Math 17C (Spring 2014)
Kouba
Exam 3

Please PRINT your name here : KEY

Your Exam ID Number _____

1. PLEASE DO NOT TURN THIS PAGE UNTIL TOLD TO DO SO.

2. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO, IN ANY WAY, ASSIST ANOTHER PERSON IN THE COMPLETION OF THIS EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO COPY ANSWERS FROM ANOTHER STUDENT'S EXAM. IT IS A VIOLATION OF THE UNIVERSITY HONOR CODE TO HAVE ANOTHER STUDENT TAKE YOUR EXAM FOR YOU. PLEASE KEEP YOUR OWN WORK COVERED UP AS MUCH AS POSSIBLE DURING THE EXAM SO THAT OTHERS WILL NOT BE TEMPTED OR DISTRACTED. THANK YOU FOR YOUR COOPERATION.

3. No notes, books, or classmates may be used as resources for this exam. YOU MAY USE A CALCULATOR ON THIS EXAM.

4. Read directions to each problem carefully. Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your getting a good score on this exam. Neatness and organization are also important.

5. Make sure that you have 8 pages, including the cover page.

6. You have until 8:50 a.m. sharp to finish the exam. Failure to stop working on your exam when time is called may lead to points deducted from your total score. Thank you for your cooperation.

1.) (6 pts.) How many distinct permutations are there using all of the letters in the word CACHINNATION (which means hysterical laughter) ?

$$\frac{12!}{2!\,2!\,2!\,3!}$$

↑ ↑ ↑ ↑

C's A's I's N's

2.) (7pts.) How many distinct factors does 36,000 have ?

$36{,}000 = 36 \times 1000 = 4 \cdot 9 \cdot 10^3$

$= 2^2 \cdot 3^2 \cdot (2 \cdot 5)^3 = 2^5 3^2 5^3$

FACTORS: $\frac{6}{2's} \cdot \frac{3}{3's} \cdot \frac{4}{5's} = 72$

3.) (6 pts. each) A bag contains 20 red and 12 blue ping pong balls. Randomly select 9 balls.

a.) How many ways can this be done ?

$C(32,9)$

b.) How many ways can this be done if there must be at least one of each color ?

$C(32,9) - C(20,9) - C(12,9)$

↑ All ↑ Red ↑ Blue

4.) (6 pts) Barack Obama selects a committee of 11 people from the United States Congress. If he picks from among Independents (I), Republicans (R), and Democrats (D), how many different political outcomes are possible (for example, 4D, 5R, and 2I) ?

$$C(11+2,2) = C(13,2)$$

5.) We are going to select 24 donuts from among caramel, chocolate, buttermilk, and maple flavors (all equally likely).

(3 pts.) a.) How many ways can this be done ?

$$C(24+3,3) = C(27,3)$$

(3 pts.) b.) How many ways can this be done if we select exactly 2 chocolate and exactly 3 caramel ?

Ch Ch Ca Ca Ca | 19 donuts, 2 flavors, 1 separators $\Big\}$ $C(19+1,1) = C(20,1)$

(4 pts.) c.) How many ways can this be done if we select at least 8 maple ?

M M M M M M M M | 16 donuts, 4 flavors, 3 separators $\Big\}$ $C(16+3,3) = C(19,3)$

6.) Let sample space $\Omega = \{q, r, s, t, u, w\}$ with all outcomes equally likely. Let events $A = \{r, s\}$, $B = \{t, u, w\}$, and $C = \{q, r, s, t\}$.

a.) (6 pts.) LIST the elements in sets $A \cup B$, $B \cap C$, and B^c .

$A \cup B = \{r, s, t, u, w\}$,

$B \cap C = \{t\}$, $B^c = \{q, r, s\}$

b.) (4 pts.) Determine the probabilities $P(A)$ and $P(\{\})$.

$$P(A) = \frac{n(A)}{n(\Omega)} = \frac{2}{6} \quad , \quad P(\{\}) = \frac{n(\{\})}{n(\Omega)} = \frac{0}{6} = 0$$

c.) (4 pts.) Determine the probabilities $P(B|C)$ and $P(B \cup C)$.

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{1/6}{4/6} = 1/4 \; ; \quad B \cup C = \{q, r, s, t, u, w\}$$

$$P(B \cup C) = \frac{n(B \cup C)}{n(\Omega)} = \frac{6}{6} = 1$$

7.) A box holds 4 peach Snapples and 6 raspberry Snapples. Randomly select 5 Snapples from the box without replacement.

a.) (5 pts.) How many ways can you do this ?

