac{dx_2}{dt} = -2x_1 \end{cases}$ $\rightarrow$ direction vector at $x_1 = 1,\ x_2 = 1$ is

$X' = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ so fig 11.20

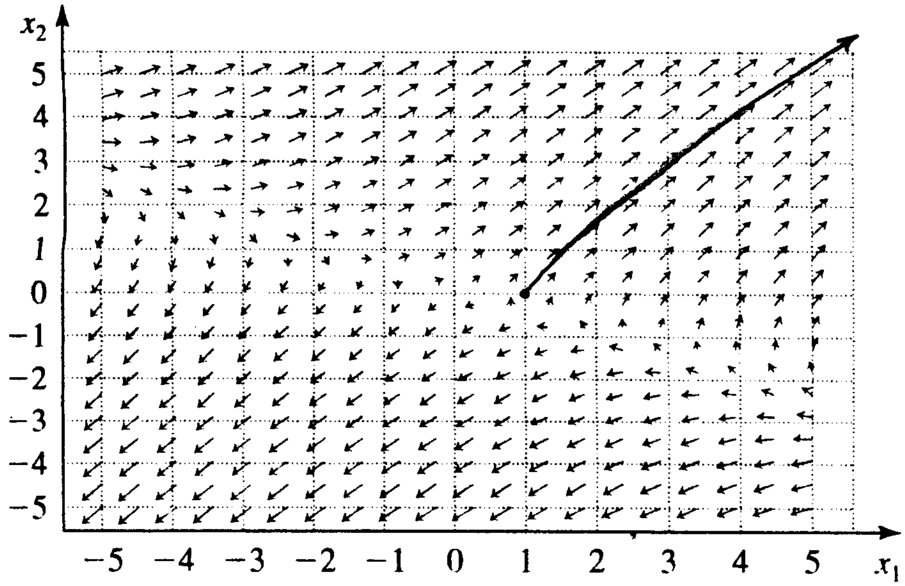
Note direction vector at $x_1 = 2,\ x_2 = -1$ in remaining 2 figures:

c.) $\begin{cases} \dfrac{dx_1}{dt} = x_1 \\ \dfrac{dx_2}{dt} = x_2 \end{cases}$ $\rightarrow$ direction vector at $x_1 = 2,\ x_2 = -1$ is

$X' = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ so fig 11.19

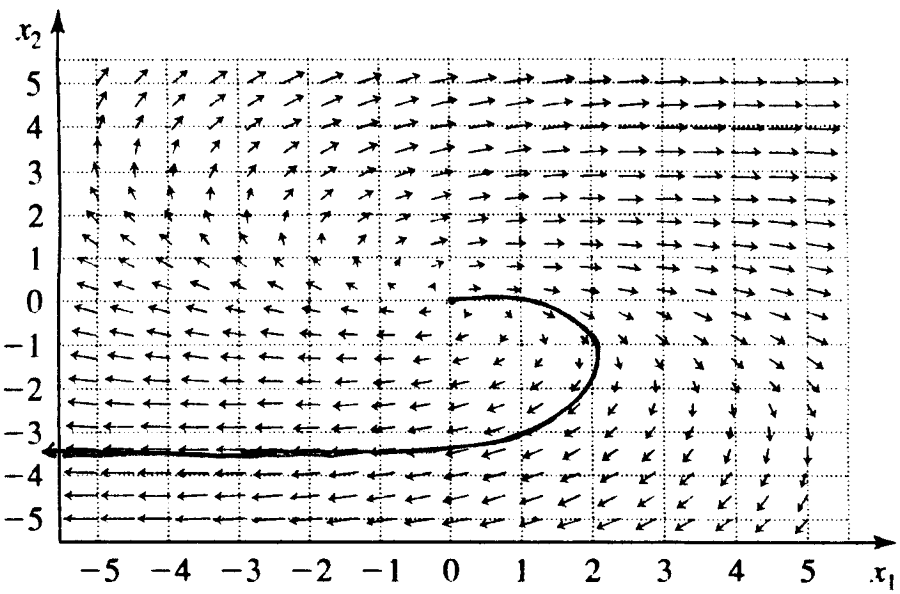
a.) fig. 11.21

10.)



