

Math 17C

Kouba

Categorizing

an Equilibrium

at (c,d) for a Nonlinear 2×2 System of D.E.'s

$$X' = f(x,y) = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \end{bmatrix} \rightarrow$$

$$X' = Df(x,y) \cdot X = AX$$

1.) Assume λ_1, λ_2 are real, nonzero, and distinct eigenvalues:

a.) If $\lambda_1 < 0, \lambda_2 < 0$ then (c,d) is a stable equilibrium, called a sink.

b.) If $\lambda_1 > 0, \lambda_2 > 0$ then (c,d) is an unstable equilibrium, called a source.

c.) If λ_1 and λ_2 are opposite in sign, then (c,d) is an unstable equilibrium, called a saddle.

2.) Assume $\lambda_1 = a+bi, \lambda_2 = a-bi$ with $a \neq 0, b \neq 0$:

a.) If $a > 0$, then (c,d) is an unstable equilibrium, called an unstable spiral.

b.) If $a < 0$, then (c,d) is a stable equilibrium, called a stable spiral.

c.) If $a = 0$, then this method is inconclusive.