

Math 21A
Kouba
Challenge Discussion Sheet 6

1.) Consider the function $f(x) = x^{3/2} + x^{1/2}$. Show that f satisfies the assumptions of the Mean Value Theorem (MVT) on the interval $[0, 1]$, and determine all values of c guaranteed by the MVT.

2.) Explain why $f(x) = x^3 - x^{1/3}$ does not satisfy all of the assumptions of the MVT on the interval $[-1, 1]$.

3.) Consider the function $f(x) = \begin{cases} x^2, & \text{if } 0 \leq x \leq 3 \\ x^3, & \text{if } -1 \leq x < 0. \end{cases}$

a.) Sketch the graph of f .

b.) Show that f satisfies the conditions of the Mean Value Theorem (MVT) over the interval $[-1, 3]$, including special attention at $x = 0$, and determine all values of c guaranteed by the MVT.

4.) Determine the domain of each function and where each function is increasing, decreasing, concave up, and concave down. Identify all relative and absolute extrema, inflection points, x - and y -intercepts, and asymptotes (vertical, horizontal, or tilted). Sketch the graph.

a.) $f(x) = x(5 - x)^4$

b.) $y = \frac{x}{x^2 + 4}$

c.) $g(x) = \frac{x^2 + 4}{x}$

d.) $f(x) = x^{2/3} + x$ on the interval $[-1, 8]$

e.) $f(x) = \sqrt{3}\sin x + \cos x$ on the interval $[0, 2\pi]$

5.) A hemisphere sits on top of a right circular cylinder of height 10 and radius r . Write the volume V of the hemisphere as a function of the volume C of the cylinder.