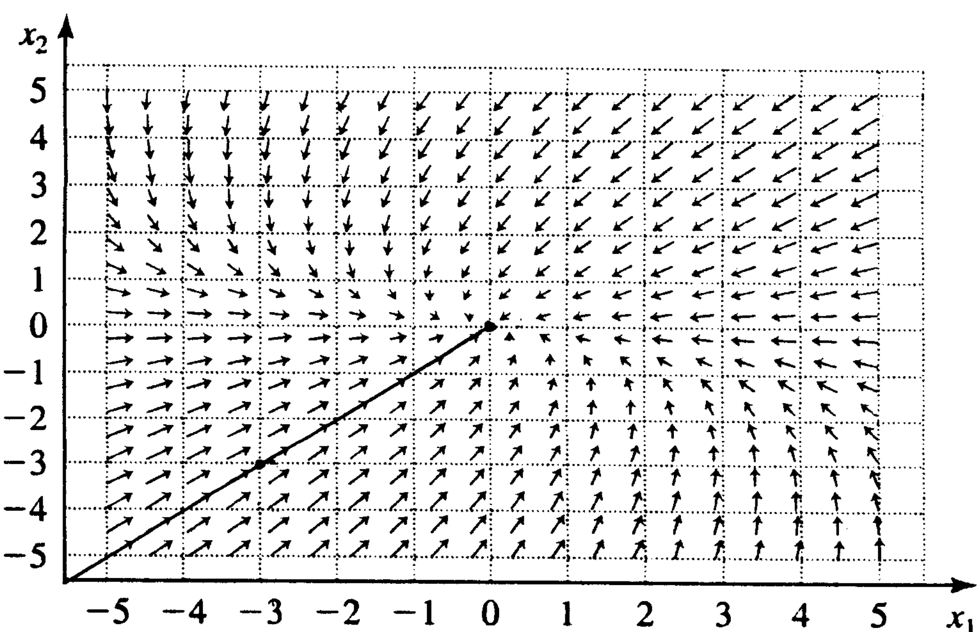
x_2
5
4
3
2
1
0
−1
−2
−3
−4
−5
−5 −4 −3 −2 −1 0 1 2 3 4 5
x_1

11.)



x_2
5
4
3
2
1
0
−1
−2
−3
−4
−5
−5 −4 −3 −2 −1 0 1 2 3 4 5
x_1

12.)



x_2
5
4
3
2
1
0
−1
−2
−3
−4
−5
−5 −4 −3 −2 −1 0 1 2 3 4 5
x_1

13.) $\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\det(A - \lambda I) = \det \begin{bmatrix} 1-\lambda & 3 \\ 5 & 3-\lambda \end{bmatrix}$

$= (1-\lambda)(3-\lambda) - 15 = \lambda^2 - 4\lambda + 3 - 15$

$= \lambda^2 - 4\lambda - 12 = (\lambda - 6)(\lambda + 2) = 0$

$\rightarrow \quad \lambda = 6, \ \lambda = -2$; solve $(A - \lambda I)X = 0$:

For $\lambda_1 = 6$: $\left[\begin{array}{cc|c} -5 & 3 & 0 \\ 5 & -3 & 0 \end{array}\right] \rightarrow -5x_1 + 3x_2 = 0 \rightarrow$

let $x_2 = t$ any # $\rightarrow \quad 5x_1 = 3t \rightarrow$

$x_1 = \frac{3}{5} t$ so $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3/5\, t \\ t \end{bmatrix} = \frac{t}{5} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

so eigenvector $V_1 = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$;

For $\lambda_2 = -2$: $\left[\begin{array}{cc|c} 3 & 3 & 0 \\ 5 & 5 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow$

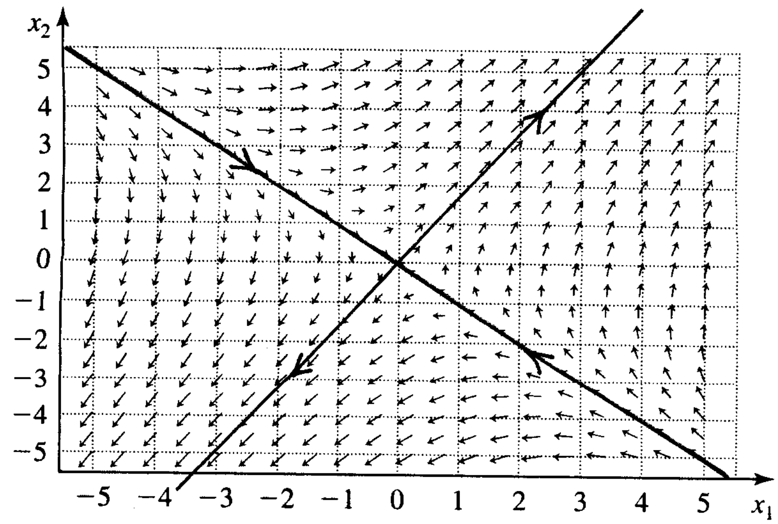
$x_1 + x_2 = 0 \rightarrow$ let $x_2 = t$ any # $\rightarrow$

