

Math 21A

Kouba

Derivatives of Inverse Trig Functions

Trig Function	Domain Restriction	Inverse Function	Derivative of Inverse
$y = \tan x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	$y = \arctan x$	$y' = \frac{1}{1+x^2}$
$y = \sin x$	$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$	$y = \arcsin x$	$y' = \frac{1}{\sqrt{1-x^2}}$
$y = \sec x$	$0 \leq x \leq \pi, x \neq \frac{\pi}{2}$	$y = \operatorname{arcsec} x$	$y' = \frac{1}{ x \sqrt{x^2-1}}$
$y = \cot x$	$0 < x < \pi$	$y = \operatorname{arccot} x$	$y' = \frac{-1}{1+x^2}$
$y = \cos x$	$0 \leq x \leq \pi$	$y = \arccos x$	$y' = \frac{-1}{\sqrt{1-x^2}}$
$y = \csc x$	$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, x \neq 0$	$y = \operatorname{arccsc} x$	$y' = \frac{-1}{ x \sqrt{x^2-1}}$

Why is  $D \arctan x = \frac{1}{1+x^2}$  ?

PROOF :  $y = \arctan x \implies x = \tan y$  (Definition of inverse tangent)

$$\implies 1 = \sec^2 y \cdot y' \quad (\text{Implicit differentiation})$$

$$\implies y' = \frac{1}{\sec^2 y} \quad (\text{Solve for } y.)$$

$$\implies y' = \frac{1}{1/\cos^2 y} \quad (\text{Definition of secant})$$

$$\implies y' = \cos^2 y$$

$$\implies y' = (\cos y)^2$$

$$\implies y' = \left( \frac{1}{\sqrt{1+x^2}} \right)^2 \quad (\text{Definition of cosine and Pythagorean Theorem from right triangle})$$

$$\implies y' = \frac{1}{1+x^2}$$

