Math 21A
Kouba
Discussion Sheet 3

1.) Evaluate the following limits.

a.) \( \lim_{x \to 4} \frac{\sqrt{5 + x} - 3}{\sqrt{x} - 2} \)

b.) \( \lim_{x \to \infty} (x - x \cos(4/x)) \)

c.) \( \lim_{x \to 0} \frac{2x}{\sin x - x} \) (HINT: \( \frac{\sin x}{x} \leq 1 \).)

2.) Use one-sided limits and limits to infinity to find all vertical and horizontal asymptotes for the following functions. Use the asymptotes together with x- and y-intercepts to sketch graphs of each function.

a.) \( y = \frac{x - 2}{x^2 - 9} \)

b.) \( y = \frac{x + 2}{x^2(x + 1)} \)

3.) Give an \( \varepsilon, \delta \)-proof for \( \lim_{x \to 1} \frac{x + 3}{1 + \sqrt{x}} = 2 \).

4.) Use the IMVT to determine if the following equation is solvable. This is a writing exercise: \( x^3 + x - \sqrt{x + 4} = 0 \).

5.) In the given diagram the smaller circle is the largest one that can be inscribed in the given semi-circle. If the larger circle has circumference \( 4\pi \) in., what is the area of the inscribed shaded square?

6.) Determine the \( x \)-values for which the following function is continuous. It is not necessary to graph the function:

\[
f(x) = \begin{cases} 
\sin 3x & \text{if } x < 0 \\
x & \text{if } x = 0 \\
3.01 & \text{if } 0 < x < 1 \\
x - 1 & \text{if } 0 < x < 1 \\
2 & \text{if } x > 1.
\end{cases}
\]

7.) Use the limit definition of derivative,

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}, \]

to differentiate each of the following functions.

a.) \( f(x) = \cos x \)

b.) \( f(x) = \frac{7 + x}{3x - 5} \)

c.) \( f(x) = \sqrt{x^2 + x} \)
8.) Let \( f(x) = \begin{cases} \sin 2x, & \text{if } x \geq 0 \\ 2x, & \text{if } x < 0 \end{cases} \). Use the limit definition of derivative to determine \( f'(0) \)

The following problem is for recreational purposes only.

9.) Without lifting your pencil, join all sixteen dots with six straight lines.