$x_1 = -x_2 = -t$ so

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -t \\ t \end{bmatrix} = t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ so eigenvector

$V_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$;

gen. sol. : $\boxed{X = c_1 \begin{bmatrix} 3 \\ 5 \end{bmatrix} e^{6t} + c_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-2t}}$



15.) $\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} -3 & 3 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\det(A - \lambda I) = \det \begin{bmatrix} -3-\lambda & 3 \\ 6 & 4-\lambda \end{bmatrix}$

$= (-3-\lambda)(4-\lambda) - 18 = \lambda^2 - \lambda - 12 - 18$

$= \lambda^2 - \lambda - 30 = (\lambda - 6)(\lambda + 5) = 0 \rightarrow$

$\lambda = 6$, $\lambda = -5$; solve $(A - \lambda I)X = 0$:

<u>For $\lambda_1 = 6$</u> : $\left[\begin{array}{cc|c} -9 & 3 & 0 \\ 6 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} -3 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow$

$-3X_1 + X_2 = 0 \rightarrow$ let $X_1 = t$ any # $\rightarrow$

$X_2 = 3X_1 = 3t$ so $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} t \\ 3t \end{bmatrix} = t \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

so eigenvector is $V_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$;

<u>For $\lambda_2 = -5$</u> : $\left[\begin{array}{cc|c} 2 & 3 & 0 \\ 6 & 9 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow$

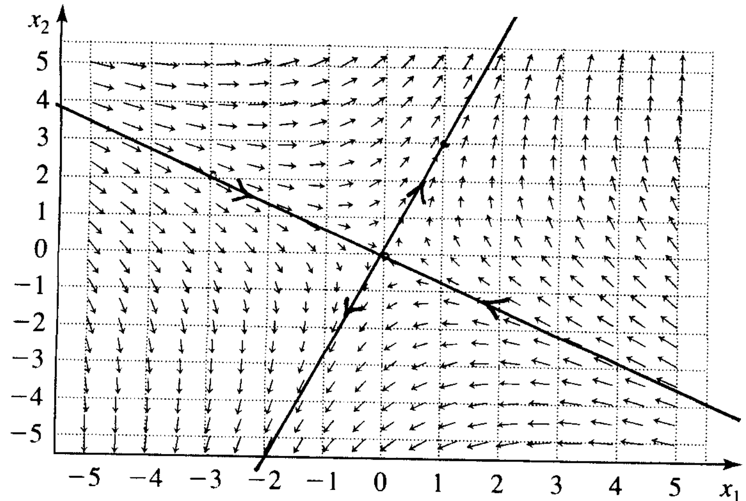
$2X_1 + 3X_2 = 0 \rightarrow$ $X_2 = t$ any # $\rightarrow$

$2x_1 = -3x_2 = -3t \rightarrow x_1 = \frac{-3}{2}t$ so

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3/2\, t \\ t \end{bmatrix} = \frac{t}{2}\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ so

eigenvector $V_1 = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$;

gen. sol. $\boxed{X = c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{6t} + c_2 \begin{bmatrix} -3 \\ 2 \end{bmatrix} e^{-5t}}$



17.) $\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\det(A - \lambda I) = \det \begin{bmatrix} -2-\lambda & 0 \\ -3 & 1-\lambda \end{bmatrix}$

$= (-2-\lambda)(1-\lambda) - 0 = -(2+\lambda)(1-\lambda) = 0 \rightarrow$

$\lambda = -2, \ \lambda = 1$; solve $(A - \lambda I)X = 0$:

<u>For $\lambda_1 = -2$:</u> $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ -3 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 0 & 0 & 0 \\ -1 & 1 & 0 \end{array}\right] \rightarrow$

$-x_1 + x_2 = 0 \to$ let $x_2 = t \to x_1 = x_2 = t$

so $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix} = t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, so

eigenvector is $V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$;

