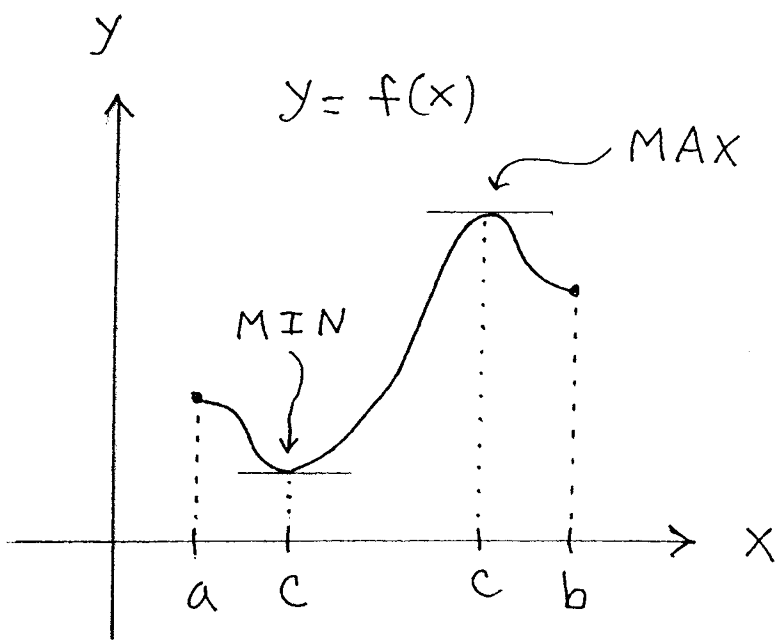
Math 21A
Kouba
The Intermediate Value Theorem (IMVT) and Other Theorems

Maximum Value Theorem : Let f be a continuous function on the closed interval $[a, b]$. Then there is at least one number c in $[a, b]$ at which f takes on its maximum value, i.e.,

$$f(c) \geq f(x) \text{ for all } x \text{ in } [a, b] \ .$$

Minimum Value Theorem : Let f be a continuous function on the closed interval $[a, b]$. Then there is at least one number c in $[a, b]$ at which f takes on its minimum value, i.e.,

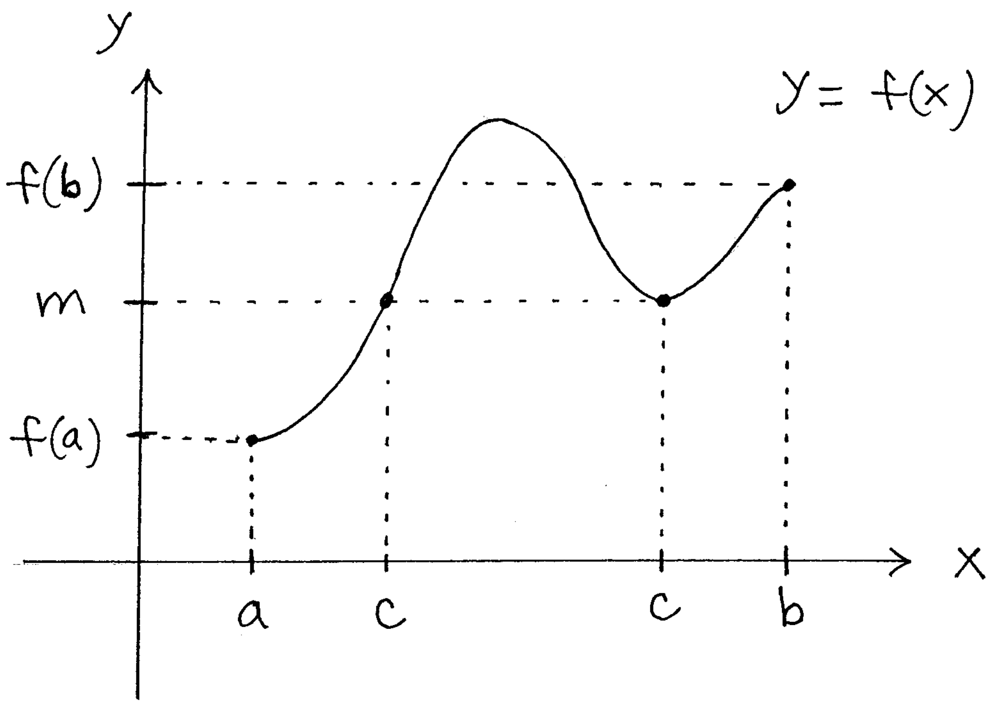
$$f(c) \leq f(x) \text{ for all } x \text{ in } [a, b] \ .$$



REMARK : If f is not continuous or if the interval is not closed, then the conclusions of the previous theorems are not guaranteed, but may sometimes be true.

Intermediate Value Theorem (IMVT) : Let f be a continuous function on the closed interval $[a, b]$. Let m be any number between $f(a)$ and $f(b)$. Then there is at least one number c in $[a, b]$ which satisfies

$$f(c) = m \ .$$



When applying the IMVT to a problem, the following five steps must be clearly established:

1. Define a function f.
2. Define a number m.
3. Establish that f is continuous.
4. Choose an interval $[a, b]$.
5. Indicate that m is between $f(a)$ and $f(b)$.

Once these five steps have been established, the conclusion of the IMVT can be invoked.

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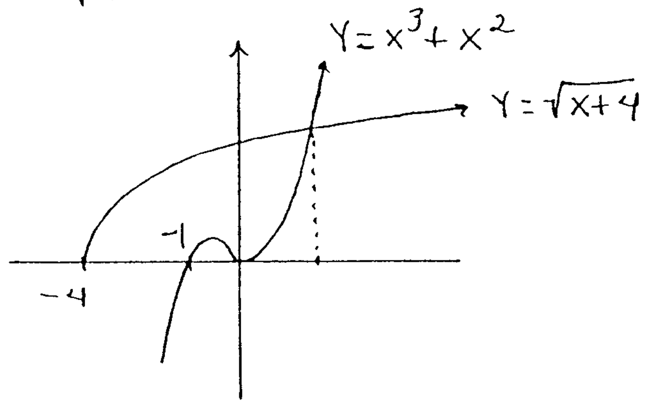
An Example Using the Intermediate Value Theorem (IMVT)

Ex: Determine if $x^3+x^2=\sqrt{x+4}$ is solvable.

Begin by sketching $y=x^3+x^2=x^2(x+1)$ and $y=\sqrt{x+4}$:



It appears that there is a solution. Now we must prove its existence. Since $x^3+x^2=\sqrt{x+4}$ then $x^3+x^2-\sqrt{x+4}=0$ so let $f(x)=x^3+x^2-\sqrt{x+4}$ and let $m=0$.
By trial and error $f(0)=-2$ and $f(5)=147$, and $m=0$ is between $f(0)$ and $f(5)$.

Thus, since f is a continuous function (It is sum and difference of continuous functions.) on a closed interval $[0,5]$, it follows from the IMVT that there is at least one number c in $[0,5]$ so that

$$f(c)=m \ , \text{ i.e.,}$$

$$c^3+c^2-\sqrt{c+4}=0 \ , \text{ i.e.,}$$

$$c^3+c^2=\sqrt{c+4} \ .$$

Thus, we have proven that the original equation is solvable.