$C(10,5)$

b.) (6 pts.) What is the probability that you select exactly 3 raspberry Snapples and 2 peach Snapple ?

$$P(3R,2P) = \frac{C(6,3)\cdot C(4,2)}{C(10,5)}$$

8.) (6 pts.) In the game of 5-card poker played with a standard deck of 52 cards, how many two-pair hands (for example, two 5's, two jacks, and the 3 of diamonds) are possible?

$C(13,2)$ · $C(4,2)$ · $C(4,2)$ · 44

↑ pick 2 face values — ↑ pick pair — ↑ pick pair — ↑ pick 5th card

9.) (6 pts.) A committee of 8 will be chosen from a group of 13 people. How many different committees are possible if both Adam and Eve must be on the committee?

Adam, Eve + 6 → $C(11,6)$

10.) Consider a bag containing 1 red, 2 yellow, and 1 blue ping pong ball. Select 3 balls from the bag (one at a time) without replacement.

a.) (2 pts.) What is the probability that the first ball is yellow ?

$$P(1st\ Y) = \frac{2}{4} = \frac{1}{2}$$

b.) (2 pts.) What is the probability that the second ball is red given that the first ball is blue?

$$P(2nd\ R \mid 1st\ B) = 1/3$$

c.) (2 pts.) What is the probability that the firstball is red and the second ball is blue?

$$P(1st\ R \text{ and } 2nd\ B) = \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \frac{1}{12}$$

d.) (4 pts.) What is the probability that the second ball is blue ?

$$P(2nd\ B) = P(RB \text{ or } YB) = P(RB) + P(YB)$$

$$= \left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{2}{4}\right)\left(\frac{1}{3}\right) = \frac{3}{12} = \frac{1}{4}$$

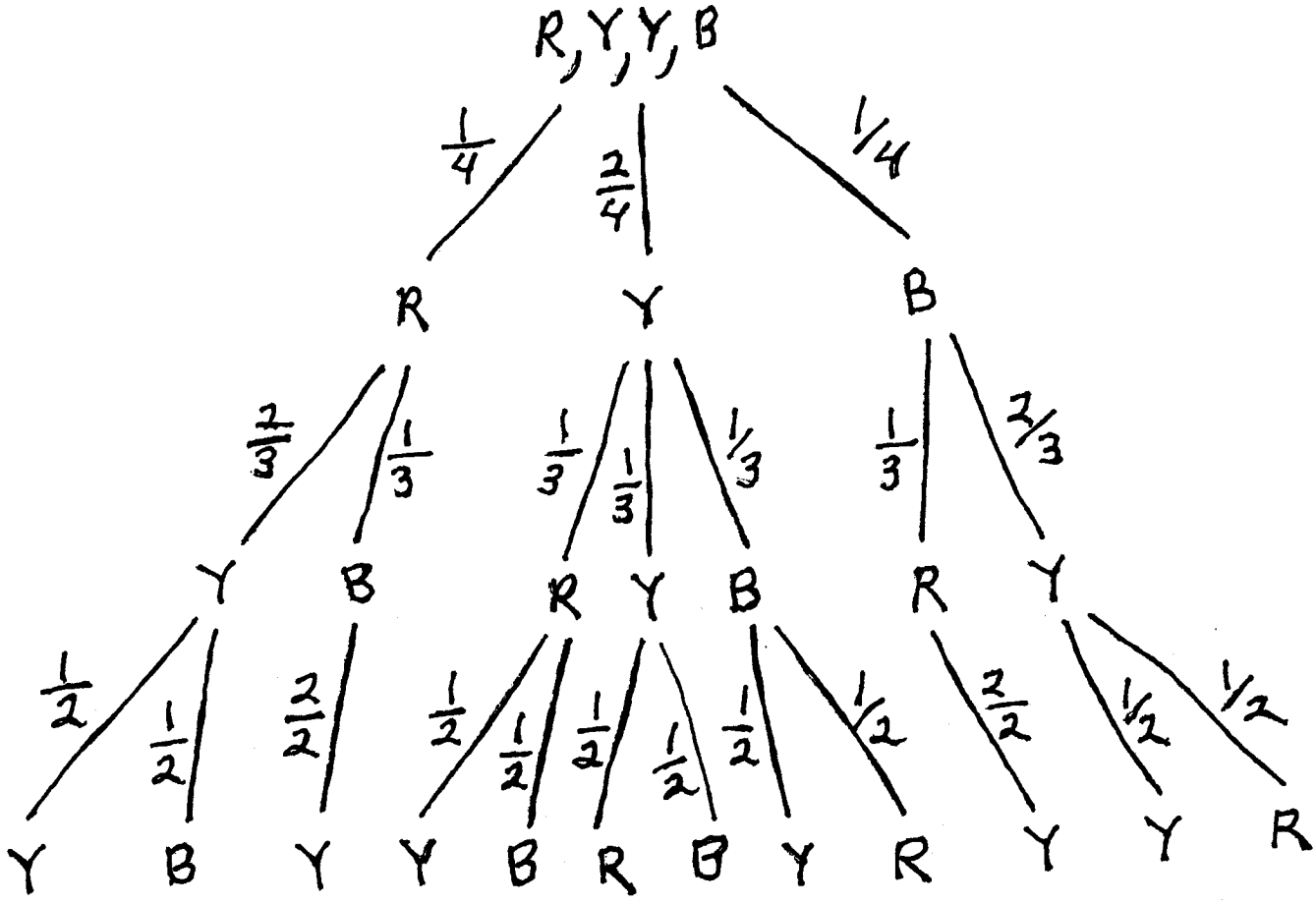
e.) (4pts.) What is the probability that the third ball is yellow given that the second ball is red ?