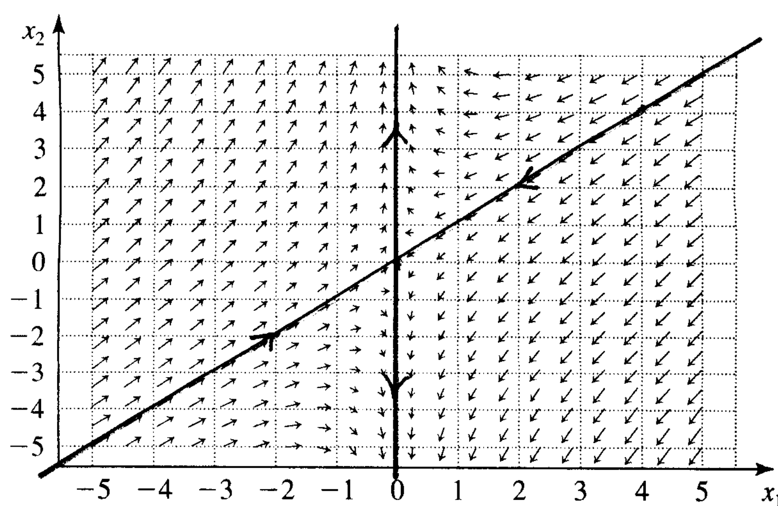
<u>For $\lambda = 1$</u>: $\left[\begin{array}{cc|c} -3 & 0 & 0 \\ -3 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \to$

$x_1 = 0$ and $x_2 = t$ any # so

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ t \end{bmatrix} = t\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ so eigenvector

$V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; gen. sol.

$$X = c_1\begin{bmatrix} 1 \\ 1 \end{bmatrix}e^{-2t} + c_2\begin{bmatrix} 0 \\ 1 \end{bmatrix}e^{t}$$



19.) $A = \begin{bmatrix} -3 & 0 \\ 4 & 2 \end{bmatrix} \to$

$\det(A - \lambda I) = \det\begin{bmatrix} -3-\lambda & 0 \\ 4 & 2-\lambda \end{bmatrix}$

$= (-3-\lambda)(2-\lambda) - 0 = -(3+\lambda)(2-\lambda) = 0 \rightarrow$

$\lambda = -3,\ \lambda = 2$; solve $(A-\lambda I)X = 0$:

For $\lambda_1 = -3$: $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 4 & 5 & 0 \end{array}\right] \rightarrow 4x_1 + 5x_2 = 0 \rightarrow$

let $x_2 = t$ any # $\rightarrow 4x_1 = -5x_2 = -5t \rightarrow$

$x_1 = -\frac{5}{4}t$ so $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -5/4\, t \\ t \end{bmatrix} = \frac{t}{4}\begin{bmatrix} -5 \\ 4 \end{bmatrix}$

so eigenvector $V_1 = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$;

For $\lambda_2 = 2$: $\left[\begin{array}{cc|c} -5 & 0 & 0 \\ 4 & 0 & 0 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow$

$x_1 = 0$ and $x_2 = t$ any # so

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ t \end{bmatrix} = t\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ so

eigenvector $V_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; gen. sol.

$$\boxed{X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = c_1\begin{bmatrix} -5 \\ 4 \end{bmatrix} e^{-3t} + c_2\begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t}}$$; and

$x_1(0) = -5,\ x_2(0) = 5$ then

$\underline{x_1 = -5c_1 e^{-3t}}$, $x_1(0) = -5 \rightarrow$

$-5 = -5c_1(1) \rightarrow c_1 = 1$ so $\underline{\underline{x_1 = -5e^{-3t}}}$;

$\underline{x_2 = 4c_1 e^{-3t} + c_2 e^{2t}}$, $x_2(0) = 5 \rightarrow$

$5 = 4(1)(1) + c_2(1) \rightarrow c_2 = 1 \rightarrow$

$\underline{\underline{x_2 = 4e^{-3t} + e^{2t}}}$; then

$$\boxed{X = \begin{bmatrix} -5 \\ 4 \end{bmatrix} e^{-3t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t}}$$

26.) $A = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \rightarrow$

$\det(A - \lambda I) = \det \begin{bmatrix} 2-\lambda & 6 \\ 1 & 3-\lambda \end{bmatrix}$

$= (2-\lambda)(3-\lambda) - 6 = \lambda^2 - 5\lambda + \cancel{6} - \cancel{6}$

$= \lambda(\lambda - 5) = 0 \rightarrow \lambda = 0, \lambda = 5;$

solve $(A - \lambda I)X = \vec{0}$:

For $\lambda_1 = 0$: $\left[\begin{array}{cc|c} 2 & 6 & 0 \\ 1 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow$

$x_1 + 3x_2 = 0 \rightarrow$ let $x_2 = t$ any # $\rightarrow$

$x_1 = -3x_2 = -3t$ so

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3t \\ t \end{bmatrix} = t \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ so

eigenvector is $V_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$;

For $\lambda_2 = 5$: $\left[\begin{array}{cc|c} -3 & 6 & 0 \\ 1 & -2 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right] \rightarrow$

$x_1 - 2x_2 = 0 \rightarrow$ let $x_2 = t$ any # $\rightarrow$

$x_1 = 2x_2 = 2t$ so

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2t \\ t \end{bmatrix} = t \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ so eigenvector

$V_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$; gen. sol.

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = c_1 \begin{bmatrix} -3 \\ 1 \end{bmatrix} e^{0t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{5t}$$

$$= c_1 \begin{bmatrix} -3 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{5t} \rightarrow$$

$$\begin{cases} x_1 = -3c_1 + 2c_2 e^{5t}, & x_1(0) = -3 \\ x_2 = c_1 + c_2 e^{5t}, & x_2(0) = 1 \end{cases} \rightarrow$$

$$\begin{cases} -3 = -3c_1 + 2c_2 & \rightarrow \quad -3 = -3c_1 + 2(1-c_1) \\ 1 = c_1 + c_2 & \rightarrow \quad c_2 = 1 - c_1 \end{cases}$$

$$\rightarrow -3 = -3c_1 + 2 - 2c_1 \rightarrow -5 = -5c_1 \rightarrow$$

$c_1 = 1$ and $c_2 = 1 - c_1 = 1 - 1 = 0 \rightarrow$

$c_2 = 0 \longrightarrow \boxed{X = \begin{bmatrix} -3 \\ 1 \end{bmatrix}}$.

35.) $A = \begin{bmatrix} 6 & -4 \\ -3 & 5 \end{bmatrix} \rightarrow$

$$\det A = \det \begin{bmatrix} 6-\lambda & -4 \\ -3 & 5-\lambda \end{bmatrix} = (6-\lambda)(5-\lambda) - 12$$

$$= \lambda^2 - 11\lambda + 18 = (\lambda - 9)(\lambda - 2) = 0 \rightarrow$$

$\lambda = 9$, $\lambda = 2$ so $(0,0)$ is a <u>source</u> (unstable equilibrium)

37.) $A = \begin{bmatrix} -3 & -1 \\ 1 & -6 \end{bmatrix} \rightarrow$

$$\det A = \det \begin{bmatrix} -3-\lambda & -1 \\ 1 & -6-\lambda \end{bmatrix} = (-3-\lambda)(-6-\lambda) - (-1)$$

$= \lambda^2 + 9\lambda + 18 + 1 = \lambda^2 + 9\lambda + 19 = 0 \longrightarrow$

$$\lambda = \frac{-9 \pm \sqrt{81 - 4(19)}}{2} = \frac{-9 \pm \sqrt{5}}{2} \longrightarrow$$

$\lambda_1 = \frac{-9 - \sqrt{5}}{2} < 0$, $\lambda_2 = \frac{-9 + \sqrt{5}}{2} < 0$ so

$(0,0)$ is a <u>sink</u> (stable equilibrium)

39.) $A = \begin{bmatrix} 0 & -2 \\ -1 & 3 \end{bmatrix} \longrightarrow$

$$\det(A - \lambda I) = \det \begin{bmatrix} 0-\lambda & -2 \\ -1 & 3-\lambda \end{bmatrix}$$

$= -\lambda(3-\lambda) - 2 = \lambda^2 - 3\lambda - 2 = 0 \longrightarrow$

$$\lambda = \frac{3 \pm \sqrt{9-(-8)}}{2} = \frac{3 \pm \sqrt{17}}{2} \longrightarrow$$

$\lambda_1 = \frac{3+\sqrt{17}}{2} > 0$, $\lambda_2 = \frac{3-\sqrt{17}}{2} < 0$

so $(0,0)$ is <u>saddle</u> (unstable equilibrium)

43.) $A = \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix} \longrightarrow$

$$\det(A - \lambda I) = \det \begin{bmatrix} 2-\lambda & -1 \\ 3 & 0-\lambda \end{bmatrix}$$

