Math 17C
Kouba
Set Theory and Probability Rules

SET THEORY

DEFINITION: A *set* is a collection of objects.

EXAMPLES: $S=\{1,2,3,7,11,20\}$, $A=\{a,b,r,s,w\}$, $B=\{red, blue, sun, snapple\}$,
$P=\{\{X,Y\},\{A,B,C,D\}\}$, $N=\{1,2,3,4,5,\cdots\}$, $E=\{\ \}$

DEFINITION: Let A and B be sets.

1.) The *union* of sets A and B is $A\cup B=\{x : x\in A \text{ or } x\in B\}$.
2.) The *intersection* of sets A and B is $A\cap B=\{x : x\in A \text{ and } x\in B\}$.

EXAMPLE: Let $A=\{1,2,3\}$, $B=\{2,3,4,5\}$, $C=\{4,5\}$. Determine

$A\cap B=$

$A\cup B=$

$B\cap C=$

$B\cup C=$

$A\cap C=$

$A\cup B=$

DEFINITION: A *sample space* , Ω, is the set of all possible outcomes.

EXAMPLE: Flip a coin twice and record H or T :

Sample Space $\Omega=\{HH, HT, TH, TT\}$

DEFINITION: A set A is a *subset* of set B if each object in set A is also in set B. We write $A\subseteq B$.

EXAMPLE: Let $A=\{a,b,c\}$ and $B=\{a,b,c,d,e\}$ then $A\subseteq B$.

FACT: Every set A is a subset of itself, i.e., $A\subseteq A$.

FACT: The empty set, $\{\ \}=\phi$, is a set containing no objects. The empty set is a subset of every set A, i.e., $\phi\subseteq A$.

DEFINITION: Let Ω be a sample space and let $A\subseteq\Omega$. Then A is called an *event* in Ω. The *complement* of A, written A^c, is the set of all objects which are in Ω but NOT in set A.

EXAMPLE: If $\Omega = \{HH, HT, TH, TT\}$ and $A = \{HH, HT, TH\}$, then $A \subseteq \Omega$ and $A^c = \{TT\} \subseteq \Omega$.

PROPERTIES of SETS:

1.) De Morgan's Laws :

a.) $(A \cup B)^c = A^c \cap B^c$
b.) $(A \cap B)^c = A^c \cup B^c$

2.) $(A^c)^c = A$

3.) $\Omega^c = \phi$ and $\phi^c = \Omega$

PROBABILITY

RULES for PROBABILITY: Let Ω be a sample space and let sets A and B be events in Ω. Let $n(A)$ represent the number of objects in set A. Then the following rules apply in the context of probability :

1.) For equally-likely outcomes the probability of events A and B are defined to be

$$P(A) = \frac{n(A)}{n(\Omega)} \quad \text{and} \quad P(B) = \frac{n(B)}{n(\Omega)} \ .$$

2.) $0 \leq P(A) \leq 1$ and $0 \leq P(B) \leq 1$

3.) $P(\phi) = \dfrac{n(\phi)}{n(\Omega)} = \dfrac{0}{n(\Omega)} = 0$ and $P(\Omega) = \dfrac{n(\Omega)}{n(\Omega)} = 1$

4.) If $A \cap B = \phi$, then $P(A \cup B) = P(A) + P(B)$.

5.) If $A \cap B \neq \phi$, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

6.) $P(A^c) = 1 - P(A)$.

EXAMPLE: Let $\Omega = \{HH, HT, TH, TT\}$ (all equally likely outcomes) and let

event A be "at least one head" $\longrightarrow A = \{HH, HT, TH\}$,

event B be "both are tails" $\longrightarrow B = \{TT\}$, and

event C be "a tail on the second flip" $\longrightarrow C = \{HT, TT\}$.

What is

a.) $P(A)$

b.) $P(B)$

c.) $P(C)$