$$P(3rd\ Y \mid 2nd\ R) = \frac{P(3rd\ Y \text{ and } 2nd\ R)}{P(2nd\ R)}$$

$$= \frac{P(YRY \text{ or } BRY)}{P(YR \text{ or } BR)} = \frac{P(YRY) + P(BRY)}{P(YR) + P(BR)}$$

$$= \frac{\left(\frac{2}{4}\right)\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{3}\right)\left(\frac{2}{2}\right)}{\left(\frac{2}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{3}\right)} = \frac{\frac{4}{24}}{\frac{3}{12}} = \frac{4}{6} = \frac{2}{3}$$

PLEASE DRAW TREE DIAGRAM HERE FOR PROBLEM 10.) ON PREVIOUS PAGE.



12.) (Lotka-Voltera Predator-Prey Model) Consider the system of differential equations given below, where $N(t)$ is the prey population at time t and $P(t)$ is the predator population at time t.

$\frac{dN}{dt} = 6N - 2PN$ and $N(0) = 3$

$\frac{dP}{dt} = PN - 4P$ and $P(0) = 4$

a.) (4 pts.) Use separation of variables (for N and P) to solve this system of differential equations. Use the initial conditions to solve for the unknown constant C.

$$\frac{dN}{dP} = \frac{\frac{dN}{dt}}{\frac{dP}{dt}} = \frac{6N-2PN}{PN-4P} = \frac{2N(3-P)}{P(N-4)} \longrightarrow$$

$$\int \frac{N-4}{N}\,dN = \int \frac{6-2P}{P}\,dP \longrightarrow$$

$$\int \left(1-\frac{4}{N}\right)dN = \int \left(\frac{6}{P}-2\right)dP \rightarrow \boxed{N-4\ln N = 6\ln P - 2P + C}$$

and $N=3$, $P=4 \rightarrow 3-4\ln 3 = 6\ln 4 - 8 + C \longrightarrow$

$$\boxed{C = 11 - 4\ln 3 - 6\ln 4}$$

b.) (2 pts.) Is the number of prey animals N increasing or decreasing when $t = 0$?

$\frac{dN}{dt} = 6(3)-2(4)(3) = 18-24 = -6$ so N is $\downarrow$

c.) (2 pts.) Is the number of predator animals P increasing or decreasing when $t = 0$?

$\frac{dP}{dt} = (4)(3)-4(4) = -4$ so P is $\downarrow$

The following two EXTRA CREDIT PROBLEM are OPTIONAL.

1.) (7 pts.) Five (5) Brazilian women and four (4) Venezuelan men are to be arranged in a row of 15 chairs. How many different ways can this be done if all of the men must sit together ?

5W, 4M, 6E

$$\underbrace{12}_{\text{place men}} \cdot \underbrace{4!}_{\text{order men}} \cdot \underbrace{P(11,5)}_{\text{seat + order women}}$$

2.) (7 pts.) Today is March 11, 2016, and our class has 90 students. What is the probability that at least two students in our class have a birthday TODAY ?

A: birthday today ← 1A ← no A

$$P(\geq 2A) = \frac{(365)^{90} - 90(364)^{89} - (364)^{90}}{(365)^{90}}$$