$= (2-\lambda)(-\lambda) - (-1)(3) = \lambda^2 - 2\lambda + 3 = 0 \longrightarrow$

$$\lambda = \frac{-(-2) \pm \sqrt{4-12}}{2} = \frac{2 \pm 2\sqrt{2}\,i}{2} = 1 \pm \sqrt{2}\,i$$

and $a = 1 > 0$ so $(0,0)$ is <u>unstable spiral</u> (unstable equilibrium)

45.) $A = \begin{bmatrix} -2 & 4 \\ -3 & -2 \end{bmatrix} \rightarrow$

$\det(A - \lambda I) = \det \begin{bmatrix} -2-\lambda & 4 \\ -3 & -2-\lambda \end{bmatrix}$

$= (-2-\lambda)(-2-\lambda) - (4)(-3)$

$= \lambda^2 + 4\lambda + 4 + 12 = \lambda^2 + 4\lambda + 16 = 0 \rightarrow$

$\lambda = \dfrac{-4 \pm \sqrt{16-64}}{2} = \dfrac{-4 \pm \sqrt{-48}}{2}$

$= \dfrac{-4 \pm 4\sqrt{3}\, i}{2} = -2 \pm 2\sqrt{3}\, i$

and $a = -2 < 0$ so $(0,0)$ is <u>stable</u> <u>spiral</u> (stable equilibrium)

52.) $A = \begin{bmatrix} 3 & -2 \\ 1 & 3 \end{bmatrix} \rightarrow$

$\det(A - \lambda I) = \det \begin{bmatrix} 3-\lambda & -2 \\ 1 & 3-\lambda \end{bmatrix}$

$= (3-\lambda)(3-\lambda) - (-2)(1)$

$= \lambda^2 - 6\lambda + 9 + 2 = \lambda^2 - 6\lambda + 11 = 0 \rightarrow$

$\lambda = \dfrac{6 \pm \sqrt{36-44}}{2} = \dfrac{6 \pm \sqrt{-8}}{2}$

$= \dfrac{6 \pm 2\sqrt{2}\, i}{2} = 3 \pm \sqrt{2}\, i$ and

$a = 3 > 0$ so $(0,0)$ is <u>unstable</u> <u>spiral</u> (unstable equilibrium)

53.) $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \rightarrow$

$\det(A - \lambda I) = \det\begin{pmatrix} 0-\lambda & -1 \\ 1 & 0-\lambda \end{pmatrix}$

$= \lambda^2 - (-1) = \lambda^2 + 1 = 0 \rightarrow \lambda^2 = -1 \rightarrow$

$\lambda = 0 \pm i$ so $a = 0$ and $(0,0)$ is a <u>center</u> (unstable equilibrium)

54.) $A = \begin{bmatrix} 0 & -3 \\ 2 & 2 \end{bmatrix} \rightarrow$

$\det(A - \lambda I) = \det\begin{bmatrix} 0-\lambda & -3 \\ 2 & 2-\lambda \end{bmatrix}$

$= -\lambda(2-\lambda) - (-6) = \lambda^2 - 2\lambda + 6 = 0 \rightarrow$

$\lambda = \dfrac{2 \pm \sqrt{4-24}}{2} = \dfrac{2 \pm \sqrt{-20}}{2} = \dfrac{2 \pm 2\sqrt{5}\,i}{2}$

$= 1 \pm \sqrt{5}\,i$ and $a = 1 > 0$ so

$(0,0)$ is an <u>unstable spiral</u> (unstable equilibrium)

55.) $A = \begin{bmatrix} 1 & 2 \\ -5 & -3 \end{bmatrix} \rightarrow$

$\det(A - \lambda I) = \det\begin{bmatrix} 1-\lambda & 2 \\ -5 & -3-\lambda \end{bmatrix}$

$= (1-\lambda)(-3-\lambda) - (-10)$

$= \lambda^2 + 2\lambda - 3 + 10 = \lambda^2 + 2\lambda + 7 = 0 \rightarrow$

$\lambda = \dfrac{-2 \pm \sqrt{4-28}}{2} = \dfrac{-2 \pm \sqrt{-24}}{2}$

$= \dfrac{-2 \pm 2\sqrt{6}\,i}{2} = -1 \pm \sqrt{6}\,i$

and $a = -1 < 0$ so $(0,0)$ is
a <u>stable</u> <u>spiral</u> (stable equilibrium)