Recall: (from Math 17B) If $\frac{dx}{dt} = 3x$ and $x(0) = 4$, then using separation of variables the solution is $x = 4e^{3t}$.

In general, the solution to $\frac{dx}{dt} = ax$ is $x = Ce^{at}$. This leads us to "guess" that the solution to the system of D.E.'s

$$X' = AX$$

takes the form

$$X = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} e^{\lambda t}$$

where $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ is an unknown vector and $\lambda$ is an unknown constant. Substituting (*) into $X' = AX$ gives

$$X' = \frac{d}{dt} \left\{ \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} e^{\lambda t} \right\} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \cdot \lambda e^{\lambda t}$$

or

$$X' = \lambda \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} e^{\lambda t}$$

and
\[ AX = A \left[ u_1 e^{\lambda_1 t} \right] = A \left[ u_2 \right] e^{\lambda_1 t} \]

Thus,

\[ X' = AX \rightarrow \]

\[ A \left[ u_1 \right] e^{\lambda_1 t} = \lambda \left[ u_1 \right] e^{\lambda_1 t} \]

(Divide both sides by \( e^{\lambda_1 t} \)) \rightarrow

\[ A \left[ u_2 \right] = \lambda \left[ u_2 \right] \rightarrow \]

\( \lambda \) is eigenvalue for \( A \) and \( \left[ u_1 \right] \) is an associated eigenvector.

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**How to solve \( X' = AX \):**

1.) Solve \( \det (A - \lambda I) = 0 \) to get distinct eigenvalues \( \lambda_1, \lambda_2 \).

2.) Solve \( (A - \lambda I)X = 0 \) for \( X \) to get distinct eigenvectors \( V_1, V_2 \).

3.) The general solution in matrix form is

\[ X = c_1 V_1 e^{\lambda_1 t} + c_2 V_2 e^{\lambda_2 t} \]