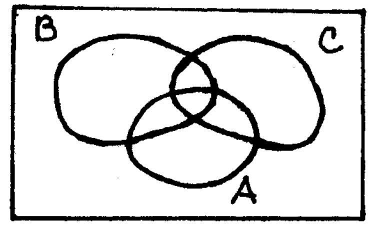
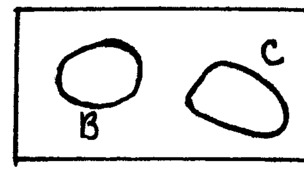
Math 17C
Kouba
The Law of Total Probability



RECALL :

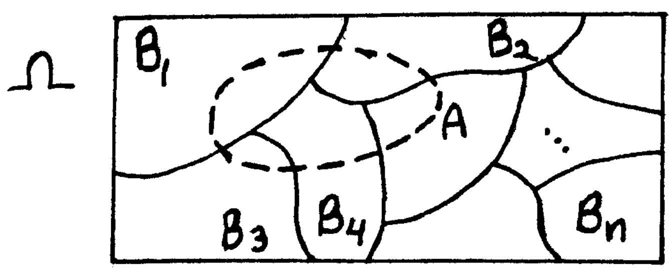
I.) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

II.) $P(B|C) = \dfrac{P(B \cap C)}{P(C)} \longrightarrow P(B \cap C) = P(B|C) \cdot P(C)$



III.) If $B \cap C = \phi$, then $P(B \cup C) = P(B) + P(C)$.

Let Ω be a sample space and let $B_1, B_2, B_3, \cdots, B_n$ be a partition for Ω. Let A be an event in Ω. Then



$$
\begin{aligned}
A &= A \cap \Omega \\
&= A \cap (B_1 \cup B_2 \cup B_3 \cup \cdots \cup B_n) \quad \text{(since } B_1, B_2, B_3, \cdots, B_n \text{ are a partition for } \Omega) \\
&= (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup \cdots \cup (A \cap B_n) \quad \text{(by RECALL I.)} \longrightarrow
\end{aligned}
$$

$$
\begin{aligned}
P(A) &= P(A \cap B_1) + P(A \cap B_2) + \cdots + P(A \cap B_n) \quad \text{(by RECALL III.)} \\
&= P(A|B_1) \cdot P(B_1) + P(A|B_2) \cdot P(B_2) + \cdots + P(A|B_n) \cdot P(B_n) \quad \text{(by RECALL II.)},
\end{aligned}
$$

i.e.,

$$P(A) = \Sigma_{i=1}^{n} P(A \cap B_i) = \Sigma_{i=1}^{n} P(A|B_i) \cdot P(B_i) \ .$$

This is called the Law of Total Probability.