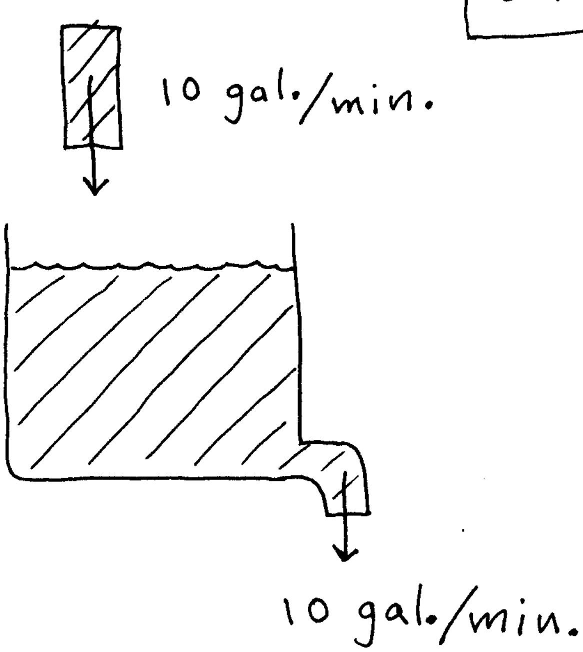
Math 17C
Kouba
Mixture Problems and Differential Equations

Consider the given tank containing 50 gallons of salt water solution in the diagram below. Let x represent the pounds of salt in the tank at time t. Initially, the tank contains 15 pounds of salt. Assume that a solution containing 1/2 pound of salt per gallon flows into the tank at the rate of 10 gal./min and the well-stirred mixture flows out of the tank at the same rate. Set up a differential equation describing the rate of change of the amount x of salt in the tank at time t with initial condition, then solve the differential equation.

Let x : lbs. of salt in tank at time t
t : minutes

$$\frac{dx}{dt} = (\text{rate in}) - (\text{rate out})$$
$$= \left(\frac{\frac{1}{2}\text{ lb.}}{\text{gal.}}\right)\left(\frac{10\text{ gal.}}{\text{min.}}\right) - \left(\frac{x\text{ lbs.}}{50\text{ gal.}}\right)\left(\frac{10\text{ gal.}}{\text{min.}}\right)$$
$$\longrightarrow \boxed{\frac{dx}{dt} = 5 - \frac{1}{5}x} \quad \left(\frac{\text{lbs.}}{\text{min.}}\right)$$



: $t=0$, $x=15$ lbs.

$$\boxed{x = 25 - 10e^{-\frac{1}{5}t}}$$